Unemployment and Nominal Volatility at low frequency: a Bayesian TVP-VAR approach

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Abstract

This paper empirically investigates the relationship between unemployment and nominal volatility at low frequency. Starting from a stylized economy featured by downward nominal wage rigidity, a long-run Phillips curve which positively relates expected unemployment to inflation volatility is derived. The relation is tested by estimating a time-varying parameters VAR with Bayesian techniques for each of the G7 countries. Taking two alternative proxies for nominal volatility, namely the Economic Policy Uncertainty (EPU) index and the CPI inflation volatility, their correlation at low-frequency with unemployment is detected through the spectral analysis of the estimates. The empirical evidence suggests that the unconditional mean of unemployment positively co-moves i) with the unconditional mean of EPU index for all G7 countries, but Germany, and ii) with the spectrum of CPI inflation at low frequency for all G7 countries, but United Kingdom.

Keywords: Long-run unemployment, Volatility, Bayesian VAR, Time-varying coefficients, Frequency domain

JEL codes: E23 E31, E52, C51

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1. Introduction

The dynamics affecting the economy in the short-run is generally studied separately from the one prevailing in the long-run. The separation comes naturally from the fundamentals of the Neoclassical Synthesis, which claim that the frictions preventing nominal variables from fluctuating around their secular trends, hold only temporarily. In the long-run the frictions should vanish, thereby prices and wages are free to fluctuate clearing the markets. This results is not guaranteed when institutional frictions affect prices and wages permanently. In such a case, still in the long-run the dynamics of real variables depends on the processes leading the nominal variables. For the real variables concerning the labor market for instance, it means that expected levels prevailing in the long-run are affected by both level and volatility of the processes leading prices and wages. In this respect, a whole strand of literature starting with Tobin (1972) and Akerlof et al. (1996, 2000), stresses on the greasing effects of a nominal variable as price inflation on labor market outcomes.[1] The most of this literature focuses on the relation between the *level* of the inflation, and the *level* of the labor margins. This paper wishes instead to investigate on the long-run relation between the *volatility* of the nominal variables and the *level* of labor margins.

In particular, this paper wishes to gain the evidence on the claim that, for an economy affected by some frictions that make nominal wages downwardly rigid, nominal volatility matters for the average levels of long-run unemployment. This claim lies on two simple premises. The first one is that nominal volatility directly affects the likelihood of nominal wages of being constrained. The more nominal wages are volatile, the more they are likely to be downwardly constrained. The second one is that labor margins have to compensate to clear the market, whenever nominal wages are not free to adjust. It follows that, the more nominal wages are constrained, the more often unemployment has to compensate the missing fall of nominal wages. In a long-run perspective, it means that high volatile nominal wages should imply *on average*, higher levels of unemployment.

This implication is firstly verified in the paper by developing a very stylized Neo Keynesian model in which the volatility of nominal wages is exclusively driven by the volatility of prices. This model economy assumes that i) labor market institutions prevent nominal wages from falling and ii) price growth dynamics is exogenously driven by the monetary authority. As a model result, a long-run Phillips curve relating positively unemployment with the price inflation volatility is derived. This kind of relation is then investigated on the data among the G7 countries. This country sample allows to restrict

[1] More recently, some contributions that stress on the long-run positive relation between inflation and employment are Benigno and Ricci (2011), Kim and Ruge-Murcia (2008), Fagan and Messina (2009), Daly and Hobijn (2014).

the analysis to developed economies, for which microeconomic literature has widely documented the presence of some nominal rigidities in the labor market.[2] As an important novelty, the nominal volatility is measured in the paper not only by the CPI inflation volatility, but also by an alternative and more generic proxy, i.e. the Economic Policy Uncertainty index (EPU index henceforth) provided by Baker et al. (2015). **This index is provided for different countries and measures the perceived volatility about the economic policies**.[3]

