Terms of Trade Shocks, Monetary Instability and Exchange Rate Regime Choice

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Abstract

This paper reexamines the case for fixed exchange rates using a microfounded model of a small open economy with nominal wage rigidities and subject to both terms of trade shocks and money demand shocks. If monetary instability is sufficiently important compared to real shocks, a fixed rate regime improves welfare over a flexible rate regime with an exogenous money supply. The relative importance of the two types of shocks can be calibrated to reproduce the observed increase in real exchange rate variability in industrialized countries under flexible rates.

Key words: exchange rate regime, terms of trade shocks, monetary instability.

JEL classification codes: F2, F31, F33

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1 Introduction

Several recent papers in the New Open Economy Macroeconomics (henceforth NOEM) tradition[1] have addressed the question of the choice of exchange rate regime.[2] They conclude that at least a limited degree of exchange rate flexibility is desirable. For example, Macklem et al. (2000) show that the macroeconomic benefits conferred by flexible exchange rates, which allow economies to adjust to asymmetrical real shocks and which offset the effects of nominal rigidities, far outweigh the microeconomic gains from reduced transactions costs provided by fixed rates. The NOEM literature generally reconfirms Mundell's (1961) celebrated result that flexible exchange rates help economies adjust to asymmetrical real shocks.

Mundell's (1961) paper can be contrasted with his later writings[3] and with his reputation as the intellectual godfather of the euro. McKinnon (2001) reviews Mundell's contributions in the light of the latter's Nobel prize. He concludes that the Mundell of the 1960s, who emphasized asymmetrical real shocks and optimal currency areas, is quite different from the less well-known Mundell of the 1970s (1973a, 1973b), who stressed the importance of expectations and monetary instability.

Here, we formalize the monetary instability argument for fixed exchange rates in the NOEM framework. We analyze a simple model of a small open economy, based on Macklem et al. (2000) and Ambler (2000), subject to shocks to its terms of trade and to money demand. Without monetary instability, flexible exchange rates are clearly welfare-improving since they reduce the deviations of aggregate employment from its Walrasian flexible-wage equilibrium. With monetary instability and an exogenous money supply, money demand shocks lead to unanticipated fluctuations in both aggregate employment and in the marginal utility of consumption, while a fixed exchange rate regime insulates the economy from the effects of these shocks.

This implies that the choice between fixed and floating regimes hinges on the relative importance of the two types of shocks. This can be pinned down in a version of the model with an imperfectly elastic demand for the economy's exports on world markets, making the real exchange rate endogenous. The model is calibrated to replicate a key feature of the data, namely the greater volatility of real exchange rates in industrialized countries under flexible exchange rate regimes.

[1] See Lane (2001) and Sarno (2001) for surveys.

[2] See for example Devereux and Engel (2003), Devereux (2000) and Macklem et al. (2000).

[3] See Mundell (2000).

The identifying assumption is that this greater volatility is due to a combination of nominal rigidities and monetary instability. Because the effects of monetary instability are completely neutralized under fixed exchange rates, the increase in real exchange rate variability under flexible rates is entirely due to money demand shocks. The results show that for plausible parameter values, welfare losses due to monetary instability under flexible exchange rates outweigh the gains that a flexible exchange rate provides by offsetting the effects of nominal rigidities in the face of real shocks.

The paper is structured as follows. The next section presents the model. The third section discusses the model's solution and calibration. The fourth section presents results for the version of the model with exogenous terms of trade shocks. The fifth section discusses the version of the model with an endogeous real exchange rate. The seventh section concludes.

2 Model

The small open economy that produces two tradable goods, an exportable and an importable. The terms of trade between the two goods are exogenous. (This assumption will be relaxed later on.) Households rent differentiated labor services to firms and derive utility from the consumption of exportables and importables, from leisure, and from real balances.

The model is similar to models developed by Devereux (2000) and by Macklem et al. (2000), but it has a different production structure. There is an importables sector, whose output price is determined on world markets. There is also an exportables sector. We present results first for a version of the model where the world price of exportables is exogenous (small open economy) as are the economy's terms of trade. Later, a downward-sloping demand curve for its exports (semi-small open economy) is introduced, which endogenizes the terms of trade. Nominal rigidities are introduced via nominal wage contracts as in Macklem et al. (2000) rather than through staggered price setting by imperfectly competitive firms as in Devereux (2000).

The choice of nominal wage rigidities instead of nominal price rigidities is motivated by the following considerations.

- We know from closed economy macroeconomic models that nominal wage rigidities are an important propagation mechanism that allows dynamic business cycle models to explain the dynamic effects of demand and supply shocks. See for example Ambler, Guay and Phaneuf (2010) and Huang and Liu (2002).

- Nominal wage rigidity allows for the explanation of some important stylized facts in international finance, such as the volatility and persistence of real exchange rate fluctuations. See Ambler and Hakizimana (2004) and Kollmann (2001).

