A Tale of Two Velocities

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November 2016

Abstract

Quantitative easing in the US has meant a massive increase in the size of the Fed's balance sheet and the monetary base without a commensurate increase in inflation. Instead, velocity has decreased dramatically. The only comparable episode in recent economic history was Japan's experiment with quantitative easing in the early 2000s, where inflation remained low or negative and which ended in 2006 when the Bank of Japan reduced the size of its balance sheet to a level compatible with the growth path it was on before quantitative easing. We show that this is precisely what we would expect in a standard New Keynesian model in response to an increase in the money supply that is expected to be temporary.

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1 Introduction

> To keep prices stable, the Fed must see to it that the quantity of money changes in such a way as to offset movements in velocity and output. Velocity is ordinarily very stable, fluctuating only mildly and rather randomly around a mild long-term trend from year to year. **So long as that is the case**, changes in prices (inflation or deflation) are dominated by what happens to the quantity of money per unit of output. (Friedman, 2003, emphasis added)

The Fed has engaged in three distinct rounds of quantitative easing (QE) in response to the financial crisis of 2008, the Great Recession, and the subsequent slow recovery. Its balance sheet has increased substantially. So far this has not translated into a sustained increase in inflation. Instead, the velocity of circulation of money (measured using the monetary base, and also but less so using broader measures of the money supply) has decreased dramatically (see Figure 1 below).

The only comparable decrease in velocity in recent economic history occurred during the Bank of Japan's experiment with QE in the early 2000s. Starting in 2006 the Bank of Japan reduced the size of its balance sheet to levels consistent with the growth path of the monetary base before QE. It did so shortly after inflation turned positive. Throughout the episode, inflation had remained negative. Wieland (2010) contends that the reduction in the size of the monetary base was anticipated by market participants. The temporary nature of the increase in the money supply largely explains the lack of a significant impact of QE on inflation and real growth.

In this paper, we show that this is precisely the result predicted by a standard New Keynesian model (modified by the introduction of a motive for money demand). An increase in the monetary base that is (correctly) expected to be temporary leads to results similar to the Japanese and US experiences. Velocity decreases and inflation remains subdued. There is little or no impact on real economic activity. In contrast, an increase in the money supply that is (correctly) expected to be permanent boosts inflation and real activity substantially and persistently in the short run.

Three main mechanisms explain the weak impact of a temporary increase in the money supply on inflation.

1. Price-setting firms know that the price level must eventually revert to its path before QE. Firms do not raise their prices much during QE in order not to be stuck with relative prices that are too high when QE ends.

2. A second set of mechanisms is in effect even in the absence of nominal price and wage rigidities. Significant inflation in response to monetary expansion would entail rapid deflation when QE ends, entailing high ex ante real interest rates. First, consumption smoothing acts as a brake on abrupt (anticipated) swings in the ex ante real interest rate. Second, a high ex ante real interest rate would drive down investment. In the presence of adjustment costs this would be very costly. These channels were first identified by Bernholz (1988), Calomiris (1988) and Sumner (1993) in the context

> of American colonial monetary history. Smith (1988), noting historical episodes of rapid money growth not accompanied by inflation, had interpreted this as a rejection of monetarism.

QE pushes down the short-term nominal interest rate, and households absorb the increase in real balances since money demand is (by assumption) highly elastic at low interest rates. Interest rates actually remain lower for longer in response to a temporary increase in money than in response to a permanent increase.

The interpretation of these results is that old-school monetarists had an incomplete view of the monetary transmission mechanism. Even if money demand can be expressed as a stable function of the current volume of transactions and the current opportunity cost of holding cash balances (the short-term nominal interest rate on government bonds), expectations of the future evolution of the money supply are crucial in determining both velocity and the nominal interest rate.[1] Woodford (Reichlin, Turner and Woodford 2013), discussing the difference between "traditional QE" and "helicopter money" (tax cuts financed with immediate and permanent monetization of the bond issues) noted:

> Under quantitative easing, people might not expect the increase in the monetary base to be permanent – after all, it was not in the case of Japan's quantitative easing policy in the period 2001–2006, and US and UK policymakers insist that the expansions of those central banks' balance sheets won't be permanent, either – and in that case,

[1] The above quote from Friedman (2003) shows that he did recognize the importance of adapting money growth to compensate for changes in velocity, but he did not take into account the endogeneity of velocity to expected future changes in monetary policy.

there is no reason for demand to increase. Perhaps in the case of helicopter money, it would be more likely that the intention to maintain a permanently higher monetary base would be believed.

This interpretation can explain the relative success of monetary expansions viewed as permanent. Examples would include the abandonment of the gold standard by the US in 1933, which led to the most rapid increase in industrial production in US history, and the stronger real effects of Abenomics (although the final verdict is not yet in on this experiment) when compared to Japan's earlier experiment with QE. It can also explain the lack of success both of QE in Japan in the early 2000s and of the more recent US episodes of QE.[2]

Section 2 presents some of the stylized facts of the US and Japanese experiences with QE. The model is presented in the third section. Section 4 presents results from perfect-foresight simulations of the model in response to temporary and permanent increases in the money supply. Section 5 interprets the results in more detail. Section 6 concludes.

2 Background

Figure 1 illustrates the time path of the US monetary base and velocity since 2000. The three main episodes of QE are clearly visible on the graph. The drop in velocity clearly mirrors these episodes except for the initial drop in 2008 which

[2]In an early paper, Krugman (1998) suggested that a permanent monetary expansion would be a reliable way for the Bank of Japan to boost inflation and aggregate demand, but did not demonstrate the result formally.

was much greater than the increase in base money. This coincided with nominal GDP falling below its trend growth path, a "nominal GDP gap" that has yet to be closed.

The initial drop in velocity in 2008 was a direct response to the financial panic in the wake of the Lehman Brothers bankruptcy, a "flight to safety" as banks and individuals sought to avoid risk and increase liquidity. The depth of the US recession has been interpreted as resulting from an insufficiently vigorous response by the Fed to the financial crisis and to the resultant increase in the demand for money and other safe assets.[3]

[3] See Hetzel (2009) and (2012) for a cogent summary of these arguments. See also Congdon (2009, 2011, 2014), Hummel (2011), Johnson (2010), and Sumner (2013, 2015) for "neo-monetarist" interpretations of the Great Recession.