More in details, the empirical strategy followed in the paper consists in two steps. First, for each of the G7 countries a vector auto-regression model with time varying parameters (TVP-VAR model henceforth) is estimated using Bayesian methods *à* la Cogley and Sargent (2005) and Cogley et al. (2005). Second, by applying the spectral analysis, the estimated time-varying variances and covariances of the endogenous variables are studied by frequencies. Actually the low-frequency co-movement between unemployment and nominal volatility is detected differently according to which proxy of nominal volatility is considered. Two specifications are taken for the TVP-VAR model. In the first specification, the TVP-VAR is estimated with EPU index, unemployment and nominal wage inflation as endogenous variables. The relation between unemployment and EPU index is detected both i) by comparing the time-varying unconditional means of the estimated series, and ii) by evaluating some measures of co-movement provided by the spectral analysis that are defined over the frequency domain. On one side, by capturing the drift components of the endogenous variables, the unconditional means well-explain the long-run dynamics of the same variables. On the other side, by disentangling the share of variances and covariances that can be associated to different frequency components, the spectral analysis allows to evaluate among others, the long-run correlation between endogenous variables. Importantly, the use of spectral analysis in these terms needs a *direct* proxy of nominal volatility as endogenous variable of the TVP-VAR model. This motives the novelty of choosing the EPU index as endogenous variable along with the unemployment rate. In the second specification, the TVP-VAR is estimated with CPI inflation, unemployment and nominal wage inflation as endogenous variables. In this case, following Cogley and Sargent (2005) and many others,[4] the

[2]For the evidence in the United States, some contributions investigating on the rigidities in nominal wages are Akerlof et al. (1996), Kahn (1997), Card and Hyslop (1997), Altonji and Devereux (2000), Lebow et al. (2003), Elsby (2009), Fagan and Messina (2009), Kim and Ruge-Murcia (2009) and Daly et al. (2012). For an international evidence on the wage rigidities in advanced economies, see among others Holden (2004), Dickens et al. (2007), Holden and Wulfsberg (2008), Knoppik and Beissinger (2009), Messina et al. (2010) and Babecky et al. (2010).

[3]One of the principal components of the EPU index is related to the use of some specific words in the articles of the main newspapers for the country considered. All these words have in common a meaning that is related to the concept of volatility.

[4]See for instance Benati and Mumtaz (2007), Benati and Surico (2008), Benati (2009), Bianchi et al. (2009) among others.

measure of volatility is recovered *indirectly* through the spectrum of the CPI inflation. The long-run relation between unemployment and nominal volatility is then **detected** by comparing over the sample period, the unconditional mean of unemployment and the spectrum at low frequency of CPI inflation.[5]

Remarkably, the empirical analysis are instructive for all the G7 countries. Although the heterogeneity in the length of the sample periods implies some caution in comparing the experiences of the G7 countries, the evidence is clearly in favor of a positive relation at low frequency between unemployment and nominal volatility. For the first specification of the TVP-VAR model, the long-run measures of both unemployment and EPU index are positively correlated for the G7 countries, with the only exception of Germany. Similarly for the second specification of the TVP-VAR model, the long-run measures of both unemployment and CPI inflation volatility are positively correlated for all G7 countries, but the United Kingdom. For the United States in particular, the positive long-run co-movement between unemployment and CPI inflation volatility is supported by the data only from the onset of the Great Moderation on, that is from a period when the lower levels of inflation have reasonably made the nominal wage downward rigidity more binding. This result, in particular **sustains** the theoretical model that finds a long-run relation between unemployment and nominal volatility insofar the nominal wages are effectively rigid.

More broadly, the focus on the long-run relation between real variables and macroeconomic volatility is shared with other contributions in the literature. However, some of them as Hairault et al. (2010) and Benigno et al. (2015), focus on the effects on labor margins, by considering only the real volatility.[6] Others instead, as Fischer (1993), Clarida et al. (1999), Judson and Orphanides (1999), consider the nominal volatility, but they study only the effects on real output.[7] By investigating then on the co-movements at low frequency between nominal volatility and the unemployment rate, this paper contributes to shed the light on a specific relation that has been less discussed in the literature so far.

The rest of the paper proceeds as follows. Section 2 describes a stylized economy featured **by prices dynamics exogenously determined**. Section 3 describes the TVP-VAR model estimated via Bayesian techniques and the measures of co-movement deriving from the spectral analysis. Sections 4 studies the estimated behavior of unemployment and nominal volatility resulting from the two TVP-VAR specifications for all the G7 countries. The section comments the results obtained firstly with US data,

[5] A similar analysis is proposed by Benigno et al. (2015), who however consider the spectrum of productivity growth along with the unconditional mean of unemployment.

[6] Starting from different theoretical models, both Hairault et al. (2010) and Benigno et al. (2015) find a positive relation between the long-run level of unemployment and the volatility of productivity.

[7] Clarida et al. (1999) study the role of inflation volatility by using US data, whereas Fischer (1993) and Judson and Orphanides (1999) provide an international comparison on the effects of inflation volatility on real growth, by using cross-country data.

secondly with Canada, UK and Japan data and thirdly with France, Germany and Italy data. Section 5 sums up and concludes.