- Obstfeld and Rogoff (2000) argue that the empirical evidence on comovements between nominal exchange rate and relative prices is incompatible with models that have nominal price rigidities and pricing to market, while evidence on exchange rate pass-through is incompatible with models that have nominal price rigidities and export-currency pricing.[4]

2.1 Households

Households supply labor to firms. There is a continuum of households on the unit interval, indexed by s. A household of type s maximizes the following intertemporal utility function:

$$U(s) = E_t \sum_{i=0}^{\infty} \beta^i u\left(C(s)_{t+i}, H(s)_{t+i}, \frac{M(s)_t}{P_t}\right), \tag{1}$$

where $C(s)_t$ is consumption and $H(s)_t$ denotes hours worked, $M(s)_t$ denotes nominal balances and P_t is the price level. Period utility of household s is given by:

$$u\left(C(s)_t, H(s)_t, \frac{M(s)_t}{P_t}\right) =$$
$$\frac{\gamma_1}{1-\gamma_1} \ln\left(C(s)_t{}^{\frac{(1-\gamma_1)}{\gamma_1}} + b_t{}^{\frac{1}{\gamma_1}}\left(\frac{M(s)_t}{P_t}\right)^{\frac{\gamma_1-1}{\gamma_1}}\right) - \frac{\sigma}{1+\gamma_2} H(s)_t{}^{(1+\gamma_2)}. \tag{2}$$

This functional form leads to a simple money demand equation where b_t has the interpretation of a money demand shock. The shock follows an exogenous AR(1) process in logs given by:

$$\ln(b_t) = \rho_b \ln(b_{t-1}) + \varepsilon_{bt}, \quad 0 < \rho_b < 1. \tag{3}$$

[4]They go on to develop a model with nominal wage rigidities and traded and nontraded goods. The model developed in this paper does not include a nontraded goods sector, since we focus on different issues. It could however easily be extended to include nontraded goods.

Households derive income from wages, from dividend payments by firms, from interest payments on their holdings of noncontingent bonds, and from government transfers. The household's period budget constraint can be written in nominal terms as follows:

$$W(s)_t H(s)_t + T(s)_t + P_t\psi_t + B(s)_{t-1} + S_t B(s)^*_{t-1} + M(s)_{t-1} =$$
$$P_t C(s)_t + \frac{B(s)_t}{(1+r_t)} + \frac{S_t B(s)^*_t}{\kappa_t(1+r^*_t)} + M(s)_t, \quad (4)$$

where $W(s)_t$ is the nominal wage rate paid to the household, $T(s)_t$ represents transfers from the government, ψ_t gives real dividend payments from firms, $B(s)_{t-1}$ is the household's holdings of domestic bonds at the beginning of period t, r_t is the real interest rate on one-period bonds, κ_t is a risk premium to be defined below, $B(s)^*_{t-1}$ is the household's holdings of foreign bonds, r^*_t is the real interest rate on those bonds, S_t is the nominal exchange rate measured in units of domestic currency per unit of foreign currency, $M(s)_{t-1}$ gives the household's holdings of money balances at the beginning of period t, and P_t is the price index for final demand, defined below. Dividend payments are assumed to be the same for all households, so ψ_t just gives aggregate profits. The government does not issue domestic bonds and they are not traded internationally, so that:

$$\int_0^1 B(s)_t ds = 0.$$

Aggregate consumption is a composite good derived from exportables and importables and is given by:

$${X_t}^{\phi}{I_t}^{(1-\phi)} = C_t = \int_0^1 C(s)_t ds, \quad (5)$$

where X_t is domestic absorption of the exportable good, I_t is domestic absorption of the importable good, and C_t is aggregate consumption. The cost-minimizing price index for the composite good is given by:

$$P_t = \left(\frac{P_{Xt}}{\phi}\right)^{\phi}\left(\frac{P_{It}}{(1-\phi)}\right)^{(1-\phi)}.$$

In order to minimize the cost of purchasing a given consumption bundle, the individual spends a fixed fraction ϕ of her total consumption expenditure on the exportable good and a fraction $(1-\phi)$ of her total consumption expenditure on the importable good:

$$P_{Xt}X(s)_t = \phi P_t C(s)_t,$$
$$P_{It}I(s)_t = (1-\phi)P_t C(s)_t.$$

2.2 Wage Setting

Aggregate labor services are given by

$$H(s)_t = \left(\int_0^1 n_t^{\frac{\theta-1}{\theta}} ds \right)^{\frac{\theta}{\theta-1}}, \tag{6}$$

so that θ gives the elasticity of substitution across different types of labor. This leads to the following conditional demand for labor of type s:

$$H(s)_t = \left(\frac{W(s)_t}{W_t} \right)^{-\theta} N_t, \tag{7}$$

where W_t is an exact wage index given by:

$$W_t = \left(\int_0^1 W(s)_t^{1-\theta} ds \right)^{\frac{1}{1-\theta}}$$

Note that aggregate hours worked in this economy are not equal to the index of aggregate labor services. Total hours worked are just the integral over all s of hours worked by each type of household. We have:

$$H_t \equiv \int_0^1 H(s)_t ds = \int_0^1 \left(\frac{W(s)_t}{W_t} \right)^{-\theta} N_t ds \equiv \Omega_t N_t.$$

The variable Ω_t is a measure of wage dispersion. It can be shown that $\Omega_t \geq 1$,[5] so that total hours are always at least equal to the aggregate labor services index, and the dispersion measure will vary in response to shocks.