Figure 1: Base and Base Velocity

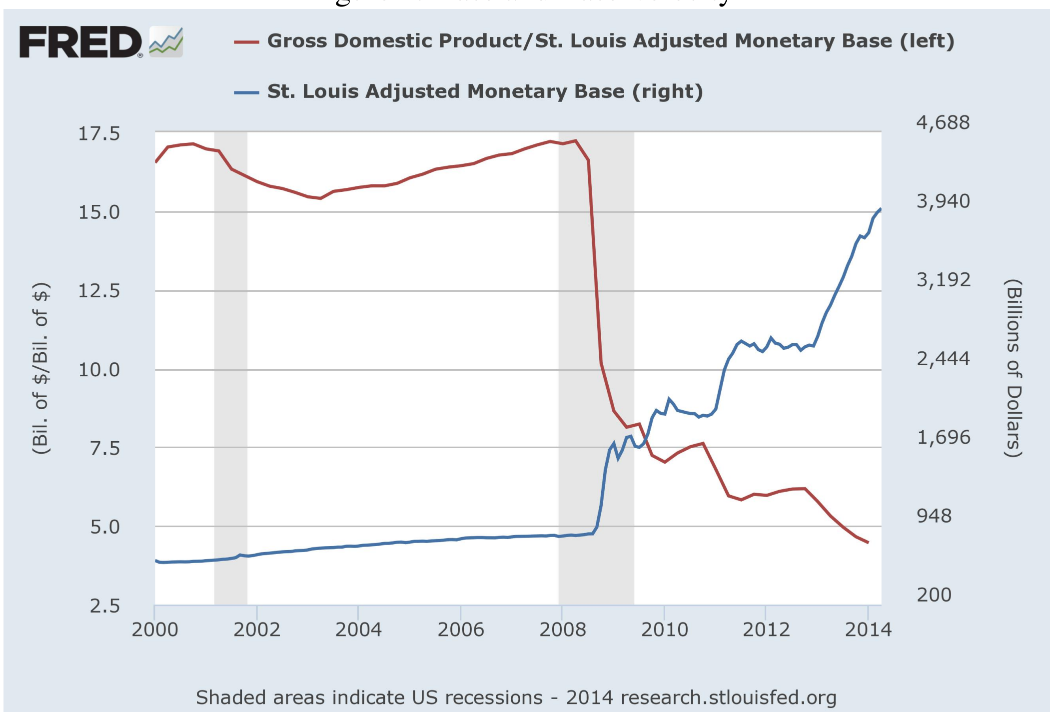


Figure 2 clearly illustrates the lack of pressure on inflation. Since the financial crisis and the Great Recession, US inflation has only risen above 2% (the Fed's official inflation target since 2012) very briefly in 2011. Current headline inflation is significantly lower than this and shows little or no sign of increasing.[4]

[4]The vertical scale of the graph measures quarterly inflation. For annualized inflation rates, multiply by four.

Figure 2: PCE Inflation

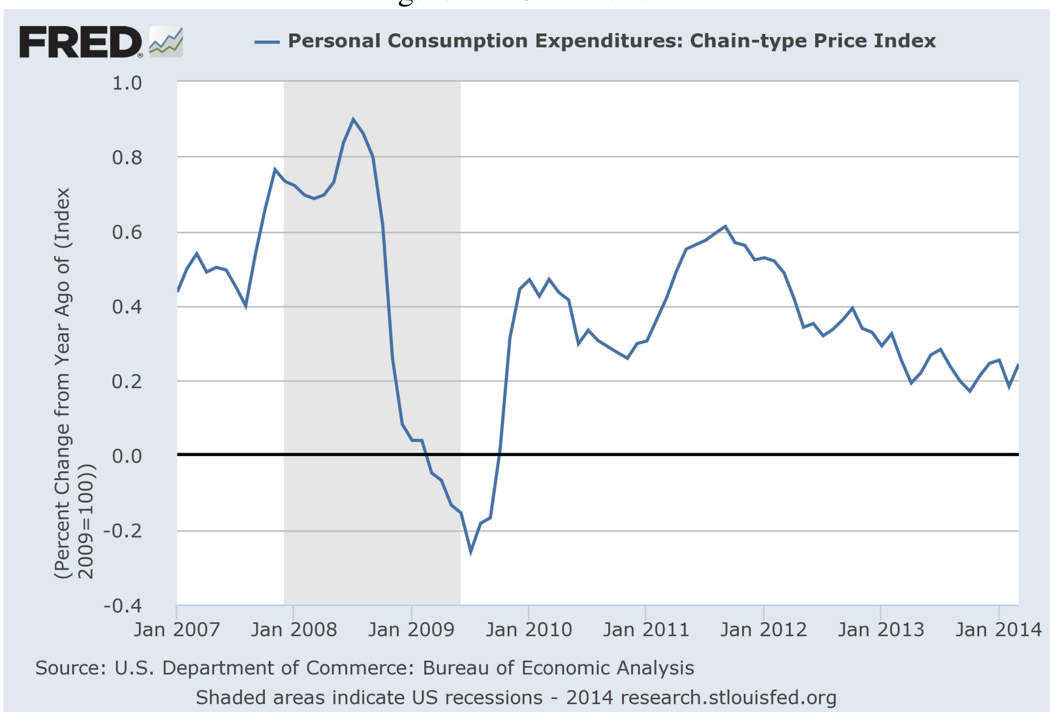
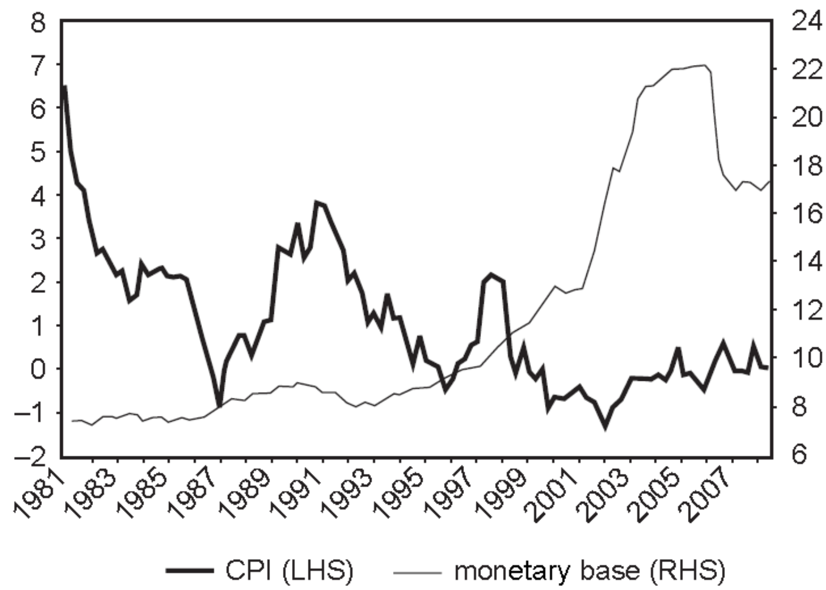


Figure 3 (from Wieland 2010) illustrates the Japanese experience with QE. The Japanese monetary base spiked up sharply in 2001 as QE was implemented. As soon as Japanese inflation turned slightly positive in 2006, the Bank of Japan shrank the monetary base so that it returned to the growth path it was on before 2001. Inflation remained subdued and the real recovery was very weak. The vertical scale for the monetary base is in levels rather than logs. It is easy to imagine a smooth exponential curve following the expansion of the monetary base up to the year 2000, and rejoining the level of the monetary base from 2007 on.