2. A simple economy with nominal wage rigidity

This section discusses the long-run implications of a stylized economy featured by downwardly rigid nominal wages and positive trend inflation. These elements are applied to a basic New Keynesian model by assuming the presence of i) a social norm in the labor market that prevents nominal wages from falling, and of ii) a monetary authority which is able to pursue a positive target on inflation, unless unpredictable shocks.[8] Given the presence of the social normal, current nominal wages W_t are not allowed to fall. Their dynamics is limited by the following non-decreasing constraint,

$$d \ln W_t \geqslant 0 \ , \tag{1}$$

Given the inflation targeting adopted by the monetary authority, the price level P_t is assumed to oscillate around a positive drift. Its dynamics is described by the following geometric Brownian motion,

$$d \ln P_t = \mu dt + \sigma_P dB_{P,t} \ , \tag{2}$$

where μ corresponds to the positive drift. Variability of the price inflation is determined by the volatility coefficient σ_P, which actually states how large the effects on price growth are of a zero mean normally distributed shock $B_{P,t}$.

The rest of the model is very basic in the Neo Keynesian fashion. A representative household with a continuum of infinitely lived members is assumed to derive utility from consuming goods C_t, and disutility from supplying labor L_t.[9] A representative firm is assumed to produce consumption goods by employing labor through the technology $C_t = L_t^\alpha$. The production function exhibits decreasing returns to scale at degree α -assumed lower than 1-. **Both goods and labor market are perfect competitive. Labor demand schedule then equates the nominal wage to the value of marginal labor productivity,**

$$W_t = \alpha P_t L_t^{\alpha-1} \ . \tag{3}$$

[8] The theoretical model here considered is a simplified version of Fasani (2016). More in general, the framework follows Benigno, Ricci (2011) and Benigno et al. (2015). It basically differes from them by the assumption of a distinct process on price inflation that isolates the nominal dynamics of the economy.

[9] As in Fasani (2016), in each period t household is assumed to face an utility function like the following one,

$$U_t = \ln C_t - \frac{{L_t}^{1+\eta}}{1+\eta} \ ,$$

where η is the inverse of the Frisch elasticity.

Labor supply schedule is not uniquely determined because of the occasionally binding constraint (1). The social norm preventing nominal wage cuts is assumed to intervene ex-post, that is after the workers have optimally chosen the wage level.[10] **Whenever optimal nominal wage is increasing, i.e. $dW_t^* > 0$, the current nominal wage at t is equal to that level, that is to the marginal rate of substitution between consumption and leisure.**

$$W_t = P_t \lambda_t^{-1} L_t^{\eta} \ , \tag{4}$$

where λ_t is the marginal utility with respect to consumption and η -assumed higher than 1- is the inverse of the elasticity of labor supply with respect to the nominal wage. In such a case equations (3) and (4) equates, thereby labor market clears at employment level $L^e = \alpha^{\frac{1}{1+\eta}}$. Differently, whenever the optimal nominal wages are decreasing, i.e. $dW_t^* < 0$, the social norm binds and the current nominal wages remain stuck at the level of the previous period.

$$W_t = W_{t-1} \ , \tag{5}$$

In this second case, nominal wage at t is higher than the optimal level. As a consequence, a gap between the labor supply prevailing with flexible wages and the labor demand emerges. This gap corresponds to the unemployment rate indicated by Gali (2011), provided that here no monopolistic competition in labor market is admitted and then unemployment without nominal rigidity is null. Rewriting equations (3) and (4) in logs, unemployment is so defined,

$$u_t = \chi \left(\ln W_t - \ln P_t \right) \ , \tag{6}$$

with $\chi \equiv \frac{1+\eta}{\eta(1-\alpha)}$ -assumed higher than 1 because of α and η-. Taking the first differences of (6), the implications of the two possible scenarios are easily detected. When current nominal wages are free to fluctuate, that is when $dW_t > 0$, they exactly off-set the price changes and unemployment is null. Conversely, when current nominal wages are rigid, i.e. $dW_t = 0$, it is the unemployment margin that compensates the price changes. When unemployment is above the zero-level prevailing with flexible wages, it follows a geometric Brownian motion with drift $-\chi\mu$ and volatility coefficient $-\chi\sigma_P$. The non-negative barrier for unemployment ensures that it actually follows a regulated

[10] In a richer model Fasani (2016) shows that theoretical findings for long-run dynamics of unemployment do not change substantially whether workers are assumed to take care of the norm in choosing the optimal wage. In that case, workers will be simply more conservative in adjusting upward the optimal wages, knowing that their choices affect the likelihood of future cuts in the optimal level. Worker prefer to limit wage increases in order to reduce the probability that social norm binds and unemployment rises.

