Real Rigidities and Endogenous Nominal Wage Rigidity

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Abstract

The paper uses a simple model of monopolistic competition in the labor market to show that nominal wage rigidity can be endogenized with modest costs of adjusting wages and without the types of real rigidities introduced in recent New Keynesian models. In contrast, models of nominal price rigidity require strong real rigidities or implausibly high adjustment costs to endogenize nominal price rigidity. The minimum size of the cost of adjusting wages remains modest even if labor supply is inelastic, different labor types are highly substitutable, or there are decreasing returns to labor in production.

Key words: Nominal wage rigidity; Nash equilibrium; New Keynesian models

JEL classification codes: E1, E3

1 Introduction

This paper uses a simple model of monopolistic competition in the labor market to show that nominal wage rigidity can be endogenized with modest fixed costs

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of adjusting wages and without any of the types of real rigidities introduced in recent New Keynesian models. The required size of the adjustment cost for which nominal wage rigidity is an equilibrium remains modest even if labor supply is inelastic, different labor types are highly substitutable, or there are decreasing returns to labor in production.

There is an extensive literature on the role of real rigidities in New Keynesian models, both as a way to endogenize price rigidity and as a source of persistent fluctuations in models with exogenous price rigidities. The literature on the latter is more recent: New Keynesian macroeconomics has put the endogenization of nominal rigidities on hold, even though it is of fundamental importance.[1] Huang and Liu (2002) compared the roles of nominal price rigidity and nominal wage rigidity in generating persistence using a model without real rigidities and with monopolistic competition in both the goods and labor markets. They concluded (p.428) that "in the absence of real rigidity, the nominal wage rigidity (in the form of staggered wage contract) tends to generate larger real effects than does the nominal price rigidity."[2] Ascari (2003) tied together results concerning nominal rigidities and persistence. Starting with a general model of price and wage rigidities, he found just two types of model that generate substantial persistence. One is the "craft union" model with households that sell different labor services (the type of model analyzed by Huang and Liu (2002) and here). The other is the "yeoman farmer" model with price staggering, in which labor is completely immobile across firms.[3] Because of the link between persistence and real rigidities in models with nominal price rigidities, Huang and Liu's results suggest the possibility of endogenous wage rigidity without real rigidities, as we show here.

Table 1 gives a selective summary of the literature on real rigidities, persistence and endogenous nominal rigidity. The current paper fills a gap in the lower left-hand corner of the the table.

The paper is structured as follows. The next section sets up a simple model of wage determination by monopolistically competitive households. The third section addresses the model's calibration, calculates the size of fixed costs necessary to sustain nominal wage rigidity as a Nash equilibrium, and investigates the robustness of the results to changes in the model's parameter values. The fourth

[1] Major exceptions include Danziger (1999) and Dotsey, King and Wolman (1999) who built general equilibrium models in which state-contingent price adjustment is an equilibrium outcome.

[2] Erceg (1997) and Andersen (1998) also showed that nominal wage rigidity can lead to substantial persistence without real rigidities.

[3] Edge (2000) also found that models with firm-specific factor inputs and nominal price rigidity can generate substantial persistence.

Table 1: Real rigidities, persistence and endogenous nominal rigidity

Type of model	Endogenous nominal rigidity?	Persistence?
Price rigidity	**no**	**no**
No real rigidities	Ball and Romer (1990)	Chari et al. (2000)
Price rigidity	**yes**	**yes**
Real rigidities	Ball and Romer (1990)	Alexopoulos (2001)
	Akerlof and Yellen (1985)	Jeanne (1998)
		Basu (1995)
		Kimball (1995)
		Dotsey and King (2002)
		Edge (2000)
		Chari et al. (2000)
Wage rigidity	**yes**	**yes**
No real rigidities	[THIS PAPER]	Huang and Liu (2002)
		Andersen (1998)
		Erceg (1997)
		Ambler et al. (2010)

section analyzes the optimal length of wage contracts as a function of the size of the fixed cost of adjusting the nominal wage in a dynamic extension to the basic model. The fifth section concludes.

2 Basic Model

The model is standard in that it does not include increasing returns to scale, intermediate inputs in production, variable capital utilization, or variable markups in response to aggregate demand shifts. It includes monopolistic competition in the labor market, with households that sell differentiated labor services.