Figure 3: Japanese Monetary Base and Inflation (Wieland 2010)

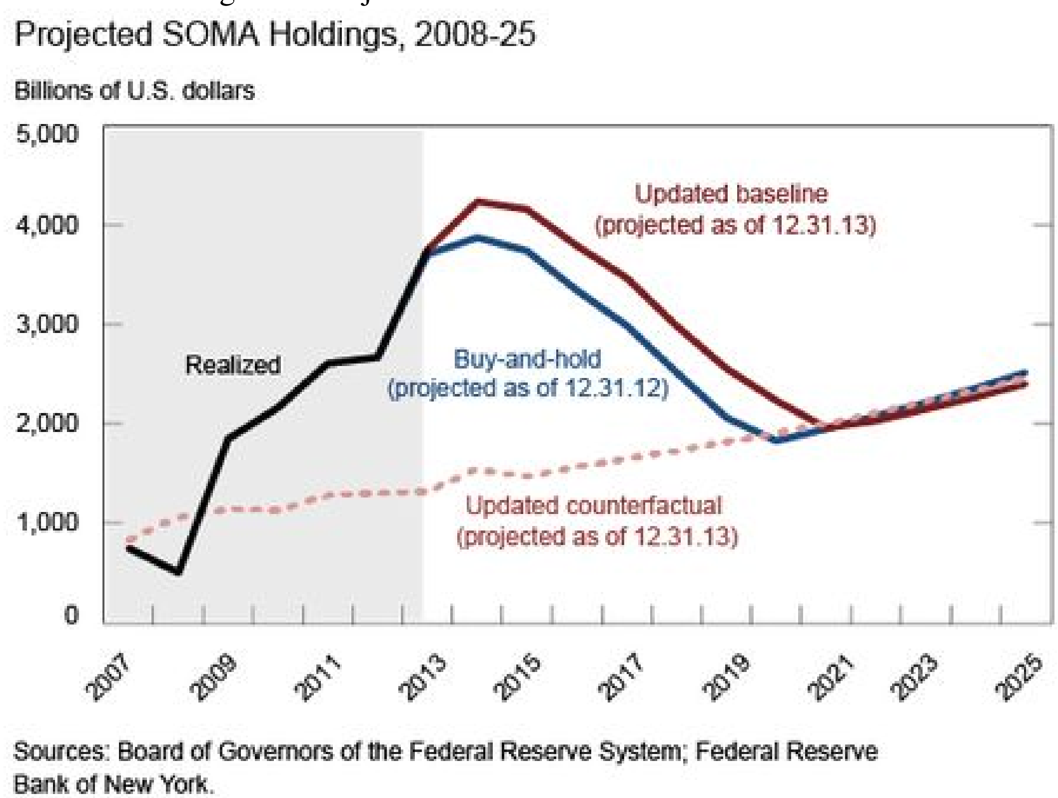
Figure 6. Base money and CPI inflation in Japan: 1981 – 2008, quarterly observations.



It is not completely clear from a reading of FOMC statements and Fed reports what the exact future path of the Fed's balance sheet will be. From statements by the Fed, "tapering" could easily be taken to mean a reduction in the growth rate of its balance sheet or an eventual reduction in its level. Given that the Fed itself is not entirely explicit about the future time path of its balance sheet, it is likely that there are differing views held by private agents. Some evidence is proveded by Figure 4 below, taken from Cambron et al. (2014). It shows projections by researchers at the Federal Reserve Bank of New York of the Fed's

SOMA holdings.[5] The red line reflects their updated projections, and shows that the size of these holdings is expected to revert to a trend line projection from 2007 that counterfactually assumes no QE by the Fed. This clearly shows that the Fed itself expects that the increase in the level of the monetary base due to QE will be temporary.

Figure 4: Projected Size of the Fed's Balance Sheet



[5]These are not exactly equivalent to the size of the Fed's balance sheet, but represents a substantial fraction of the Fed's total assets.

3 Model

The model is the simplest formal framework that illustrates the less formal arguments of Bernholz (1988), Calomiris (1988) and Sumner (1993). It is a vanilla New Keynesian model with few modifications. Rather than modelling monetary policy without money as in Woodford (2003), households' utility functions include real balances (consumption and real balances are non-separable) in such a way that money demand is highly elastic at low interest rates while not giving rise to a true liquidity trap. The monetary base is the central bank's instrument for the conduct of monetary policy. The model incorporates capital accumulation with an exogenous wedge between the rate of return on capital and on short-term bonds and convex adjustment costs. This provides one of the important mechanisms preventing temporary increases in money from having strong effects on inflation and real variables.

The model abstracts completely from the financial sector, broader monetary aggregates, and the money multiplier.[6] It is the simplest possible framework to illustrate the distinction between the effects of temporary and permanent monetary expansion. For simplicity, the level of the monetary base is constant except for once and for all (permanent or temporary) shifts in its level and there is no real growth. Adding positive long-term money growth (and trend inflation) along with real growth would complicate the algebra by requiring normalizations but would not change the main results.

[6] See Ireland (2013) for a New Keynesian model with a banking sector and a role for deposits in addition to base money.

3.1 Households

A representative household that works, consumes, and holds real balances that yield utility.[7] The household's savings can be allocated to real balances, one-period nominal bonds, and capital. The household rents capital to firms and its capital accumulation is subject to convex adjustment costs.

Representative household utility is given by

$$U = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{\gamma}{\gamma - 1} \ln \left(C_t^{\frac{\gamma-1}{\gamma}} + u_t^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) + \eta \ln (1 - h_t) \right\}, \quad (1)$$

where C_t is real consumption expenditure, M_t is nominal balances, P_t is the price level, and h_t is hours worked.

The household is subject to the following period-t budget constraint:

$$C_t + I_t + CAC_t + \frac{B_t}{P_t}\frac{1}{R_t} + \frac{M_t}{P_t} \leq$$

$$w_t h_t + (q_t - \tau_t) K_t + \frac{B_{t-1}}{P_{t-1}}\frac{1}{\pi_t} + \frac{M_{t-1}}{P_{t-1}}\frac{1}{\pi_t} + \frac{D_t}{P_t} + T_t, \quad (2)$$

where I_t is real investment expenditure, B_t is the quantity of one-period nominal bonds, R_t is the short-term (gross) interest rate on nominal bonds, CAC_t represents capital adjustment costs, q_t is the gross return to capital, τ_t is a risk premium shock that drives a wedge between the returns on capital and the one-period bond,[8]

[7]As noted by Buiter (2014), money balances must yield benefits other than its pecuniary rate of return not only for money to be willingly held when dominated in rate of return but also for permanent QE to be effective.

[8]Following Amano and Shukayev (2012), the risk premium shock is the only exogenous shock capable of driving the return on one-period bonds to its zero lower bound for a plausible size of its

π_t is the gross rate of inflation P_t/P_{t-1}, D_t is (nominal) dividend payments from firms, and T_t represents real lump-sum transfers and monetary injections, and finally u_t is a money demand shock.[9]

The money demand shock u_t follows the process given by:

$$\ln(u_t) = (1-\rho_u)\ln(\bar{u}) + \rho_u \ln(u_{t-1}) + \varepsilon_{ut}. \tag{3}$$

The risk premium shock follows the process given by

$$\ln(\tau_t) = (1-\rho_\tau)\ln(\bar{\tau}) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_{\tau t}. \tag{4}$$

Capital accumulation is given by

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{5}$$

where δ is the (constant) depreciation rate of capital.

Capital adjustment costs are given by

$$CAC_t = \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{6}$$

Subtracting δ from the gross investment rate ensures that no adjustment costs are

standard deviation.

[9]It is clear from the household's balance sheet that the central bank can affect the quantity of money either through open market operations or in cooperation with the Treasury, involving a change in lump sum taxes financed by new bond issues that are sterilized via open market operations. Both types of monetary policy will have equivalent effects in our simple model.

paid in the deterministic steady state.