Brownian motion. Given the negative sign of the drift, standard results guarantee a stationary distribution for unemployment that looks like the following,[11]

$$f(x) = \frac{2}{\chi}\frac{\mu}{\sigma_P^2}e^{\frac{2}{\chi}\frac{\mu}{\sigma_P^2}x} \ . \tag{7}$$

From this invariant distribution (7), it is straightforward to derive the expected value for long-run unemployment,

$$E\left[u_\infty\right] = \frac{\chi}{2}\frac{\sigma_P^2}{\mu} \ , \tag{8}$$

which depends positively on inflation volatility and negatively on inflation drift. Equation (8) can be read as a long-run Phillips curve relating unemployment to inflation. Differently from a standard Phillips curve, in addition to a *relation* between the levels of the two variables, it still predicts a *cross-relation* between level and volatility of the same variables. This further relation basically motives the empirical analysis below.

This model shows that a very basic framework with simple assumptions on wages and prices, is able to prescribe a positive association between the unemployment level prevailing in the long-run and the inflation volatility, which generally is considered only in a business cycle perspective. Intuitively, the presence of downward nominal rigidity implies that a high volatility in price inflation makes nominal wages more volatile too and then on average, more affected by those rigidities. A higher volatility coefficient σ_P implies that, for any negative realization of the normally distributed zero-mean price shock, nominal wages are more likely to hit the lower constraint and unemployment is more likely to be positive. In average terms, a positive relation between unemployment and inflation volatility in a longer horizon comes out naturally. Although the relation is not exclusive to this specific kind of volatility,[12] the assumption of a process with non-negligible volatility on price inflation has the advantage to specify easily the source of nominal variability in an otherwise standard Neo Keynesian model. The empirical analysis developed in the next sections will be however more agnostic on this point. In addition to the CPI inflation volatility, it will also consider a wider measure of nominal volatility, namely the Economic Policy Uncertainty.

[11] See for more details Harrison (1985).

[12] Benigno and Ricci (2011) for instance, find a positive relation between expected long-run unemployment and nominal spending volatility. In their framework, an exogenous process with non-negligible volatility is assumed to hold for nominal spending and not for price inflation. By definition however, nominal spending includes both real and nominal components, so differently from the model here developed, an analysis of the single contribution of nominal volatility is prevented.

3. The empirical analysis

The third part of the paper introduces the empirical analysis. The outline is the following. Sections 3.1 describes the time-varying parameters VAR model estimated with Bayesian methods, and gives a brief overview on the data used. Section 3.2 illustrates some measures of co-movement coming from the spectral analysis of the times series. In the Appendix, further details about the Bayesian estimation and the spectral theory are provided. Appendix A.1 discusses the Markov chain Monte Carlo algorithm used to evaluate the posterior distribution. Appendix A.2 presents in-depth the data used in the estimations. Appendix A.3 gives an interpretation of some spectral quantities defined over the frequency domain in terms of leads and lags in the time domain.

Before moving to the empirical analysis, it is worth to emphasize that it contemporaneously proceeds along two dimensions, namely the time and the frequency dimension. With the time dimension, the analysis wishes to take into account how the patterns of unemployment and nominal volatility have evolved during the sample periods considered. A VAR version with time varying parameters and heteroskedastic covariance matrix is indeed estimated to get some information about the relation between unemployment and nominal volatility at each point of time. Meanwhile, the introduction of the frequency domain allows to differentiate between long- and short-run components of the estimated series. In this way, it is possible to focus the analysis only on the relation between the long-run components of unemployment and nominal volatility.

3.1 A time-varying parameters VAR model

This paper estimates through Bayesian techniques a VAR quarterly model with time-varying parameters to get the posterior distributions of several objects. From the model estimation, some approximations of trend and volatility of the endogenous variables are recovered. For the former, as explained in this section, the estimates guarantee a local linear approximation to the variable mean, evaluated at the posterior mean of VAR coefficients. This quantity, known in literature as the unconditional mean of the endogenous variables, is time-varying given the VAR parameters and allows to take care of the drift component of the variables. Some measures of variance and covariance between the endogenous variables are instead provided through the spectral analysis of the estimates. These measures are however explained in Section 3.2.