Each household sets its nominal wage and supplies labor to satisfy demand by the representative firms in the two sectors. At the beginning of each period, each household receives with probability $(1-d)$ (following Calvo, 1983) a shock that leads it to revise its nominal wage. Households that do not receive a shock maintain a fixed nominal wage.

All households setting their wage at time t choose the same wage, denoted by W_t^*. The wage index follows the following nonlinear first order difference equation:

$$W_t^{1-\theta} = dW_{t-1}^{1-\theta} + (1-d)W_t^{*1-\theta}. \tag{8}$$

Ω_t obeys the following nonlinear first-order difference equation:

$$\Omega_t = (1-d) \left(\frac{W_t^*}{W_t} \right)^{-\theta} + d \left(\frac{W_t}{W_{t-1}} \right)^{\theta} \Omega_{t-1}. \tag{9}$$

[5] See Schmitt-Grohe and Uribe (2005).

2.3 Firms

There is a representative competitive firm in the exportables sector and a representative competitive firm in the importables sector. The production functions of the two goods are given by:

$$Y_{Xt} = A_X {N_{Xt}}^{\alpha_X}, \tag{10}$$

$$Y_{It} = A_I {N_{It}}^{\alpha_I}, \tag{11}$$

where N_{it} is the aggregate labor input employed in sector i, the A_i are constants,[6] and the α_i are constants such that $0 < \alpha_i < 1$. Diminishing returns to labor in each sector imply a convex production possibility frontier.[7]

Aggregate labor input, which is split between the two productive sectors of the economy, is given by

$$N_t = N_{Xt} + N_{It}. \tag{12}$$

With exogenous terms of trade, the foreign-currency prices of exportables and importables are given. We relax this assumption below. The domestic-currency price of the two goods is

$$P_{it} = S_t P_{it}^* \quad i = X, I,$$

where S_t is the spot nominal exchange rate. Profit maximization gives the following demand functions for labor in each sector:

$$N_{it} = \left(\frac{W_t}{P_{it}} \frac{1}{\alpha_i A_i} \right)^{-1/(1-\alpha_i)}$$

Conditional demand for each type of labor input is the sum of demands by the representative firms in the two sectors, and is given by:

$$N(s)_t = \left(\frac{W(s)_t}{W_t} \right)^{-\theta} (N_{Xt} + N_{It},) \tag{13}$$

where W_t is an index of average wages given by

$$W_t \equiv \left(\int_0^1 (W(s)_t)^{(\theta-1)} \right)^{1/(\theta-1)},$$

so that $-\theta$ is the elasticity of labor demand with respect to household $s's$ nominal wage.

[6]Suitable normalizations of these constants allow us to determine the comparative advantage of the economy and to determine which of the two goods is exported in the steady state.

[7]Also, pure profits in each sector are positive. One possible interpretation of these profits is as a payment to sector-specific factors which are in fixed supply and which are owned by households.

2.4 International Financial Markets

Domestic agents can lend and borrow on international financial markets. Following Senhadji (2003), the risk premium on foreign bonds depends on the economy's level of net foreign indebtedness according to

$$\kappa_t = \exp\left(\frac{-\varphi S_t B^*_{t-1}}{P_t Y_t}\right), \tag{14}$$

where Y_t is real GDP. The risk premium depends negatively on the economy's net foreign asset position relative to GDP. This specification captures the effects of default risk (on sovereign debt and other types of debt) on interest rates. It also has the consequence that the economy's steady state equilibrium is unique, with a domestic real interest that depends only on the subjective discount rate. Since we solve the model by approximating its dynamic equations around an initial steady state equilibrium, the approximation could break down if a sequence of small temporary shocks led to large changes in the steady state.[8]

2.5 Government Policy

Under a flexible exchange rate regime, the central bank fixes the exchange rate, and the money supply is determined exogenously. In order to restrict the number of different stochastic shocks in the model, monetary uncertainty is captured by money demand shocks alone, and the aggregate money supply itself is perfectly predictable. Under a fixed exchange rate regime, the exchange rate itself is exogenous, and the money supply becomes endogenous as the central bank stands ready to intervene in the market for foreign currency to support the fixed rate. Changes in foreign exchange reserves directly affect the monetary base. That is to say, foreign exchange market intervention is not sterilized. The government's period budget constraint is given by:

$$M_{t+1} - M_t = T_t. \tag{15}$$

2.6 Shocks

There are two sources of stochastic uncertainty in the model. First, there is the money demand shock b_t. Second, the terms of trade are stochastic. There are

[8]See Schmitt-Grohe and Uribe (2003) for a discussion of different ways of obtaining a unique steady state in small open economy models.

unified world markets for exportables and importables, so that the law of one price holds for each good. We normalize the price of importables expressed in foreign currency to equal one, so that

$$P_{It}^{*} = 1,$$

$$P_{It} = P_{It}^{*} S_t = S_t,$$

$$P_{Xt}^{*} = \rho_t P_{It}^{*}$$

$$\Rightarrow P_{Xt} = S_t P_{Xt}^{*} = S_t \rho_t P_{It}^{*} = S_t \rho_t.$$

The terms of trade shock obeys the following stochastic process:

$$\log(p_t) = \rho_p \log(p_{t-1}) + \varepsilon_{pt}, \quad 0 < \rho_p < 1.$$

The innovations ε_{bt}, and ε_{pt} are uncorrelated white noise shocks with constant variance.