The household's problem can be written as the following Lagrangian:

$$\max_{C_{t+i},h_{t+i},M_{t+i},B_{t+i},K_{t+i+1},\lambda_{t+i}} \mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\frac{\gamma}{\gamma-1} \ln \left(C_t{}^{\frac{\gamma-1}{\gamma}} + u_t{}^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) \right.\right.$$

$$+\eta \ln (1-h_t)$$

$$\lambda_{t+i} \left(w_{t+i} h_{t+i} + (q_{t+i} - \tau_{t+i}) K_{t+i} + \frac{B_{t+i-1}}{P_{t+i-1}} \frac{1}{\pi_{t+i}} + \frac{M_{t+i-1}}{P_{t+i-1}} \frac{1}{\pi_{t+i}} \right.$$

$$+ \frac{D_{t+i}}{P_{t+i}} + T_{t+i} - C_{t+i} - K_{t+i+1} + (1-\delta) K_{t+i}$$

$$\left.\left.\left. - \frac{\varphi}{2} \left(\frac{K_{t+i+1} - (1-\delta) K_{t+i}}{K_{t+i}} - \delta \right)^2 K_{t+i} - \frac{B_{t+i}}{P_{t+i}} \frac{1}{R_{t+i}} - \frac{M_{t+i}}{P_{t+i}} \right) \right] \right\}. \tag{7}$$

The complete first order conditions for the household's problems are given in the appendix. Using the first order conditions for the choice of consumption and for the choice of bonds, the first order condition for money balances can be simplified to yield the money demand function given by

$$\Rightarrow \frac{M_t}{P_t} = u_t C_t \left(1 - \frac{1}{R_t} \right)^{-\gamma}. \tag{8}$$

This is a monetarist demand-for-money equation. The demand for real balances is a simple function of real transactions measured by aggregate consumption expenditures and of opportunity cost given by the short-term nominal interest rate (and of the exogenous shock variable u_t). Despite this, the velocity of circulation of money will not be stable in response to fluctuations in the money supply that are expected to be temporary.

3.2 Goods Production

A representative, competitive producer uses intermediate goods to produce a final good using the production function given by

$$Y_t = \left[\int_0^1 Y_t(i)^{\theta - 1 \theta} di \right]^{\frac{\theta}{\theta - 1}}. \tag{9}$$

There is a continuum of intermediate goods on the unit interval that enter final goods production symetrically.

Maximization of profits by the final goods producer gives the following conditional intermediate good demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t. \tag{10}$$

The exact price index for final production is given by

$$P_t = \left[\int_0^1 P_t(l)^{1-\theta} dl \right]^{\frac{1}{1-\theta}} \tag{11}$$

There is a continuum of monopolistically competitive firms on the unit interval indexed by l that produce Intermediate goods production by

$$Y_t(l) = A_t H_t(l)^{\alpha} K_t(l)^{(1-\alpha)}, \tag{12}$$

where H_t is aggregate per capita hours worked.

The aggregate technology shock process is given by

$$\ln(A_t) = (1 - \rho_A)\ln\left(\bar{A}\right) + \rho_A \ln(A_{t-1}) + \varepsilon_{At}. \tag{13}$$

We assume Calvo pricing. Each firm has a fixed probability $(1-d)$ of being able to re-optimize its price in any period. If allowed to reset its price in period t, the firm maximizes the discounted sum of expected future profits:

$$\max E_t \sum_{i=0}^{\infty} d_{t+i}\left(\beta^i \frac{\lambda_{t+i}}{\lambda_t}\right)\left(\frac{D_{t+i}(l)}{P_{t+i}}\right), \tag{14}$$

where D_{t+i} represents nominal dividends in period $t+i$, $\left(\beta^i \frac{\lambda_{t+i}}{\lambda_t}\right)$ is the stochastic discount factor used by shareholders to value profits at date $t+i$, and d_{t+i} is the probability that the price set in time t will still be in force at time $t+i$. Under Calvo pricing, firms have a constant probability $(1-d) \in (0,1)$ of being able to reset their price optimally in any given period, so that

$$d_{t+i} = d^i, \quad 0 \leq i \leq \infty.$$

Nominal dividends $D_{t+i}(l)$ are given by

$$D_{t+i}(l) = P_t^*(l)Y_{t+i}(l) - w_{t+i}P_{t+i}H_{t+i}(l) - q_{t+i}P_{t+i}K_{t+i}(l), \tag{15}$$

where $P_t^*(l)$ is the price set by the firm in period t, w_t is the real wage rate, and

q_t is the real rental rate of capital. The first-order conditions of the firm's problem with respect to $K_t(l)$, $H_t(l)$ and $P_t^*(l)$ are given by

$$q_t = (1-\alpha)\psi_t(l)\frac{Y_t(l)}{K_t(l)}, \tag{16}$$

$$w_t = \alpha\psi_t(l)\frac{Y_t(l)}{H_t(l)}, \tag{17}$$

$$P_t^*(l) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{i=0}^{\infty}(d\beta)^i\frac{\lambda_{t+i}}{\lambda_t}\psi_{t+i}(l)Y_{t+i}(P_{t+i})^{\theta}}{E_t\sum_{i=0}^{\infty}(d\beta)^i\frac{\lambda_{t+i}}{\lambda_t}Y_{t+i}(P_{t+i})^{\theta-1}}, \tag{18}$$

where $\psi_t(l)$ denotes the real marginal cost at date t associated with firm l's maximization problem. According to equations (16) and (17), the marginal products of labor and capital both exceed their respective marginal costs. Equation (18) is the firm's optimal price equation, derived from the equalization of marginal cost with marginal revenue in a dynamic context.

As is well known, this equation can be linearized under the assumption of zero steady-state inflation to obtain the standard New Keynesian Phillips curve, or under non-zero steady-state inflation to obtain an extended version of the New Keynesian Phillips curve (on the latter, see Bakhshi et al. 2007). However, linearizing the model involves ignoring fundamental nonlinearities of the model when the nominal interest rate approaches its lower bound. The appendix shows that the representative firm's optimal pricing equation can be written in the form of two first-order forward-looking nonlinear difference equations, with a static relationship linking the two. This follows Schmitt-Grohé and Uribe (2007). We simulate the response of variables to changes in the money supply under perfect foresight, and this allows us to keep potentially important nonlinearities as part of the solution.

3.3 Aggregation

The aggregate resource constraint is given by

$$Y_t^s = (C_t + I_t + CAC_t)\, S_t, \tag{19}$$

where Y_t^s is aggregate supply and where S_t is the inefficiency wedge due to price dispersion across firms. It is defined as follows:

$$S_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di. \tag{20}$$

The wedge follows the nonlinear law of motion given by

$$S_t = d{\pi_t}^{\theta} S_{t-1} + (1-d){p_t^*}^{-\theta} \tag{21}$$

3.4 Solution Method

We solve the model using *Dynare*.[10]. We are interested primarily in perfect foresight simulations. In such cases, *Dynare* allows the user to impose terminal conditions on the state variables of the model and solves a system of nonlinear equations for the model's dynamics between the initial and final periods. The solution

[10]See Adjémian et al. (2011)

algorithm preserves the nonlinearities of the model's dynamics.

In order for the dynamic paths of variables to be smooth, it is necessary to impose terminal conditions that are compatible with the solution to the model's steady state and also to simulate over a large enough number of periods so that the dynamic paths are arbitrarily close to those steady states at the terminal date.

3.5 Calibration

The model is calibrated to a quarterly frequency. The parameter values are standard in the literature, and are given in Table 1 below. Galí (2014) calibrates a similar New Keynesian model and uses parameter values similar to the ones used here. The parameters related to the risk premium shock are taken from Amano and Shukayev (2012). The weight on leisure in the utility function is calibrated so that households spend one third of their time endowment working. With $d = 0.75$, the average nominal price rigidity is four quarters. The semi-elasticity of money demand, γ, is compatible with Ball (2001).

The calibrated values give a steady-state nominal interest rate of two percent. In the absence of trend inflation this means an annual real interest rate of two percent in the deterministic steady state.