The linear multivariate model here estimated, admits time variation for both VAR parameters and VAR innovations. This assumption allows to take care of respectively, the lag structure changes and the shock heteroskedasticity. The measurement equation of the model is the following, for all $t = 1, ...T$,

$$\begin{aligned} Y_t &= B_{0,t} + B_{1,t}Y_{t-1} + ... + B_{p,t}Y_{t-p} + \varepsilon_t \\ &= X_t'\theta_t + \varepsilon_t \ , \end{aligned} \tag{9}$$

where Y_t is a $N \times 1$ vector containing the endogenous variables; $X_t' \equiv I_N \otimes [1, Y_{t-1}', ..., Y_{t-p}']$ is a matrix collecting the first p lags of Y_t; $\theta_t \equiv vec\,(B_{0,t}, B_{1,t}, ..., B_{p,t})$ is a vector stacking the time-varying $N \times 1$ vector $B_{0,t}$ and the $N \times N$ matrices $B_{s,t}$, with $s = 1, ..., p$; ε_t is a $N \times 1$ vector of reduced-form residuals, which are assumed independent and normally distributed, with zero mean and covariance matrix Ω_t, that is $\varepsilon_t \overset{iid}{\sim} N\,(0, \Omega_t)$. The order p of TVP-VAR model is 2, as commonly assumed in the related literature.

The first source of time variability derives from the autoregressive parameters of the measurement equation, which are assumed to evolve as,

$$p\,(\theta_t|\theta_{t-1}, Q) = I\,(\theta_t)\, f\,(\theta_t|\theta_{t-1}, Q)\ . \tag{10}$$

The law of motion of lag parameters depends on their past values and on the covariance matrix Q of the parameter innovations. Since the explosive representations for the endogenous variables are excluded, parameters dynamics faces a reflecting barrier given by the indicator function $I\,(\theta_t)$. This function assumes the value of 0 when the draws are unstable, that is when the roots of the associated VAR polynomial are inside the unit circle. Whenever the stationary of the VAR is preserved, lag coefficients are free to move as $f\,(\theta_t|\theta_{t-1}, Q)$ dictates, that is as

$$\theta_t = \theta_{t-1} + \nu_t\ , \tag{11}$$

where the parameter innovations ν_t are assumed to be independent and normally distributed, with zero mean and covariance matrix Q, that is $\nu_t \overset{iid}{\sim} N\,(0, Q)$.

The second source of time variability derives from the covariance matrix Ω_t of the measurement equation innovations, which is modelled adopting a multivariate version of the stochastic volatility model of Jacquier et al. (1994). Time-varying matrix Ω_t is factored as follows,

$$\begin{aligned} \Omega_t &= A^{-1} H_t \left(A^{-1}\right)' \\ &= A^{-1} H_t^{1/2} \epsilon_t \epsilon_t' \left(H_t^{1/2}\right)' \left(A^{-1}\right)'\ , \end{aligned} \tag{12}$$

where ϵ_t are standard normal innovations of the measurement equation, i.e. $\epsilon_t \sim (0, I)$, and matrices A and H_t are defined as,

$$A = \begin{pmatrix} 1 & 0 & 0 \\ \alpha_{2,1} & 1 & 0 \\ \alpha_{3,1} & \alpha_{3,2} & 1 \end{pmatrix} \quad \text{and} \quad H_t = \begin{pmatrix} h_{1,t} & 0 & 0 \\ 0 & h_{2,t} & 0 \\ 0 & 0 & h_{3,t} \end{pmatrix}\ . \tag{13}$$

As in Cogley and Sargent (2005) and Cogley et al. (2005), contemporaneous relations among variables are time-invariant. The free elements of matrix A are thereby modelled as constant, stating that simultaneous correlation between endogenous variables does not change over time. The assumption is not trivial, because it implies that a variable innovation has a time invariant impact over the other variables. But since the focus of the paper is on the long-run relations, such limitation should not seriously affects the low frequency dynamics. It helps instead to contain the otherwise much higher number of parameters to be estimated. The matrix H_t includes the time-varying univariate stochastic volatilities along the main diagonal. Vector h_t in particular, stacks these stochastic volatilities. Its dynamics is specified by the following random walk process,

$$\ln h_t = \ln h_{t-1} + \eta_t \ , \tag{14}$$

where η_t is a vector of independent zero-mean innovations normally distributed with a diagonal covariance matrix, whose elements are σ_i, for $i = 1, ..., N$. For model tractability, it is assumed that state and measurement innovations are all uncorrelated among each other, that is $E_t [\epsilon_t \eta_t] = E_t [\epsilon_t \nu_s] = E_t [\eta_t \nu_s] = 0$ for all t and s.