2.1 Production

There is a continuum of monopolistically competitive firms on the unit interval. Goods prices are flexible. Firms have identical production functions, that depend on a composite labor input. Firm i produces its output subject to the following production function:

$$Y_{it} = {N_{it}}^{\alpha} \tag{1}$$

where Y_{it} is the output of firm i, N_{it} is the quantity of composite labor employed by firm i, and $\alpha > 0$ captures returns to scale. Composite labor is an aggregate of different types of labor, each of which is supplied by an individual household. There is a continuum of households on the unit interval, and

$$N_t = \left(\int_0^1 {N(s)_t}^{(\theta-1)/\theta} ds \right)^{\theta/(\theta-1)}, \tag{2}$$

where N_t is the aggregate labor index, $N(s)_t$ is the quantity of labor supplied by household s, and $\theta > 1$ is the elasticity of substitution of different labor types in production. Aggregate employment is just the sum of employment at the different firms in the economy:

$$N_t \equiv \int_0^1 N_{it} di. \tag{3}$$

Profit maximization leads to the following conditional demand function for labor of type s:

$$N(s)_t = \left(\frac{W(s)_t}{W_t} \right)^{-\theta} N_t, \tag{4}$$

where $W(s)_t$ is the nominal wage set by the household s and where W_t is the average wage index defined by:

$$W_t \equiv \left(\int_0^1 {W(s)_t}^{(1-\theta)} ds \right)^{1/(1-\theta)}.$$

2.2 Households

Each household s is a monopoly supplier of a particular type of labor. It maximizes the following utility function:

$$U(s)_t = \frac{1}{1-\gamma_1} {C(s)_t}^{(1-\gamma_1)} - \frac{\phi}{1+\gamma_2} {N(s)_t}^{(1+\gamma_2)}, \tag{5}$$

where γ_1 and γ_2 are positive parameters. The aggregate consumption good is a bundle of the goods produced by different firms and is given by

$$C_t = \left(\int_0^1 {Y_{it}}^{(\eta-1)/\eta} di \right)^{\eta/(\eta-1)} = \int_0^1 C(s)_t ds. \tag{6}$$

The exact consumption price index is

$$P_t \equiv \left(\int_0^1 P_{it}{}^{(1-\eta)} di \right)^{1/(1-\eta)}, \tag{7}$$

and the conditional demand for the output of firm i is given by

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t. \tag{8}$$

Households derive income from wages, from dividend payments by firms, and from government transfers. The household's period budget constraint is

$$\frac{W(s)_t}{P_t} N(s)_t + \pi_t + \frac{M(s)_{t-1}}{P_t} + T(s)_t = C(s)_t + \frac{M(s)_t}{P_t}, \tag{9}$$

where $M(s)_t$ denotes the household's holdings of nominal money balances, π_t denotes dividend payments from firms (assumed to be equal across households), and $T(s)_t$ denotes the real value of transfers from the government. The household's maximization problem leads to the following rule for setting its nominal wage:

$$-\frac{\partial U}{\partial N(s)_t} = \frac{W(s)_t}{P_t} \frac{\theta - 1}{\theta} \frac{\partial U}{\partial C(s)_t}. \tag{10}$$

2.3 Symmetrical Flexible-Wage Equilibrium

The model is closed with the following cash-in-advance constraint:

$$C_t = \frac{M_t}{P_t} \tag{11}$$

where M_t is the aggregate money stock. All households in the model are identical, so all will choose the same nominal wage and consumption level. Imposing a symmetrical equilibrium and aggregating, the model can be solved for the equilibrium level of output which is given by

$$Y_t = \left(\frac{\alpha}{\phi} \frac{\theta - 1}{\theta} \frac{\eta - 1}{\eta} \right)^{\alpha/(\alpha\gamma_1 + \gamma_2 + (1-\alpha))}. \tag{12}$$

The optimal level of output for a social planner who maximizes the utility of the representative household is given by:

$$Y_t^* = \left(\frac{\alpha}{\phi} \right)^{\alpha/(\alpha\gamma_1 + \gamma_2 + (1-\alpha))}. \tag{13}$$

Equilibrium output is lower than the first-best optimum because of households' and firms' monopoly power, and converges to the first best as θ and η tend to infinity.