Table 1: Model Calibration (Base Case)

Parameter	Meaning	Value
β	discount rate	0.995
α	labor share	0.667
d	probability of no price adjustment	0.750
γ	semi-elasticity of money demand	0.060
δ	capital depreciation rate	0.025
θ	elasticity of substitution between intermediates	6.000
φ	capital adjustment cost parameter	3.000
H	steady-state hours	0.333
η	implied value of weight on leisure	2.670
$\bar{A}$	steady-state technology level	1.000
ρ_A	technology shock persistence	0.950
σ_{ε^A}	technology shock standard deviation	0.010
$\bar{u}$	steady-state value of money demand shock	0.062
ρ_u	money demand shock persistence	0.950
σ_{ε_u}	money demand shock standard deviation	0.010
$\bar{\tau}$	steady-state risk premium	0.016
ρ_τ	risk premium shock persistence	0.950
$\sigma_{\varepsilon^\tau}$	risk premium shock standard deviation	0.010

4 Perfect Foresight Simulation Results

We simulate the response of the economy to a large large ($\mu_t = 0.5$) temporary (8 quarters) increase in M from its steady state:

$$\mu_t = -\mu_{t+k} = 0.5, \quad k = 8.$$

The increase in the size of the money stock is $\exp(0.5) = 67\%$. We compare this to the response of the economy to a **permanent** monetary expansion of the same size. In this case

$$\mu_t = 0.5, \quad k = \infty.$$

The results for the price level are shown in Figure 5 below. In response to a permanent expansion of the money supply, prices begin to rise steadily towards their new steady-state level. The increase is not immediate due to the presence of rigid nominal prices. In response to a temporary increase in the money supply, prices move very little and are very close to their initial value when the money supply expansion ends after eight quarters.

The response of inflation is shown in Figure 6. In response to a permanent monetary expansion, inflation increases quickly and peaks at almost six percent. The response of inflation to a temporary increase in the money supply is weak, and inflation becomes negative before the monetary expansion ends, approaching its steady-state value of zero from below.

Figure 7 shows the response of the nominal interest rate in the two scenarios. With a permanent increase in base money, the interest rate returns almost to its steady-state level after only one period. In contrast, in the case of a temporary increase the nominal interest rate remains close to the zero lower bound for as long as the monetary base is higher than its long-run level. Since the model is solved under the assumption of perfect foresight, the interest rate at horizons longer than one period is just the average of the expected future short-term rate. This means that longer-term interest rates are more strongly (negatively) affected by a temporary increase in the monetary base than by a permanent increase.

Figure 8 shows the response of consumption. A permanent monetary expansion has strong real effects. Consumption initially decreases in order to satisfy households' Euler equation.[11] The response becomes strongly positive thereafter and is quite persistent.

Figure 9 shows the response of output. Output changes very little in response to a temporary monetary shock. (Investment increases in the first period to compensate for the decrease in consumption.) Output increases substantially and persistently in response to a permanent increase in base money.

Figure 10 shows the response of the velocity of money in the two different scenarios. Velocity decreases substantially and persistently in response to a temporary monetary shock. In contrast, in response to a permanent increase in base money, velocity spikes upward for one period and then returns to a level very close to its steady state.

[11] Introducing habit persistence into the utility function would smooth out the initial response in consumption.

5 Interpretation

There are three main channels that explain why a temporary increase in the money supply has such a weak effect on prices.

The first hinges on the presence of nominal price rigidity. Firms that reset prices just before the money stock reverts to its initial level will not want to be stuck with relative prices that are too high and that will lead to low demand for their output. For this reason, they set their prices close to the steady-state price level. Firms that reset their prices just before these firms will not want to set prices that are too high relative to firms who will set prices in the following period. By backward induction, firms that set prices in the same period as the temporary monetary expansion takes place will not want to increase their prices by very much.

The other two channels operate via the real interest rate. They hold whether or not there are nominal rigidities. As noted in the introduction, they were identified by authors in the context of colonial American monetary history (Bernholz, 1988; Calomiris, 1988; Sumner, 1993). If prices increased substantially in response to a temporary money supply increase, they would have to decrease again quickly as the money supply fell back to its initial level or growth path. This deflation would be perfectly anticipated and would mean a high real interest rate. A high real interest rate necessitates a high marginal product of capital. Investment must become negative in order to achieve this, and this is costly because of the convex adjustment costs in the model. Consumption smoothing also pushes inflation and prices not to change abruptly. If prices were expected to decrease rapidly towards the end of a temporary QE episode, this implies a high real interest rate since the nominal interest rate cannot fall below zero. This would in turn mean a sharply rising path for consumption, and so a low level of consumption in the periods before the monetary base is reduced to its initial level. This is feasible for an individual household, but in aggregate it implies a large imbalance between savings and investment. The real interest rate must be low enough to equation savings and investment.

The interest rate drop in response to the temporary increase in base money induces households to hold higher real balances. Money demand becomes very interest-sensitive at low interest rates and equilibrates the money market in a situation where neither prices nor real output respond very much.

5.1 Main Implications

The model and the simulation results have a number of implications for monetary policy and for our understanding of the observed weak response of economies to unconventional monetary policy, or more specifically QE, since the financial crisis of 2008.

1. QE must be expected to lead to a permanent increase in the money supply in order to be effective in stimulating demand and output.

2. The end point or final level of the central bank's balance sheet must be clearly communicated to the public, and this must be credible, in order to

create the expectation of a permanent increase in the monetary base.

3. A credible way of achieving this would be to adopt some form of level targeting such as price-level-path targeting or nominal GDP path targeting.[12]

4. Because only a permanent increase in the monetary base is effective, it is necessary that QE lead to a rate of inflation that is temporarily higher than the target rate of inflation. The model studied here has an inflation target (and trend inflation) equal to zero, but effective QE leads to a once-and-for-all increase in the price level. With positive trend inflation, there would be a once-and-for-all shift upwards in the price level path in response to a permanent upward shift in the path of the money supply: inflation would have to be temporarily higher than the target for prices to reach the higher path.

5. Because inflation must overshoot the target rate in response to QE, there might be a danger of inflation expectations becoming unanchored. It has certainly been the case that inflation hawks in both the US and in Europe have been predicting rapid and even uncontrollable inflation in response to QE by the Fed and the European Central Bank. Some form of level target would help calm these fears and help keep medium-term inflation expectations anchored in addition to making the permanence of money supply increases more credible.

6. Temporary QE is very ineffective in boosting demand and output, but it has

[12] Rowe (2009) develops this argument.

an unambiguously stronger impact on nominal interest rates. The short term interest rate decreases for longer in response to a temporary increase in the base. Under perfect foresight, longer-term interest rates are just averages of future short-term rates. This means that a temporary QE policy must exert stronger downward pressure on the term structure than a permanent QE policy.

7. A corollary of the previous point is that an econometrician looking for evidence of the effectiveness of QE by estimating the impact of changes in the central bank's balance sheet on the term structure could come to very misleading conclusions. Econometric estimates take it as given that the appropriate way to measure the impact of an increase in base money is as the response to an innovation to a given stochastic process. By construction such estimates cannot pick up a change in the degree of persistence of a given increase in base money.

8. Because the demand for money function is invertible, it would in principle be possible to reverse engineer the same result using the short-term interest rate as the instrument of monetary policy by announcing a time path for the interest rate. This would be equivalent to forward guidance.[13] However, the central bank would require a very precise knowledge of the parameters of the money demand function to achieve the same path via interest rate

[13] With a model similar to the one studied here (but without money), Eggertsson and Woodford (2003) show how to use forward guidance to stimulate demand with a policy rate stuck at its zero lower bound.

changes.