This assumption greatly simplifies the Markov chain Monte Carlo algorithm used to simulate the posterior distribution. As better explained in the Appendix A.1, the algorithm simulates the posterior density taking as given the priors and the history of observations Y^T. Such a posterior can be written as,

$$p\left(\theta^T, Q, \alpha, h^T, \sigma | Y^T\right) \ , \tag{15}$$

where θ^T represents the history of the time-varying measurement equation parameters, Q the covariance matrix of parameter innovations, α the vector containing the free elements of matrix A, h^T the history of the time-varying vector containing the diagonal elements of matrix H_t, σ the vector containing the variances of volatility innovations.

As a first outcome of model estimation, it is recovered a measure of the drift level of each endogenous variable. As in Cogley and Sargent (2005), the time series of the variable drifts are derived as local-to-date t approximations to variable mean evaluated at the posterior mean $E(\theta_{t|T})$. More clearly, once the model has been estimated, the measurement equation is rewritten in companion form,

$$Z_t = C_{t|T} + D_{t|T} Z_{t-1} + \xi_t \ , \tag{16}$$

where Z_t contains the current and the first $p - 1$ lag values of Y_t, while $C_{t|T}$, $D_{t|T}$, and ξ_t are respectively the time-varying intercepts, time-varying autoregressive coefficients and time-varying innovations written in conformable way. Managing on equation (16), unconditional means of the variables are computed as

$$E(Z_t) = (I - D_{t|T})^{-1} C_{t|T} \ , \tag{17}$$

where the N elements of vector $E(Z_t)$ are interpreted as the drifts of the endogenous variables, according to the order assigned in the VAR specification.

As mentioned above, two specifications of the TVP-VAR model are run, differing between for one variable. The vectors of endogenous variable in the two cases are respectively, $Y_t^a = [EPU_t,\ u_t,\ \pi_t^W]'$ and $Y_t^b = [\pi_t^P,\ u_t,\ \pi_t^W]'$. EPU_t indicates the EPU index, u_t the unemployment rate, π_t^W the growth rate of nominal wages, π_t^P the growth rate of the CPI index. Data sources for these variables change according to the country considered, with the only exception for the EPU index, whose monthly series are collected for all G7 countries from the database at www.PolicyUncertainty.com. Quarterly data for unemployment, price and wage inflation are instead collected from the FRED database for US, and from the OECD database for the rest of the G7 countries. More details about the data are given in the Appendix A.2.

3.2 Co-movements in the frequency domain

Introducing the frequency domain in the empirical analysis allows to disaggregate the overall dynamics of economic variables into singular cycle components of different duration. This paper is particularly interested in the contribution of the components explaining the long-run behavior of the economic variables. Variability and cross-variability of endogenous variables are evaluated through some measures of long-run co-movements provided by the spectral analysis. To get these measures, elements like spectrum and cross-spectrum of the variables need to be defined. **Roughly speaking**,[13] the spectrum of a variable over an interval of frequencies corresponds to the portion of the variance of that variable, which can be associated to that interval of frequencies. Analogously, the cross-spectrum between two variables over an interval of frequencies corresponds to the portion of the covariance between the same variables, which can be associated to that interval of frequencies. In this paper, spectrums and cross-spectrums of the endogenous variables are not directly estimated, but are recovered indirectly from the TVP-VAR model estimates.[14] Once spectrums and cross-spectrums are defined as in the theory, they are evaluate at the VAR estimates.

More in details, spectrums and cross-spectrums of a N-dimension random process $\{Y_t\}_{t=-\infty}^{\infty}$derive from the Fourier transform of the covariance matrix $\mathbf{\Omega}(s)$ of the residuals. This matrix is assumed to be absolutely summable for any lead and lag s, with $-\infty \leqslant s \leqslant \infty$. For each frequency ω, such transformation gives a square complex matrix $\mathbf{S}(\omega)$ of order N with the spectrums $s^{kk}(\omega)$, with $k = 1, ..., N$, as diagonal elements and the cross-spectrums $s^{kj}(\omega)$, with $k, j = 1, ..., N$ but $k \neq j$, as off-diagonal elements.

[13] For a deeper discussion on the spectral theory and its measures of co-movements see among others, Grenger and Hatanaka (1964), Priestley (1981), Sargent (1987), Hamilton (1994).

[14] For example Mallick (2015) studies the long-run Phillips curve with Australian data, by estimating directly some spectral quantities as coherence, gain and phase between unemployment and inflation rate.