3 Endogenous Fixed Nominal Wages

This section performs a thought experiment similar to the one used by Ball and Romer (1990) to analyze price adjustment by firms. The economy is initially in an equilibrium compatible with full wage and price flexibility. Then, the money supply decreases unexpectedly. All households but one maintain a constant nominal wage. For the remaining household, it is possible to calculate the level of utility it can attain if it maintains a constant nominal wage and its utility if it adjusts its wage optimally in reaction to the shock, ignoring any fixed costs of doing so. The utility loss from not adjusting its wage is expressed as a compensating variation. The size of the compensating variation measures the size of the fixed adjustment cost that would leave the household indifferent between adjusting its nominal wage and leaving it constant, the minimum size of adjustment cost necessary for wage rigidity to be an equilibrium outcome.[4]

Table 2 gives parameter values for the base-case scenario. The values of γ_1 and γ_2 are standard in the literature. In particular, the value chosen for γ_2 gives a Frisch elasticity of labor supply with respect to the real wage of 0.5. King and Rebelo (2000) note that the standard real business cycle model with log utility implies a Frisch elasticity of labor supply of 4.0. They also note that microeconomic estimates surveyed by Pencavel (1986) give an upper bound for men's labor supply elasticity of one. We show below that the results are insensitive to labor supply elasticity. The ϕ parameter normalizes the level of employment and output but does not otherwise affect the results. The value of θ gives a markup of 25% over marginal cost. The value of η is close to the upper end of the range of values estimated by Griffin (1992, 1996). With these parameter values, the flexible-wage equilibrium gives a level of output, employment, and consumption of 0.894, less than the first-best level of output with flexible wages, which is 1.0. With $\alpha = 1$, the production function has constant returns to scale: as shown below, the results are even stronger with diminishing returns to labor.

In response to a 3% drop in the money supply, if all households but one maintain a constant nominal wage, output and employment fall to 97% of their levels

[4]This calculation ignores benefits in future periods from wage adjustment. This simplifying assumption is dropped in section 4 below.

Table 2: Calibration

Parameter	Value
γ_1	2.0
γ_2	2.0
θ	5.0
η	5.0
ϕ	1.0
α	1.0

in the flexible wage/price equilibrium. For the base-case calibration, the compensating variation amounts to 0.13% of the initial level of consumption. Using a similar model, Romer (2005) showed that a firm's loss from not adjusting its output price in response to a three percent drop in aggregate demand was equal to 25% of its revenue. Ball and Romer (1990) (in the version of their model with a competitive labor market) found the firm's loss to be 38% of revenue given a five percent decrease in aggregate demand. The main implication of the thought experiment is clear: the size of the adjustment cost necessary to endogenize nominal wage rigidity is an order of magnitude smaller than the adjustment cost needed endogenize nominal price rigidity.

For values of the adjustment cost greater than the size of the compensating variation, nominal price rigidity will be a Nash equilibrium. It is possibly not the only Nash equilibrium. An appendix shows that if the adjustment cost is not too much greater than the size of the compensating variation for a particular shock, there will be two symmetrical Nash equilibria with pure strategies. One is the equilibrium in which all households maintain a fixed nominal wage. The other is an equilibrium in which all households adjust completely to the demand shock so that money is neutral in the short run. The question of equilibrium selection is beyond the scope of this paper.

3.1 Interpretation

Although the model contains none of the features from the literature that lead to real rigidities, nominal wage rigidity gives rise to a real rigidity in the following sense. Labor is the only factor of production, and there are constant returns to scale. Since firms are assumed to be price takers in the labor market, each firm's marginal cost curve is flat. In response to variations in aggregate demand, if nominal wages are fixed then nominal prices remain constant as well given the constant markup over marginal cost. The real wage is completely rigid in response to changes in output. In contrast, Romer's (2005) model has monopolistic competition only in the goods market. In order for aggregate output to change, employment must change, and furthermore employment must lie on the aggregate labor supply curve. If labor supply is relatively inelastic, the nominal wage is highly sensitive to variations in output and employment.

Figure 1 reinterprets the discussion in Romer (2005). It illustrates the incentives of an individual firm to reduce its price in the face of a negative aggregate demand shock, given that other firms do not adjust their prices. For a given nominal wage, the firm's marginal cost curve is completely flat, since labor is the only factor of production and there are constant returns to scale. If aggregate employment falls, this entails a downward movement along the aggregate labor supply curve. From the individual firm's point of view, there is an exogenous decrease in the nominal wage at which it can hire workers. Its marginal cost curve shifts down, and its incentive to adjust its price is captured by the shaded triangular area. In Romer's numerical example, the loss is very large.