9. Subject to the qualification noted in the previous point, a policy of forward guidance that promises that the policy rate will remain “lower for longer” to boost demand may backfire because interest rates do in fact remain lower for longer under a temporary monetary expansion than under a permanent monetary expansion.

10. The simulation results point to a corollary to Friedman’s (1998) dictum that short-term nominal interest rates are generally a poor indicator of the stance of monetary policy. In the context of Japanese monetary policy in the 1990s, Friedman wrote, ”Low interest rates are generally a sign that money has been tight, as in Japan; high interest rates, that money has been easy.” He called this the “interest rate fallacy.” The results here show that low interest rates can also be sign that monetary policy is expected to be tight in the future.

6 Conclusions

The main policy implication of the model is that expansionary monetary policy is only effective if it is expected to be permanent. Central banks must commit to generating inflation that is temporarily higher than their targets. The commitment must also be credible. This would make monetary policy history-dependent in the sense of Woodford (2003, 2012).

One way of committing to a higher price level in the face of a negative shock would be to adopt level targeting. Either price-level-path targeting or nominal GDP level targeting would ensure that some part of an increase in the money supply would be expected to be permanent.[14] Level targeting of some kind would also help anchor inflation expectations.

Finally, the model helps us interpret recent monetary history. Low short-term nominal interest rates have not been a sign that monetary policy has been easy. Rather they are a sign that monetary policy is expected to be restrictive in the future.

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[14]Without going all the way to level targeting, average inflation targeting would also ensure that monetary policy is history-dependent to some extent.

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Appendices

A Household’s First Order Conditions

The household’s first order conditions with respect to its choice variables (we omit the FOC with respect to λ_t) at time t are given by the following equations.

$$C_t: \qquad \frac{C_t^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + u_t^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} - \lambda_t = 0;$$

$$h_t: \qquad -\frac{\eta}{1-h_t} + \lambda_t w_t = 0;$$

$$B_t: \qquad -\lambda_t + \beta \mathrm{E}_t\left(\lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right) = 0;$$

$$M_t: \qquad \frac{u_t^{\frac{1}{\gamma}} \frac{M_t}{P_t}^{-\frac{1}{\gamma}} \frac{1}{P_t}}{C_t^{\frac{\gamma-1}{\gamma}} + u_t^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} - \lambda_t \frac{1}{P_t} + \beta \mathrm{E}_t\left(\frac{1}{P_t}\frac{1}{\pi_{t+1}}\right) = 0;$$

$$K_{t+1}: \qquad -\lambda_t\left[1 + \varphi\left(\frac{I_t}{K_t} - \delta\right)\right] +$$

$$\beta E_t\left\{\lambda_{t+1}\left[1 + q_{t+1} - \delta + \varphi\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)\frac{K_{t+2}}{K_{t+1}} - \frac{\varphi}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2\right]\right\} = 0;$$

B Optimal Pricing Equations

Equation (18) gives the intermediate firm's optimal price as a function of the ratio of two infinite sums. It is reproduced here for convenience:

$$P_t^*(l) = \left(\frac{\theta}{\theta - 1}\right) \frac{E_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \psi_{t+i}(l) Y_{t+i} (P_{t+i})^{\theta}}{E_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} Y_{t+i} (P_{t+i})^{\theta-1}}.$$

The equation can be simplified by quasi differencing the two infinite sums and by introducing two artificial variables.

First, we drop the (l) argument and divide both sides of the equation by P_t to get

$$\frac{P_t^*}{P_t} \equiv p_t^* = \left(\frac{\theta}{\theta - 1}\right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \psi_{t+i} Y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} Y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}.$$

We now work with the numerator of this transformed equation. Define

$$x_t \equiv \mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \psi_{t+i} Y_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}$$

$$\Rightarrow \lambda_t {P_t}^{\theta} x_t = \mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \lambda_{t+i} \psi_{t+i} Y_{t+i} {P_{t+i}}^{\theta}.$$

Leading this equation by one period, multiplying both sides by $d\beta$, and taking expectations conditional on information available at time t gives

$$d\beta \mathrm{E}_t \left(\lambda_{t+1} {P_{t+1}}^{\theta} x_{t+1} \right) = \mathrm{E}_t \sum_{i=1}^{\infty} (d\beta)^i \lambda_{t+i} \psi_{t+i} Y_{t+i} {P_{t+i}}^{\theta}.$$

Subtracting this equation from the preceding one and simplifying gives

$$x_t = Y_t \psi_t + d\beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} {\pi_{t+1}}^{\theta} x_{t+1} \right\}.$$

Manipulating the denominator in similar fashion gives

$$z_t = Y_t + d\beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} {\pi_{t+1}}^{(\theta-1)} z_{t+1} \right\},$$

with

$$\frac{\theta}{\theta - 1} x_t = p_t^* z_t.$$

C Equation System

The complete system of equations used to simulate the model is given by the following system.

$$\lambda_t = \frac{C_t^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + u_t^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}}; \tag{A-1}$$

$$\left(\frac{u_t C_t}{(M_t/P_t)}\right)^{\frac{1}{\gamma}} = 1 - \frac{1}{R_t}; \tag{A-2}$$

$$\lambda_t = \beta E_t \left(\lambda_{t+1} \frac{R_t}{\pi_{t+1}}\right); \tag{A-3}$$

$$\lambda_t w_t = \frac{\eta}{1 - h_t}; \tag{A-4}$$

$$\lambda_t \left[1 + \varphi \left(\frac{I_t}{K_t} - \delta\right)\right] = \beta E_t \left\{\lambda_{t+1} \left[1 + q_{t+1} - \delta + \varphi \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right) \frac{K_{t+2}}{K_{t+1}} - \frac{\varphi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2\right]\right\}; \tag{A-5}$$

$$q_t = \psi_t (1 - \alpha) \frac{Y_t^s}{K_t}; \tag{A-6}$$

$$w_t = \psi_t \alpha \frac{Y_t^s}{H_t}; \tag{A-7}$$

$$x_t = \psi_t Y_t + d\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}{}^{\theta} x_{t+1}\right); \tag{A-8}$$

$$z_t = Y_t + d\beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}{}^{(\theta-1)} x_{t+1}\right); \tag{A-9}$$

$$p_t^* z_t = \frac{\theta}{\theta - 1} x_t; \tag{A-10}$$

$$Y_t = C_t + I_t + \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t; \tag{A-11}$$

$$K_{t+1} = (1-\delta)K_t + I_t; \tag{A-12}$$

$$Y_t^s = A_t {K_t}^{(1-\alpha)} {H_t}^{\alpha}; \tag{A-13}$$

$$Y_t^s = Y_t S_t; \tag{A-14}$$

$$1 = (1-d) p_t^{*(1-\theta)} + d{\pi_t}^{(\theta-1)}; \tag{A-15}$$

$$S_t = d{\pi_t}^{\theta} S_{t-1} + (1-d) p_t^{*-\theta}; \tag{A-16}$$

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{A,t}; \tag{A-17}$$

$$\ln(M_t) = \ln(M_{t-1}) + \mu_t; \tag{A-18}$$

$$P_t = P_{t-1}\pi_t. \tag{A-19}$$

The endogenous variables are λ_t, C_t, R_t, π_t, w_t, H_t, I_t, K_t, Y_t, Y_t^s, S_t, q_t, ψ_t, p_t^*, A_t, x_t, z_t, M_t and P_t.