2.7 National Accounting

Adding together the budget constraints of the households and of the government, and assuming that all profits are distributed by firms to households, we get:

$$W_t N_t + P_t \pi_t + \frac{r_t^{*}}{(1 + r_t^{*})} S_t B_{t-1}^{*} - P_t C_t =$$

$$P_{Xt} A_X {N_{Xt}}^{\alpha_X} + P_{It} A_I {N_{It}}^{\alpha_I} + \frac{r_t^{*}}{(1 + r_t^{*})} S_t B_{t-1}^{*} - P_t C_t =$$

$$S_t \left(B_t^{*} - B_{t-1}^{*} \right)$$

The economy's current account is given by the difference between national income (or GNP) and aggregate consumption. The current account is equal to the change in the economy's net foreign asset position. The latter is measured by $S_t B_{t-1}^{*}$.

3 Calibration and Simulation

3.1 Calibration

The numerical values used to conduct stochastic simulations are summarized in Table 1. They are taken from Devereux (2000), Macklem et al. (2000), and Ambler and Hakizimana (2004).

Dynare (Juillard, 2005) was used to simulate the model using a second-order approximation to its equilibrium conditions. Since the main focus of the paper is on evaluating the impact of exchange rate regime choice on welfare, second-order approximations are necessary in order to avoid the possibility of spurious welfare reversals highlighted by Kim and Kim (2003). As shown by Schmitt-Grohe and Uribe (2004), solving dynamic stochastic general equilibrium models using second-order approximations allows the variance of shocks to have an effect on the stochastic means of variables as well as their second moments.

3.2 Simulation Results

Table 2 gives measures of the variability of consumption, total hours, real balances, and other macroeconomic aggregates under the two different exchange rate regimes, along with measures of expected utility, for different values of the variances of the two kinds of shocks.

The superiority of a fixed exchange rate regime over a flexible rate regime depends on the relative importance of money demand shocks versus shocks to the terms of trade.

One way of calibrating the relative importance of the two types of shocks is by noting the important increase in real exchange rate variability subsequent to the collapse of the Bretton Woods system in 1973. As is well known, the collapse of Bretton Woods is not a clean natural experiment since 1973 coincided with the so-called first oil price shock. However, studies such as Mussa (1986), Baxter and Stockman (1989) and Monacelli (2004) show that other switches from fixed to floating exchange rate regimes were also accompanied by significant increases in real exchange rate/terms of trade volatility. We will use the identifying assumption that all or most of increased real exchange rate variability can be attributed to the change in exchange rate regime. This will allow us to parameterize the relative variances of the two types of shocks.

4 Endogenous Terms of Trade

Following Kollmann (2001), foreign demand for domestic goods is given by the following constant-elasticity demand function:

$$X_t = \left(K\rho_t \frac{e_t P^*_{Xt}}{P_{Xt}} \right)^{\mu}, \tag{16}$$

where μ gives the elasticity of export demand and K is a constant that is chosen to adjust the level of export demand in the economy's steady state. The variable ρ_t now has the interpretation of an external demand shock which shifts the demand schedule for the economy's exports. As before, ρ_t is stochastic, with its log following an AR(1) process. The domestic price of exportables, P_{Xt} is now endogenous. The market-clearing condition for exportables is given by

$$A_X {N_{Xt}}^{\alpha_X} = \left(Kp_t \frac{S_t P^*_{Xt}}{P_{Xt}} \right)^{\mu} + \phi \cdot \left(\frac{P_t}{P_{Xt}} \right) \cdot C_t. \tag{17}$$

As before, the foreign price of importables is normalized to equal one, and the domestic price of importables is given by:

$$P_{It} = S_t P^*_{It} = S_t.$$

4.1 Simulation Results with Endogenous Terms of Trade

Comparing the effects of a given external demand shock under fixed and floating rates, the nominal exchange rate appreciation in response to an increase in ρ_t dampens the increase in the internal nominal price of exportables. There is less of an increase in the demand for labor in the exportables sector, less of a positive response in the output of exportables, and therefore the final increase in the relative price of exportables is actually higher than under fixed rates. Comparing real exchange rate volatility across exchange rate regimes, part of the increase comes from a larger volatility of real exchange rates in response to real demand shocks. However, this increase is not quantitatively important.

5 Monetary Policy Rule

The basic model supposes an extremely simple money supply process. Under flexible rates the money stock is exogenous and fixed, while under fixed rates the nominal exchange rate becomes the exogenous variable and the money supply is completely endogenous. It is well known that the optimal money supply rule in this type of model can reproduce the flexible wage equilibrium by stabilizing the markup of prices over marginal costs.[9] The fully optimal money supply rule entails some response to exchange rate fluctuations, especially in response to velocity shocks, but in general some degree of exchange rate flexibility is desirable. The next-to-last section of the paper shows that this is possible only with a reaction function that makes monetary policy a function of nominal prices and levels of output in the exportables and importables sectors. If these variables are observed only with error or only subject to a delay, which seems realistic, the degree of exchange rate variability under optimal monetary policy (subject to information restrictions) is considerably reduced.