The situation is very different for a household considering whether or not to adjust its nominal wage. Figure 2 gives a graphical representation of this case. The first order condition for the household's choice of $W(s)$ can be rewritten as follows:

$$W(s)_t \frac{\theta - 1}{\theta} = -P_t \frac{\left(\frac{\partial U}{\partial N(s)_t}\right)}{\left(\frac{\partial U}{\partial C(s)_t}\right)}$$

The left hand side gives the household's nominal marginal revenue from decreasing its nominal wage by a small amount. The right hand side gives the monetary equivalent of the marginal cost of the household's foregone leisure. Marginal cost is an increasing function of $N(s)_t$ because of the increasing disutility of work. The aggregate price level acts as a shift variable for the household's marginal cost curve, just as the aggregate nominal wage rate acts as a shift variable for the firm's marginal cost curve in Figure 1. With constant returns to scale in production, there is endogenous price rigidity. If other households maintain a constant nominal wage, firms maintain constant prices and there is no shift in the marginal cost curve. The incentive for an individual household to adjust its wage in response to a drop in aggregate demand is given by the shaded triangular area in Figure 2. The numerical calculations of the previous section show the size of this triangle to be quite small.

The model also implies that, given equilibrium nominal wage rigidity, it would be possible also to support nominal price rigidity, in line with Blanchard and Kiyotaki's (1987) results concerning an equilibrium with simultaneous price and wage rigidities. Nominal wage rigidity is sufficient to lead to nominal price rigidity. In addition, some form of real rigidity (of which nominal wage rigidity can be considered an example) is necessary for nominal price rigidity. On the other hand, nominal price rigidity is not necessary for nominal wage rigidity to be an equilibrium. In this sense, nominal wage rigidity provides a more compelling underpinning to New Keynesian models of the business cycle.

3.2 Sensitivity Analysis

This subsection analyzes the effects of changes in the values of some of the model's key structural parameters on the size of the loss to a single household if it does not adjust its nominal wage. The thought experiment is the same as in the previous subsection: starting from an equilibrium compatible with full wage and price flexibility, the money supply decreases by three percent, and all other households maintain a constant nominal wage.

Table 3 shows how the size of the compensating variation changes in response to changes in the value of γ_2, which measures the inverse of the elasticity of labor supply. The table shows that the costs of not adjusting the nominal wage increase as labor supply elasticity decreases. In this respect, the results are qualitatively similar to those of Ball and Romer (1990). If we interpret a high labor supply elasticity to mean high real wage rigidity, then more real wage rigidity means that smaller costs of adjusting nominal wages can support nominal wage rigidity as an equilibrium outcome. However, the results are quantitatively very different. Even with a very low labor supply elasticity ($\gamma_2 = 10$ corresponds to a labor supply elasticity of 0.1), the costs of not adjusting the real wage amount to only 0.33% of initial consumption.

Table 4 shows how the compensating variation changes with the value of θ, the elasticity of substitution between different types of labor. The table shows that the household's costs are not very sensitive to the degree of substitutability between labor types in the model.

Table 5 shows how the compensating variation changes with the value of α, the returns to scale parameter in the production function. With $\alpha < 1$, firms' real marginal costs are increasing with their output. Their optimal response to a decrease in aggregate demand if the average nominal wage does not change is to

Table 3: Sensitivity to γ_2

Value of γ_2	Value of CV in %
0.25	0.08%
0.50	0.09%
1.00	0.10%
2.00	0.13%
10.00	0.33%

Table 4: Sensitivity to θ

Value of θ	Value of CV in %
1.25	0.04%
2.00	0.08%
5.00	0.13%
10.00	0.14%
20.00	0.15%
40.00	0.15%

lower their price. The aggregate supply curve becomes upward-sloping. A three percent drop in aggregate demand leads to a three percent decrease in nominal output, split between a drop in real output and a drop in the price level depending on the slope of the aggregate supply curve. With the functional forms used in our model, employment drops by three percent in response to a three percent decrease in the money supply. This is the same as with constant returns to scale, while consumption drops less because output drops less. The increase in consumption required to restore period utility to its level before the shock is lower for lower values of α. Period utility with wage adjustment is at an intermediate level between period utility without adjustment and period utility without the shock. The increase in consumption required to give the household this intermediate level of utility is also lower for lower values of α.