D Steady State

The nominal money supply is stationary since we allow for only temporary increases (a positive μ_t is offset by a negative μ_{t+k} after k periods. In the case of a permanent increase in the money supply, there is a different steady state with the same level of all real variables (monetary neutrality) with all nominal variables changing in proportion to the money supply.

We first of all set $h = 1/3$. Equation (A-4) can then be used to back out the value of η compatible with a steady state in which households devote one third of their time endowment to active work. The long-term values of τ, M and A are pinned down by their laws of motion.

Equation (A-3) then pins down the value of the riskless nominal rate of interest:

$$R_t = \frac{1}{\beta}.$$

Given this, the Euler equation for capital (A-5) then gives the steady-state level of q:

$$q = (R - 1) + \tau + \delta.$$

With a stationary price level in the long run, we have

$$p^* = 1$$

and

$$S = 1,$$

which in turn implies that

$$Y = Y^s$$

Equation (A-10) then gives

$$x = \frac{\theta - 1}{\theta} z.$$

This can be used to substitute out x and use equations (A-6), (A-8), (A-9) and

(A-13) to get

$$q = (1-\alpha)\left(\frac{\psi Y}{K}\right);$$

$$\frac{\theta-1}{\theta}z = \left(\frac{\psi Y}{K}\right)\left(\frac{K}{h}\right)h + d\beta\frac{\theta-1}{\theta}z;$$

$$z = \left(\frac{\psi Y}{K}\right)\left(\frac{K}{h}\right)h/\psi + d\beta z;$$

$$\left(\frac{\psi Y}{K}\right) = \psi A\left(\frac{K}{h}\right)^{-\alpha}$$

in transformed variables. The first of the four equations can be used to solve for $\left(\frac{\psi Y}{K}\right)$. The last equation can be used to substitute out ψ to get

$$\psi = \left(\frac{\psi Y}{K}\right) = \left(\frac{K}{h}\right)^{\alpha}/A.$$

Equations (A-8) and (A-9) together give

$$\psi = \frac{\theta}{\theta-1} = \left(\frac{\psi Y}{K}\right) = \left(\frac{K}{h}\right)^{\alpha}/A$$

$$\Rightarrow \left(\frac{K}{h}\right) = \left(\frac{\theta}{\theta-1}A/\left(\frac{\psi Y}{K}\right)\right)^{\frac{1}{\alpha}}$$