Devereux (2000) proposed a feedback rule for the central bank in which the instrument of monetary policy is the short-term nominal interest rate, and in which monetary policy reacts to the price level, sectoral prices, output, and the exchange rate. He showed that optimal monetary policy in the version of his model where the law of one price holds (which is also the case in the model developed in this paper) involves stabilizing the markup of prices over marginal costs, which is constant under flexible prices and wages.[10]

There are two difficulties with the optimal monetary policy rules in models such as Devereux's and the one here. First, such rules tie monetary policy to the values of current macroeconomic aggregates such as prices and output levels. In practice, such variables are imperfectly measured and observed only with a lag. Second, once the prices compatible with the flexible wage/price equilibrium are calculated, the models assume that monetary policy acts sufficiently quickly and precisely that these targets can be hit exactly withing the period: the effects of changes in the monetary policy instruments are not subject to long and variable lags à la Friedman.

In this section, we specify a general monetary policy reaction function in which the right-hand-side variables are observed only with error.

[9]Since there are two sectors in our model, achieving the flexible wage equilibrium involves stabilizing an average markup over marginal costs. Some real wage adjustment is necessary to facilitate the reallocation of resources across sectors.

[10]This result was first derived in the context of a closed economy model by Goodfriend and King (1997). Because of monopolistic competition in the labor market (in the goods market in Devereux's model), the flexible wage and price equilibrium is not Pareto-efficient, but monetary policy by itself cannot achieve the first-best social optimum.

[SECTION INCOMPLETE]

6 Conclusions

Our case for fixed exchange rates can be summarized as follows. While flexible rates help insulate economies from the effects of asymmetric real shocks, fixed rates insulate economies from the effects of monetary instability. Empirical evidence on real exchange rate volatility shows that real rates vary much more under flexible rates than under fixed rates: this evidence suggests that monetary instability is empirically important. In principle, optimal monetary policy rules can allow open economies to offset the effects of monetary instability while still allowing the nominal exchange rate to insulate the economy from the effects of asymmetric real shocks. In practice, if monetary policy must react to macroeconomic aggregates that are measured with error, the case becomes much stronger for reacting to a financial variable such as the exchange rate that immediately transmits the effects of exogenous shocks.

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Appendix A: First Order Conditions

The first order conditions for utility maximisation by household s can be written as follows. The variable $\lambda(s)_t$ is the Lagrange multiplier associated with the household’s budget constraint written in real terms (after dividing by the price level P_t).

$$C(s)_t: \quad \frac{C(s)_t^{-1/\gamma_1}}{\left(C(s)_t^{\frac{(1-\gamma_1)}{\gamma_1}} + b_t^{\frac{1}{\gamma_1}}\left(\frac{M(s)_t}{P_t}\right)^{\frac{\gamma_1-1}{\gamma_1}}\right)} - \lambda(s)_t = 0;$$

$$M(s)_t: \quad \frac{b_t^{1/\gamma_1}\left(\frac{M(s)_t}{P_t}\right)^{-1/\gamma_1}\frac{1}{P_t}}{\left(C(s)_t^{\frac{(1-\gamma_1)}{\gamma_1}} + b_t^{\frac{1}{\gamma_1}}\left(\frac{M(s)_t}{P_t}\right)^{\frac{\gamma_1-1}{\gamma_1}}\right)}$$

$$-\frac{1}{P(s)_t}\lambda(s)_t + \beta E_t \lambda(s)_{t+1}\frac{1}{P_{t+1}} = 0;$$

$$B(s)_t: \quad -\frac{\lambda(s)_t}{(1+r_t)P_t} + \beta E_t \frac{\lambda(s)_{t+1}}{P_{t+1}} = 0.$$

$$B^*(s)_t: \quad -\frac{\lambda(s)_t S_t}{\kappa_t(1+r_t^*)P_t} + \beta E_t \frac{\lambda(s)_{t+1}S_{t+1}}{P_{t+1}} = 0.$$

$$W(s)_t: \quad E_t \sum_0^\infty (\beta d)^i \Bigg(-\sigma H(s)_{t+i}{}^{\gamma_2}\frac{\partial H(s)_{t+i}}{\partial W(s)_t}$$

$$+\frac{\lambda(s)_{t+i}}{P_{t+i}}\left(H(s)_{t+i} + W(s)_t\frac{\partial H(s)_{t+i}}{\partial W(s)_t}\right)\Bigg) = 0$$

Appendix B: Wage Equation

We show in this appendix how to transform the first order condition for the choice of the nominal wage into a three-equation system that gives the dynamics of average wages and the contract wage.