Table 5: Sensitivity to α

Value of α	Value of CV in %
1.00	0.13%
0.90	0.12%
0.80	0.10%
0.70	0.09%
0.60	0.07%
0.50	0.06%

4 Optimal Contract Length

The results derived so far have been static, having to do with the cost of not adjusting the nominal wage in the same period that a shock to aggregate demand occurs. In this section, we consider a related question in a dynamic context.

The static analysis of the basic model understates the benefit to households of adjusting wages to the extent that aggregate demand shocks are persistent. Here, we modify the model so that each household fixes its nominal wage and faces a fixed probability $(1-d)$ at the beginning of each subsequent period of being allowed to adjust its nominal wage, following Calvo (1983). It is then possible to calculate the socially optimal average contract length that maximizes expected utility for a representative household given the stochastic process generating the money supply and given the size of the fixed cost of adjusting the nominal wage. We show that with small adjustment costs this gives wage contracts that are at least as long on average as in the U.S. data.

Household s maximizes an intertemporal utility function given by

$$U = E_t \sum_{i=0}^{\infty} \beta^i U\left(C(s)_{t+i}, N(s)_{t+i}\right),$$

with period utility as given in equation (5). Its period budget constraint is given by

$$a(s)_t + C(s)_t + \frac{M(s)_t}{P_t} + \frac{B(s)_t}{R_t P_t}$$
$$= \frac{M(s)_{t-1}}{P_t} + \frac{B(s)_{t-1}}{P_t} + \pi_t + \left(\frac{W(s)_t}{W_t}\right)^{-\theta} N_t^d W(s)_t,$$

where $W(s)_t$ is the nominal wage set optimally by the household if it is allowed to do so or its nominal wage from the previous period if it cannot reset, and $a(s)_t$ is equal to the fixed adjustment cost a if it is allowed to reset its wage and zero otherwise. R_t is the discount on the purchase of a noncontingent nominal payment of $B(s)_t$ at the beginning of the following period and has the interpretation of the gross nominal interest rate.[5] Its purchases of consumption goods are subject to the cash-in-advance constraint given by

$$M(s)_{t-1} = C(s)_t P_t.$$

The household's first order conditions with respect to its choice of consumption, nominal balances and noncontingent bonds $B(s)_t$ are given by:

$$U_c\left(C(s)_t, N(s)_t\right) - \lambda(s)_t - \psi(s)_t = 0; \tag{14}$$

$$-\frac{\lambda(s)_t}{P_t R_t} + \beta E_t \frac{\lambda(s)_{t+1}}{P_{t+1}} = 0; \tag{15}$$

$$-\frac{\lambda(s)_t}{P_t} + \beta E_t \left(\frac{\lambda(s)_{t+1}}{P_{t+1}} + \frac{\psi(s)_{t+1}}{P_{t+1}}\right) = 0, \tag{16}$$

where $\lambda(s)_t$ is the multiplier associated with its budget constraint and $\psi(s)_t$ is the multiplier associated with its cash-in-advance constraint. If it is allowed to reset its wage, the first order condition for this choice is given by

$$\begin{aligned} E_t \sum_{i=0}^{\infty} (\beta d)^i (-\theta) U_h\left(C(s)_{t+i}, N(s)_{t+i}\right) \left(\frac{W(s)_t}{W_{t+i}}\right)^{-\theta} \frac{N_{t+i}}{W(s)_t} \\ + E_t \sum_{i=0}^{\infty} (\beta d)^i (1-\theta) \lambda(s)_{t+i} \left(\frac{W(s)_t}{W_{t+i}}\right)^{-\theta} \frac{N_{t+i}}{P_{t+i}} = 0, \end{aligned} \tag{17}$$

where N_t is total labor demand by firms at time t.

We use the *Dynare*[6] to calculate second-order approximations to the equilibrium conditions of the system around its deterministic steady state. We are interested in evaluating welfare, and it is now well known that in models where the nonstochastic steady state is not a Pareto optimum first-order approximations

[5]Households have access to complete contingent asset markets so that (because of the separability of the utility function) their marginal utilities of consumption are equalized. We do not include other assets in the budget constraint in order to simplify the notation.