The matrix $\mathbf{S}(\omega)$ is so defined as,

$$\mathbf{S}(\omega) = \frac{1}{2\pi} \sum_{s=-\infty}^{+\infty} \mathbf{\Omega}(s) e^{-i\omega s} \ , \tag{18}$$

For the covariance-stationary TVP-VAR model considered in this paper, the following local linear approximation of matrix $\mathbf{S}(\omega)$ is considered,[15]

$$\mathbf{S}_{t|T}(\omega) = (I - D_{t|T} e^{-i\omega})^{-1} \frac{\Omega_{t|T}}{2\pi} \left[(I - D_{t|T} e^{-i\omega})^{-1} \right] \ , \tag{19}$$

where according to the notation above, $D_{t|T}$ and $\Omega_{t|T}$ are respectively, the matrix of the estimated autoregressive coefficients and the matrix of the estimated residual covariances. The diagonal elements of $\mathbf{S}_{t|T}(\omega)$ are real numbers corresponding to the spectrums, while the off-diagonal elements are complex numbers corresponding to the cross-spectrums. The generic cross-spectrum between variables k and j is indeed complex-valuated with a real and an imaginary component as follows,

$$s^{kj}(\omega) = co_{t|T}^{kj}(\omega) + i \cdot qu^{kj}(\omega) \ . \tag{20}$$

The real component $co_{t|T}^{kj}(\omega)$ is named *co-spectrum*, whereas the imaginary component $qu_{t|T}^{kj}(\omega)$ is named *quadrature spectrum.* Alternatively, the cross-spectrum can be rewritten in polar form,

$$s^{kj}(\omega) = R^{kj}(\omega) e^{i\theta^{kj}(\omega)} \ , \tag{21}$$

where $R^{kj}(\omega)$ and $\theta^{kj}(\omega)$ are respectively the *amplitude spectrum* and the *phase spectrum*, which are defined as

$$\begin{aligned} R^{kj}(\omega) &\equiv \left(\left(co^{kj}(\omega) \right)^2 + \left(qu^{kj}(\omega) \right)^2 \right)^{1/2} \ , \\ \theta^{kj}(\omega) &\equiv \tan^{-1} \left(\frac{qu^{kj}(\omega)}{co^{kj}(\omega)} \right) \ . \end{aligned}$$

By integrating spectrums and cross-spectrums over all the frequency domain, one obtains the unconditional covariance matrix of the endogenous variables. The diagonal elements of this matrix, i.e. the unconditional variances, can be used to normalize the

[15] According to the spectral theory, the matrix $\mathbf{S}(\omega)$ for a stationary VAR process of order p can be written as,

$$\mathbf{S}(\omega) = \left[I - B_{1,t} e^{-i\omega} - ... - B_{p,t} e^{-ip\omega} \right]^{-1} \frac{\Omega}{2\pi} \left[I - B_{1,t}' e^{-i\omega} - ... - B_{p,t}' e^{-ip\omega} \right] \ .$$

variables spectrums at different frequencies.[16] Given however the interest on long-run dynamics, the paths of unconditional variances and covariances are not *per se* particularly meaningful for the aim of the paper. More attention is instead reserved to the estimated covariances at low frequencies, which are detected by considering only the elements of matrix $\mathbf{S}_{t|T}$ with cycle duration of at least 8 years. Henceforth, any element of matrix $\mathbf{S}_{t|T}$ at *low frequency* refers to the average value of that element over the frequencies with cycle duration of 8 years and over. Conversely, any element of matrix $\mathbf{S}_{t|T}$ at *high frequency* refers to the average value of that element over the frequencies with cycle duration of less than 8 years. Once having recovered spectrums and cross-spectrums at high and low frequency, the co-movements between endogenous variables are evaluated by four measures.

The first one is the *coherence*,[17] which is defined as,

$$coh_{t|T}^{kj}(\omega) = \frac{\left(co_{t|T}^{kj}(\omega)\right)^2 + \left(qu_{t|T}^{kj}(\omega)\right)^2}{s_{t|T}^{kk}(\omega)\, s_{t|T}^{jj}(\omega)} = \frac{\left|s_{t|T}^{kj}(\omega)\right|^2}{s_{t|T}^{kk}(\omega)\, s_{t|T}^{jj}(\omega)} \; . \tag{22}$$

The coherence $coh_{t|T}^{kj}(\omega)$ is real-valued and symmetric with respect to the frequencies. As it indicates how much of the variance of one variable at a given frequency ω, is explained by the variance of another variable at the same frequency ω, it can be interpreted as the coefficient of determination or R^2 between the two variables k and j. For any given non-zero spectrums of variables k and j, the coherence lies in the interval $[0, 1]$. High values of coherence suggest that the two variables are well-correlated at a given frequency, although no indications are provided regarding to the sign of correlation.