The first order conditon can be transformed as follows.

$$E_t \sum_0^\infty (\beta d)^i (-\sigma) H(s)_{t+i}{}^{(1+\gamma_2)}\frac{\partial H(s)_{t+i}}{\partial W(s)_t}\frac{W(s)_t}{H(s)_{t+i}}\frac{1}{W(s)_t}$$

$$+E_t \sum_0^\infty (\beta d)^i \frac{\lambda(s)_{t+i}}{P_{t+i}} H(s)_{t+i}\left(1 + \frac{\partial H(s)_{t+i}}{\partial W(s)_t}\frac{W(s)_t}{H(s)_{t+i}}\right)$$

$$= 0$$

$$\Rightarrow \sigma E_t \sum_0^\infty (\beta d)^i H(s)_{t+i}{}^{(1+\gamma_2)}\theta$$

$$E_t \sum_0^\infty (\beta d)^i \lambda(s)_{t+i} \frac{W(s)_t}{P_{t+i}} H(s)_{t+i} (1-\theta)$$
$$= 0$$
$$\Rightarrow \frac{\theta}{(\theta-1)} \sigma E_t \sum_0^\infty (\beta d)^i H(s)_{t+i}{}^{(1+\gamma_2)}$$
$$= E_t \sum_0^\infty (\beta d)^i \lambda(s)_{t+i} \frac{W(s)_t}{P_{t+i}} H(s)_{t+i}.$$

We can simplify these two infinite sums by quasi-differencing. First, define

$$x_{1t} \equiv \sigma E_t \sum_0^\infty (\beta d)^i H(s)_{t+i}{}^{(1+\gamma_2)}.$$

We have

$$x_{1t+1} = E_{t+1} \sum_0^\infty (\beta d)^i \bar{H}(s)_{t+1+i}{}^{(1+\gamma_2)}$$

where

$$\bar{H}(s)_{t+1+i} \equiv \left(\frac{W(s)_{t+1}}{W_{t+1+i}} \right)^{-\theta} N_{t+1+i}.$$

This implies

$$x_{1t} - \beta d E_t x_{t+1} \left(\frac{H(s)_{t+1}}{\bar{H}(s)_{t+1}} \right)^{(1+\gamma_2)} = \sigma H(s)_t{}^{(1+\gamma_2)}.$$

Using the conditional demand functions for labor gives

$$x_{1t} = \sigma \left(\frac{W(s)_t}{W_t} \right)^{-\theta(1+\gamma_2)} N_t{}^{(1+\gamma_2)}$$

$$+\beta d E_t \left(\left(\frac{W(s)_t}{W_{t+1}} \right)^{-\theta(1+\gamma_2)} \left(\frac{W(s)_{t+1}}{W_{t+1}} \right)^{\theta(1+\gamma_2)} x_{1t+1} \right).$$

All wage-setters who are able to set their wage will choose the same wage. Call the wage set at time $t+i$ W^*_{t+i}. We have

$$x_{1t} = \sigma \left(\frac{W_t^*}{W_t} \right)^{-\theta(1+\gamma_2)} N_t{}^{(1+\gamma_2)}$$

$$+\beta d E_t\left(\left(\frac{W_t^*}{W_{t+1}}\right)^{-\theta(1+\gamma_2)}\left(\frac{W_{t+1}^*}{W_{t+1}}\right)^{\theta(1+\gamma_2)} x_{1t+1}\right)$$

$$\Rightarrow x_{1t}=\sigma\left(\frac{W_t^*}{W_t}\right)^{-\theta(1+\gamma_2)} N_t{}^{(1+\gamma_2)}$$

$$+\beta d E_t\left(\left(\frac{W_{t+1}}{W_t}\right)^{\theta(1+\gamma_2)}\left(\frac{W_t^*}{W_t}\right)^{-\theta(1+\gamma_2)}\left(\frac{W_{t+1}^*}{W_{t+1}}\right)^{\theta(1+\gamma_2)} x_{1t+1}\right).$$

We will use $\left(\frac{W_{t+1}}{W_t}\right)$ (average wage inflation) and $\left(\frac{W_t^*}{W_t}\right)$ (the relative contract wage) to simulate the model.

Proceeding in similar fashion with the other infinite sum, we have

$$x_{2t} \equiv E_t \sum_0^{\infty}(\beta d)^i \lambda(s)_{t+i} \frac{W(s)_t}{P_{t+i}} H(s)_{t+i}.$$

We have

$$x_{2t+1}=E_{t+1} \sum_0^{\infty}(\beta d)^i \lambda(s)_{t+1+i} \frac{W(s)_{t+1}}{P_{t+1+i}} \bar{H}(s)_{t+1+i}$$

$$\Rightarrow x_{2t}-\beta d E_t x_{2t+1}\left(\frac{H(s)_{t+1}}{\bar{H}(s)_{t+1}}\right)\left(\frac{W(s)_t}{W(s)_{t+1}}\right)=\lambda(s)_t \frac{W(s)_t}{P_t} H(s)_t.$$