[6]See Juillard (2005).

can give misleading welfare comparisons.[7] Using a second-order approximation allows aggregate demand shocks to affect not only the dispersion of the model's endogenous variables but also their stochastic means.

Because of the higher-order approximation, it is necessary to distinguish between total hours worked given by

$$N_t^o = \int_0^1 N(s)_t ds,$$

and total demand for labor (equal to aggregate employment) which is given by equation (2) above. Because there is wage dispersion that results from shocks, labor allocation is inefficient, and aggregate employment is less than or equal to the total number of hours worked.

The money supply follows a first-order autoregressive process given by:

$$\ln(M_t) = \omega \ln(M_{t-1}) + \varepsilon_t$$

A complete list of the equations that characterize the equilibrium of this economy are in an Appendix. For the simulations, the following parameter values were used. We set $\omega = 0.9$ and the standard deviation for the money supply shocks $\sigma_\varepsilon = 0.01$. The fixed cost of adjusting the nominal wage was set equal to 0.10% of the steady-state level of per-period consumption under flexible wages. The subjective discount rate β was set equal to 0.99.[8] We simulated the model for different values of the probability that a household is allowed to revise its nominal wage. The sample period was set equal to 10,000.

We then calculated the unconditional expected utility for a representative household that works at and owns shares in all firms, so that it pays just the per capita amount of adjustment costs, equal to the fraction of households adjusting their wage times the adjustment cost.

The results, presented in Table 6, are quite intuitive. Unconditional expected utility follows an inverse "U" shape as a function of the probability that a given wage contract remains in force at the beginning of each period. Average contract length is equal to $1/(1-d)$. For small values of d (a very short average contract length), an increase in d improves unconditional expected utility because the savings in aggregate adjustment costs swamp the loss due to the increased volatility

[7] See Kim and Kim (2003) for a discussion.

[8] β is the only parameter of the model that depends on the length in calendar time of one period. We take one period to be a quarter. The interpretation of the size of the fixed cost of adjusting the nominal wage as a fraction of annual output also depends on the length of the period.

of consumption, which arises from the magnified impact of monetary shocks on output as contracts increase in length. As average contract length increases, the marginal adjustment cost savings from further increases in contract length fall, while the marginal costs due to increased consumption volatility increase. With $d = 0.8$, the two effects offset each other, and unconditional expected utility is maximized. Given the opportunity to choose d once and for all, a social planner would choose $d = 0.80$, or an average contract length of five quarters. The consensus view (see Taylor, 1999) is that average contract length in the U.S. is equal to about four quarters. With 20% of households adjusting their wage each period, total per capita adjustment costs would amount to 0.02% of GDP. Given this, interpreting the observed average length of wage contracts as an equilibrium outcome in the face of fixed renegotiation costs is well within the realm of plausibility.

Table 6: Unconditional Expected Utility as a Function of d

Value of d	Expected Utility
0.1	-1.3575
0.2	-1.3574
0.3	-1.3573
0.4	-1.3573
0.5	-1.3572
0.6	-1.3571
0.7	-1.3571
0.8	-1.3571*
0.9	-1.3574

*: maximum value as a function of d

It is quite possible that the equilibrium contract length in a decentralized economy is greater than the social optimum due to the presence of externalities, as argued by Ball (1987). A decision by one household to increase its contract length implies that the average wage level responds more slowly to aggregate demand shocks, and hence the price level responds more slowly as well. This makes it easier for other households to forecast future prices when setting their nominal wage, which is a positive externality. On the other hand, the real money supply is more variable, which implies a greater variability of aggregate demand and therefore the demand for other firms' products. The second effect is a negative externality. Ball (1987) shows that if the net externality is negative, the contract length chosen in a symmetric Nash equilibrium will be greater than the social optimum.[9]

The dynamic model here has no important intertemporal links other than wage rigidity. Chari, Kehoe and McGrattan (2000) showed that introducing capital accumulation and interest-elastic money demand reduces the ability of models with sticky prices to generate persistence. In contrast, Huang and Liu (2002) showed that this is not the case with nominal wage rigidity. Models with nominal wage rigidity can explain persistence with or without these intertemporal links. This conclusion is ratified by Ambler, Guay and Phaneuf (2010) and by Christiano, Eichenbaum and Evans (2005) in closed-economy models and by Ambler and Hakizimana (2003) in an open economy model. These papers all showed that wage rigidities generate substantial persistence in fully dynamic models with capital and with interest-elastic money demand. We conjecture that nominal wage rigidities would be an equilibrium outcome in a dynamic model with these features.[10]