Figure 5: Response of Prices

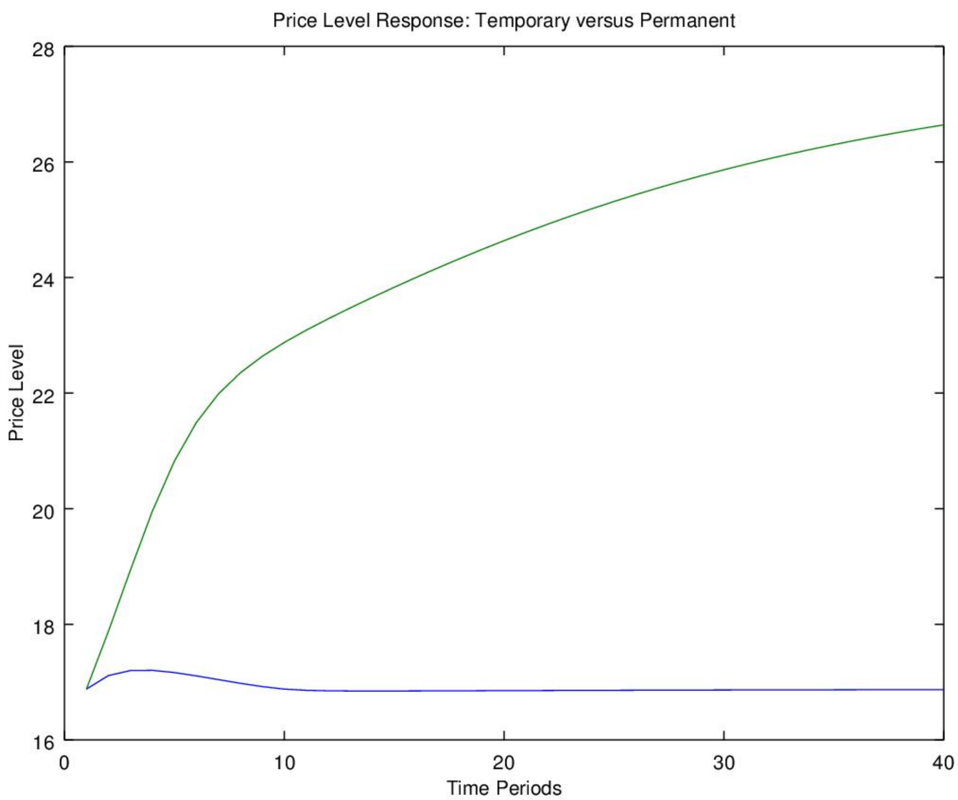

Figure 6: Response of Inflation

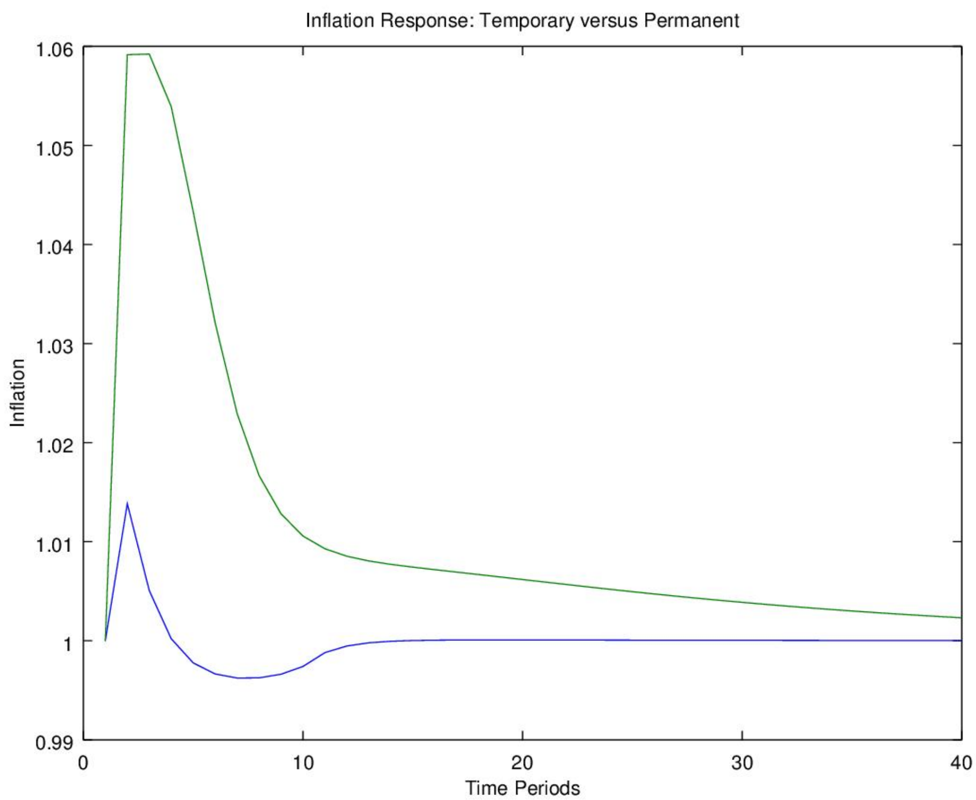

Figure 7: Response of the Interest Rate

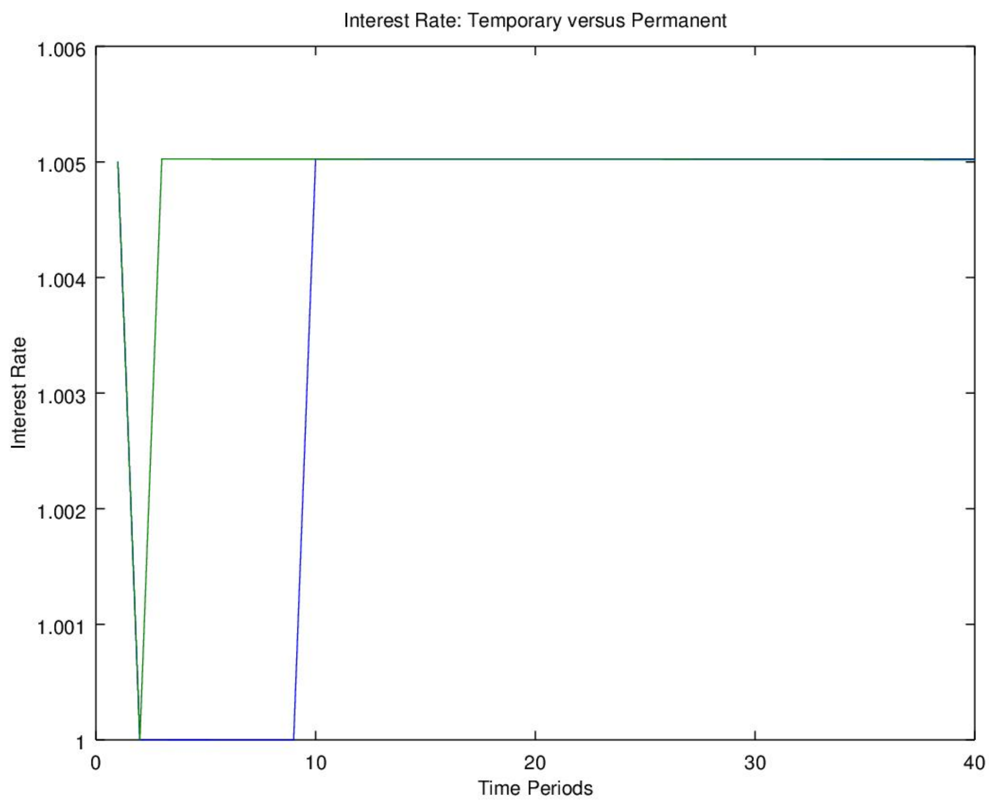

Figure 8: Response of Consumption

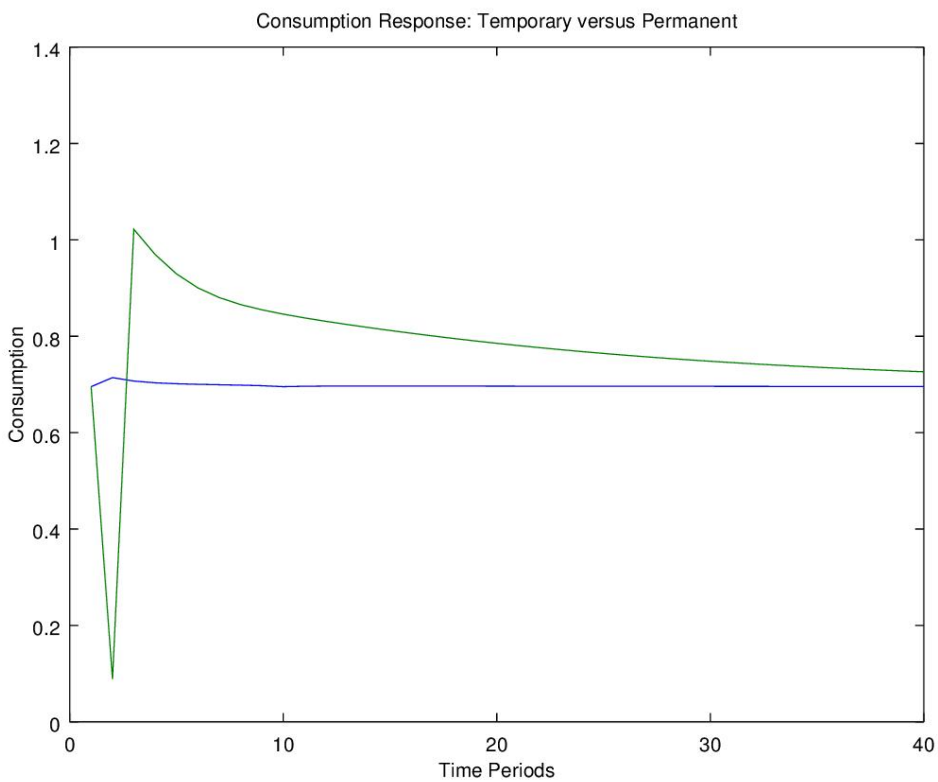

Figure 9: Response of Output

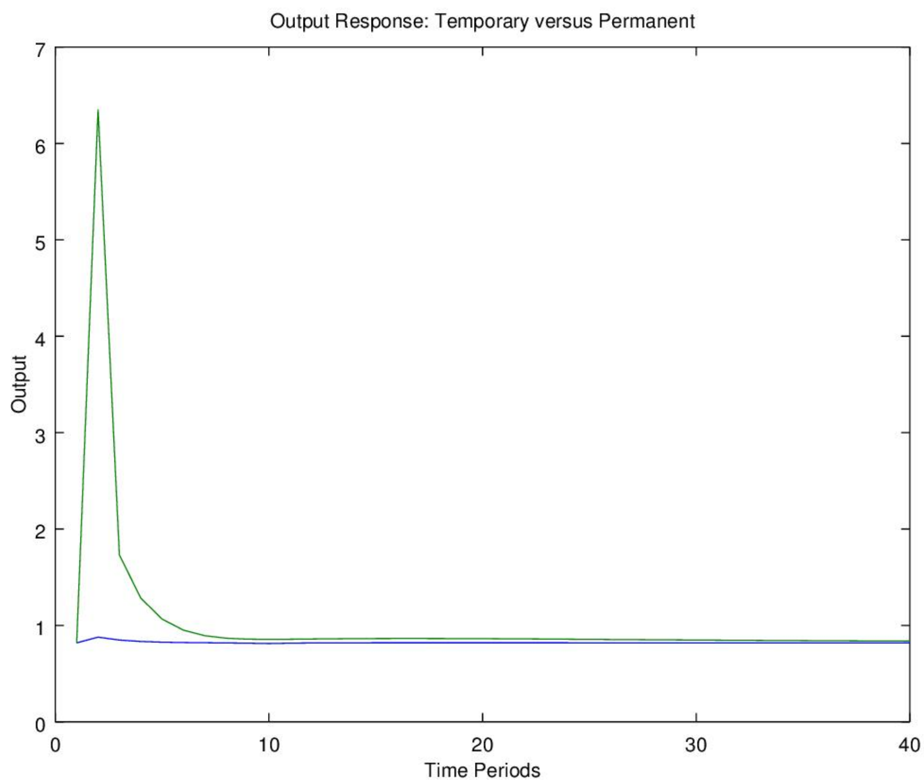

Figure 10: Response of Velocity