Using the conditional demand functions for labor,substituting W_t^* for $W(s)_t$, and noting that the marginal utility of consumption is the same across households gives

$$x_{2t}=\lambda_t \frac{W_t^*}{P_t}\left(\frac{W_t^*}{W_t}\right)^{-\theta} N_t$$

$$+\beta d E_t\left(\left(\frac{W_{t+1}}{W_t}\right)^{(\theta-1)}\left(\frac{W_t^*}{W_t}\right)^{(1-\theta)}\left(\frac{W_{t+1}^*}{W_{t+1}}\right)^{(\theta-1)} x_{2t+1}\right).$$

x_{1t} and x_{2t} are related by the simple identity

$$x_{2t}=\frac{\theta}{(\theta-1)} x_{1t}.$$

Appendix C: Equation System

Here we give the complete system of equations used to simulate the model. The model's equilibrium can be calculated using the following seventeen equations.

$$\frac{C_t^{-1/\gamma_1}}{\left(C_t^{\frac{(1-\gamma_1)}{\gamma_1}} + b_t^{\frac{1}{\gamma_1}}\left(\frac{M_t}{P_t}\right)^{\frac{\gamma_1-1}{\gamma_1}}\right)} - \lambda_t = 0,$$

which follows from aggregating the household's first order condition for consumption.

$$\left(\frac{M_t}{P_t}\right) = b_t C_t \left(\frac{r_t}{1+r_t}\right)^{-\gamma_1},$$

which follows from aggregating the household's first order condition for money balances and simplifying. The equation has the traditional form of a money-demand function where γ_1 is the semi-elasticity of money demand. The Euler equation for domestic bonds gives:

$$-\frac{\lambda_t}{(1+r_t)P_t} + \beta E_t \frac{\lambda_{t+1}}{P_{t+1}} = 0.$$

The Euler equation for foreign bonds gives:

$$-\frac{\lambda_t S_t}{\kappa_t(1+r_t^*)P_t} + \beta E_t \frac{\lambda_{t+1} S_{t+1}}{P_{t+1}} = 0.$$

As shown in Appendix B, the household's first order condition for the choice of its nominal wage gives the following three-equation system:

$$\Rightarrow x_{1t} = \sigma \left(\frac{W_t^*}{W_t}\right)^{-\theta(1+\gamma_2)} N_t^{(1+\gamma_2)}$$

$$+\beta d E_t \left(\left(\frac{W_{t+1}}{W_t}\right)^{\theta(1+\gamma_2)} \left(\frac{W_t^*}{W_t}\right)^{-\theta(1+\gamma_2)} \left(\frac{W_{t+1}^*}{W_{t+1}}\right)^{\theta(1+\gamma_2)} x_{1t+1}\right);$$

$$x_{2t} = \lambda_t \left(\frac{W_t}{P_t}\right) \left(\frac{W_t^*}{W_t}\right)^{(1-\theta)} N_t$$

$$+\beta d E_t \left(\left(\frac{W_{t+1}}{W_t}\right)^{(\theta-1)} \left(\frac{W_t^*}{W_t}\right)^{(1-\theta)} \left(\frac{W_{t+1}^*}{W_{t+1}}\right)^{(\theta-1)} x_{2t+1}\right);$$

$$x_{2t} = \frac{\theta}{\theta - 1} x_{1t}.$$

The first order conditions for labor demand by firms give:

$$N_{Xt} = \left(\frac{(W_t/P_t)}{S_t p_t} \frac{1}{\alpha_X A_X} \right)^{-1/(1-\alpha_X)};$$

$$N_{It} = \left(\frac{(W_t/P_t)}{S_t} \frac{1}{\alpha_I A_I} \right)^{-1/(1-\alpha_I)}.$$

Total aggregate labor services are given by:

$$N_t = N_{Xt} + N_{It}.$$

The national income identity is given by:

$$S_t p_t A_X {N_{Xt}}^{\alpha_X} + S_t A_I {N_{It}}^{\alpha_I} - P_t C_t + S_t B_{t-1} - \frac{S_t B_t}{\kappa_t (1 + r_t^*)} = 0.$$

The risk premium is determined by:

$$\kappa_t = \exp\left(\frac{-\varphi S_t B_{t-1}^*}{P_t Y_t} \right).$$

There are two equations for the stochastic processes determining the evolution of p_t and b_t:

$$\ln(b_t) = \rho_b \ln(b_{t-1}) + \varepsilon_{bt};$$

$$\log(p_t) = \rho_p \log(p_{t-1}) + \varepsilon_{pt}.$$

The price level is given by:

$$P_t = S_t {p_t}^{\psi} \psi^{-\psi} (1 - \psi)^{-(1-\psi)}.$$

The equation for the average wage can be written as:

$$1 = d \left(\frac{W_t}{W_{t-1}} \right)^{(\theta-1)} + (1 - d) \left(\frac{W_t^*}{W_t} \right)^{(1-\theta)}$$

The following simple identity relates the rate of wage inflation to price inflation and the evolution of the real wage:

$$\frac{W_t}{W_{t-1}} = \frac{(W_t/P_t)}{(W_{t-1}/P_{t-1})} \frac{P_t}{P_{t-1}}.$$

The endogenous variables are: b_t, p_t, N_t, N_{Xt}, N_{It}, the real wage (W_t/P_t), the ratio of the optimal wage set by wage-setters to the average wage (W_t^*/W_t), wage inflation (W_t/W_{t-1}), λ_t, C_t, x_{1t}, x_{2t}, P_t, r_t, κ_t, the level of net foreign assets B_t^*, and either S_t under flexible exchange rates or M_t under fixed exchange rates.