5 Conclusions

Without important real rigidities, price adjustment costs have to be huge to support endegenous nominal price rigidity. We have shown that it is much more plausible to justify endogenous nominal wage rigidity in the presence of small macroeconomic frictions. This result is insensitive to changes in labor supply elasticity, the degree of substitutability of different labor types, and to decreasing returns to labor in production. In a dynamic extension to the model, with a persistent monetary shocks an adjustment cost of 0.1% of per-period consumption, a contract length of five quarters maximizes the unconditional expected utility of the representative household and entails aggregate adjustment costs that amount to only 0.02% of GDP.

The model provides microfoundations for New Keynesian models of nominal wage rigidity, and therefore justifies including Keynesian features in dynamic general equilibrium models of the business cycle. In some respects, however, the analysis in this paper has anti-Keynesian implications. Since the welfare losses

[9] In his model, firms rather than households choose their optimal contract length, and wages are determined by an expected market clearing rule.

[10] A fully endogenous wage-setting model would also endogenize the decision to stagger. See Fethke and Policano (1986) for a model that does this.

from longer nominal wage rigidities are quite modest (even before factoring in the welfare gains from reduced transactions costs), this limits the size of welfare gains from stabilization policy. Developing models with rigorous foundations for nominal rigidities and a quantitatively important role for stabilization policy remains a challenge.[11]

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Appendix A: Symmetrical Nash Equilibria

We have shown that the outcome where all households choose not to adjust their nominal wage in response to an aggregate demand shock is a Nash equilibrium, as long as the fixed cost of adjustment exceeds the loss from not doing so. It is possibly not the only Nash equilibrium. If we consider only symmetrical equilibria in which all households adjust their nominal wages by the same amount, and if we exclude mixed strategies, the possibilities are illustrated by Figure 3. The vertical axis measures the change in the nominal wage of household s. The horizontal axis measures the change in the average nominal wage assuming that all other households adjust by the same amount. The straight line labelled BRF is the best response function of household s in response to a positive aggregate demand shock, when there is a positive shock to the money supply and in the absence of adjustment costs.

It can easily be shown that the slope of the individual household's best response function is less than one, that the vertical shift in the function depends directly on the size of the change in the money stock, and that it intersects the 45-degree line where the proportional change in the average wage is equal to the proportional change in the money stock. This means that, in the absence of fixed costs of adjusting the nominal wage, the only symmetrical Nash equilibrium is the one in which all households adjust fully to the change in the money stock, and money is neutral.

It is also possible to show that the cost to household s of not adjusting its wage increases with the size of the wage adjustment of other households. Let $\underline{c}$ be the cost of not adjusting when $\Delta W = 0$, and let $\overline{c}$ be the size of not adjusting when $\frac{\Delta W}{W} = \frac{\Delta M}{M}$. We know that $\underline{c} < \overline{c}$. Denote the real fixed cost of adjusting the nominal wage by a. We can analyze the best response function of the household that takes the fixed cost into account.

We have three possible scenarios for the size of the adjustment cost. If $a < \underline{c}$, then there is a point such as A in Figure 3 to the left of the origin such that the household's best response function coincides with the horizontal axis to the left of point A and then jumps up to the BRF line at point A. The only symmetrical Nash equilibrium is the one in which all households adjust fully to the change in the money supply and money is neutral. If $\underline{c} < a < \overline{c}$, then there is a point such as B in Figure 3 between the origin and the point where $\frac{\Delta W}{W} = \frac{\Delta M}{M}$ such that the household's best response function coincides with the horizontal axis to the left of point B and the jumps up to the BRF line at point B. In this case, there are two symmetrical Nash equilibria. Either all households adjust fully and money is neutral, or all households maintain a constant nominal wage. Finally, if $\overline{c} < a$, then there is a point such as C in Figure 3 to the right of the point where $\frac{\Delta W}{W} = \frac{\Delta M}{M}$ such that the household's best response function coincides with the horizontal axis to the left of C and jumps up to BRF at C. In this case, the only symmetrical Nash equilibrium is the one where all households maintain a constant nominal wage.

It is clear that the number of symmetrical Nash equilibria will depend on the relative sizes of the monetary shock and the fixed adjustment cost a. For a given size of a, small shocks will lead to no adjustment as the only Nash equilibrium, larger shocks will lead to both nonadjustment and adjustment as possible equilibria, and still larger shocks will lead to adjustment as the only possible Nash equilibrium. The model therefore contains a fundamental nonlinearity in the the response of wages, prices and output to monetary shocks of different sizes. The question of equilibrium selection in the case of multiple Nash equilibria is an interesting one that we do not address here.

Appendix B: Dynamic Model

The following system of equations characterizes equilibrium in the dynamic model:

$$\lambda_t = R_t \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}};$$

$$\lambda_t = \beta E_t \frac{P_t}{P_{t+1}} \left(\lambda_{t+1} + \psi_{t+1}\right);$$

$$C_t{}^{-\gamma} = \lambda_t + \psi_t;$$

$$N_t^o = S_t N_t;$$

$$S_t = (1-d) \left(\frac{\tilde{W}_t}{W_t}\right)^{-\theta} + d \left(\frac{W_{t-1}}{W_t}\right)^{-\theta} S_{t-1};$$

$$W_t{}^{(1-\theta)} = (1-d) \tilde{W}_t^{(1-\theta)} + d W_{t-1}{}^{(1-\theta)};$$

$$f_t^1 = \beta d E_t f_{t+1}^1 \left(\frac{\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{1+\theta}$$

$$-\phi \frac{\theta}{\theta - 1} S_t{}^{\gamma_2} \left(C_t + (1-d)a\right)^{(1+\gamma_2)/\alpha} \left(\frac{W_t}{\tilde{W}_t}\right)^{\theta} \tilde{W}_t^{-1}$$

$$f_t^2 = \beta d E_t f_{t+1}^2 \left(\frac{\tilde{W}_{t+1}}{\tilde{W}_t}\right)^{\theta} + \left(C_t + (1-d)a\right)^{1/\alpha} \left(\frac{W_t}{\tilde{W}_t}\right)^{\theta} \lambda_t \frac{C_t}{M_t}$$

$$f_t^1 + f_t^2 = 0;$$

$$M_t = P_t C_t;$$

$$N_t{}^{\alpha} = C_t + (1-d)a$$

$$P_t = \frac{1}{\alpha} \frac{\eta}{(\eta - 1)} W_t N_t{}^{(1-\alpha)}$$

$$\ln(M_t) = \omega \ln(M_{t-1}) + \varepsilon_t.$$

Here, $\tilde{W}_t$ is the nominal wage set by those households that are allowed to adjust their wage at time t. The equations in the artificial variables f_t^1 and f_t^2 come from a recursive representation of the first order condition with respect to the choice of the nominal wage. The two infinite sums in equation (17) can be reduced to two nonlinear first order difference equations in these two variables plus an equation that relates the two. The artificial variable S_t measures the wedge between aggregate labor supply and total hours that arises due to wage dispersion across groups of workers. See Schmitt-Grohe and Uribe (2005) for details.

The model's deterministic steady state equilibrium is characterized by the following set of equations.

$$\frac{W}{P}\frac{\eta}{\eta-1}Y^{\frac{1-\alpha}{\alpha}};$$

$$Y = N^{\alpha};$$

$$Y = C + (1-d)a;$$

$$\frac{W}{P}\frac{\theta-1}{\theta}\beta C^{-\gamma_1} = \phi N^{\gamma_2}.$$

The last equation is a straightforward consequence of the household's first order condition for the choice of its nominal wage. If we assume that adjustment costs are a fraction k of steady state consumption, $a = k \cdot C$, we have

$$Y = (1 + k \cdot (1-d))C \equiv KC,$$

where $K > 1$ We then have the following solution for the steady-state level of output:

$$Y = \left(\frac{\alpha}{\phi}\frac{\theta-1}{\theta}\frac{\eta-1}{\eta}K^{\gamma_1}\right)^{\alpha/(\alpha\gamma_1+\gamma_2+(1-\alpha))}.$$

Employment and output are higher when the adjustment cost is explicitly taken into account because of a negative wealth effect on labor supply.