Once the equation system is solved, it is easy to evaluate total hours using equation (9) and unconditional expected utility using equations (1) and (2).

Appendix D: Deterministic Steady State

The deterministic steady state of the model can be found by setting the shock innovations to zero, by dropping time subscripts from all variables, and by solving the resulting nonlinear equation system.

The system of equations is almost but not quite recursive. The Euler equation for domestic bonds implies immediately that:

$$(1+r) = \frac{1}{\beta},$$

which give the long run domestic real interest rate. The Euler equation for foreing bonds gives:

$$\kappa(1+r*) = \frac{1}{\beta},$$

which gives the long run value of κ given the (exogenous) long run level of the world real interest rate. The equation for the risk premium implies:

$$\frac{SB}{PY} = -\frac{\ln(\kappa)}{\varphi},$$

which gives a solution for the ratio of the economy's net foreign asset position to GDP. The national income identity gives:

$$PY - PC + SB - \frac{SB}{(1+r^*)\kappa}$$

$$\Rightarrow \frac{C}{Y} = 1 + SB\left(1 - \frac{1}{(1+r^*)\kappa}\right),$$

which gives the long run ratio of consumption to output. The price level equation gives:

$$\frac{S}{P} = p^{-\psi}\psi^{\psi}(1-\psi)^{(1-\psi)}$$

Putting together the two equations giving the demand for labor services by the importables firm and the exportables firm gives:

$$N = \left(\frac{W}{P}\frac{S}{P}\frac{1}{p}\frac{1}{\alpha_X A_X}\right)^{-1/(1-\alpha_X)} + \left(\frac{W}{P}\frac{S}{P}\frac{1}{p}\frac{1}{\alpha_I A_I}\right)^{-1/(1-\alpha_I)}.$$

This equation for labor market equilibrium can be used to solve for the long run real wage. We calibrate the model by backing out a value of θ so that $N = 1/3$. Therefore, when we solve for the model's steady state, we can impose the value of aggregate labor services and then back out the value of θ that supports this as an equilibrium. An analytical solution is not available, but the equation can easily be solved numerically. Given the real wage it is possible to solve for N_X and N_I. Then, we can calculate real GDP, since

$$PY = SpA_X {N_X}^{\alpha_X} + SA_I {N_I}^{\alpha_I}$$

$$\Rightarrow Y = \frac{S}{P} pA_X {N_X}^{\alpha_X} + \frac{S}{P} A_I {N_I}^{\alpha_I}.$$

Given real GDP, we can solve for real money balances using the money demand equation which implies:

$$\frac{M}{P} = Yb\frac{C}{Y}\left(\frac{1+r}{r}\right)^{-\gamma_1}.$$

The household's first order condition for consumption can be used to solve for λ as follows:

$$\frac{C^{-1/\gamma_1}}{\left(C^{\frac{(1-\gamma_1)}{\gamma_1}} + b^{\frac{1}{\gamma_1}}\left(\frac{M}{P}\right)^{\frac{\gamma_1-1}{\gamma_1}}\right)} = \lambda$$

$$\Rightarrow \frac{\left(\frac{C}{Y}\right)^{-1/\gamma_1} Y^{-1/\gamma_1}}{\left(\left(\frac{C}{Y}\right)^{\frac{(1-\gamma_1)}{\gamma_1}} Y^{\frac{(1-\gamma_1)}{\gamma_1}} + b^{\frac{1}{\gamma_1}}\left(\frac{M}{P}\right)^{\frac{\gamma_1-1}{\gamma_1}}\right)} = \lambda$$

The three-equation system that follows from the first-order condition for the household's choice of its optimal wage can be solved to yield:

$$\frac{W}{P} = \frac{\theta}{\theta-1}\frac{\theta N^{\gamma_2}}{\lambda}.$$

$$\Rightarrow \theta = \frac{\theta-1}{\theta}\lambda\left(\frac{W}{P}\right)/N^{(1+\gamma_2)}$$

The law of one price can be used to pin down the domestic prices of exportables and importables. We normalize the world price of exportables so that:

$$P_{Xt}^* = 1.$$

The domestic price of exportables is then given by:

$$P_{Xt} = S_t P_{Xt}^* = S_t,$$

and the domestic price of importables is:

$$P_{It} = S_t p_t$$

Table 1: Model Calibration

Parameter	Value
β	0.990
γ_1	0.250
γ_2	1.000
σ	0.520
ψ	0.030
d	0.750
φ	0.060
θ	6.000
α_X	0.667
α_I	0.667
A_X	1.000
A_I	1.000
ρ_b	0.950
ε_b	0.010
ρ_p	0.950
ε_p	0.010
M	1.000
r^*	0.010

Table 2: Variability of Aggregates, Utility

	Flexible	Fixed
Money demand shock		
σ_{C_t}		
σ_{H_t}		
σ_{M_t/P_t}		
$E(U(\cdot))$		
Money demand shock		
σ_{C_t}		
σ_{H_t}		
σ_{M_t/P_t}		
$E(U(\cdot))$		