On the latter point, it is more instructive a second index of co-movement, i.e. the *dynamic correlation* proposed by Croux et al. (2011).[18] The dynamic correlation is a real-valued and symmetric index like the coherence, but the values that it can assume lay between -1 and 1. In formula, the dynamic correlation $\rho_{t|T}^{kj}(\omega)$ is

$$\rho_{t|T}^{kj}(\omega) = \frac{co_{t|T}^{kj}(\omega)}{\left(s_{t|T}^{kk}(\omega)\right)^{1/2}\left(s_{t|T}^{jj}(\omega)\right)^{1/2}} \; . \tag{23}$$

[16] For any given frequency ω, the normalized spectrums are the diagonal elements of the following matrix,

$$\mathbf{S}_{t|T}^{N}(\omega) = \frac{\mathbf{S}_{t|T}(\omega)}{\int_{-\pi}^{\pi} \mathbf{S}_{t|T}(\omega)\, d\omega} \; .$$

[17] Sometimes in the literature, it is called as *squared coherence*.

[18] Other contributions that analyze the co-movement between two variables in the frequency domain through the dynamic correlation index are Tripier (2002, 2006) and more recently, of Akram and Mumtaz (2015).

Values of dynamic correlation near to the bound levels, suggest a close relation between the two variables at a given frequency. Such relation can be positive or negative according to the sign of $\rho^{kj}_{t|T}(\omega)$. The relation is positive for values of dynamic correlation near to 1 and negative for values near to -1. Contrary to the coherence, the dynamic correlation is dependant upon the time lag between the series one wants to analyze.[19] However, it is not informative about the lead-lag relationship between the two variables.

To get some insights into it, a third measure needs to be introduced. It is the *phase spectrum* $\theta^{kj}_{t|T}(\omega)$ defined above, which is used in time series analysis to interpret the angle difference between the sinusoids describing two variables at a given frequency, in terms of leads and lags in the time domain of one variable with respect to another.[20] As a periodic function however, the phase is uniquely determined only within the radian interval $[-\pi, \pi]$. It follows that, for frequency range wider than that interval, the phase assumes the same value for more frequencies. This implies some ambiguity in the interpretation of the phase.[21] Remarkably, both Grenger and Hatanaka (1964) and Engle (1967) emphasize that this limit can be ignored whether the analysis is focused on the low frequencies.

As the phase, still the fourth measure considered, i.e. the *gain*, is informative about which kind of relation exists between two variables. The gain *from* variable j to variable k, is defined as the ratio among the amplitude between k and j and the spectrum of k,

$$G^{k}_{t|T}(\omega) = \frac{R^{kj}_{t|T}(\omega)}{s^{kk}_{t|T}(\omega)} \ . \tag{24}$$

The gain can be interpreted as the absolute value of the OLS-regression coefficient of one variable with respect to another. Basically, it allows to evaluate how much of the behavior of one variable at a given frequency is explained by the behavior of the other at the same frequency. However, given that it is real- and positive-valued, no hints are provided about the sign of the relation between the variables.[22]

In the rest of the paper the measures here defined are applied to the estimates of the TVP-VAR model. A *quantitative* evaluation of the relation between long-run unemployment and nominal volatility is feasible with the first specification of the TVP-VAR

[19] More specifically, as Tripier (2002) emphasizes, the dynamic correlation is affected by the absolute value of the time shift between two series, that is $\rho_{t,kj}(\omega) \neq \rho_{t,kz(\tau)}(\omega)$ where $z_t(\tau) = j_{t-\tau}$, but not by the direction of the shift, that is it holds $\rho_{t,kz(\tau)}(\omega) = \rho_{t,kz(-\tau)}(\omega)$.

[20] Appendix A4 shows how real variables can be represented by a sum of periodic functions over the frequency domain and argues on the correspondence between the cross-spectrum phase and the angle difference.

[21] More clearly, adding or subtracting an entire cycle does not modify the value assumed by the phase spectrum. Thereby by looking at the angle differences, one deduces that for some frequencies a variable would lead another variable, while for other frequencies the same variable would lag the other.

[22] The gain is real and positive by definition because both the amplitude and the spectrum are real and non-negative quantities.

model, where a direct proxy of the nominal volatility, i.e. the EPU index, is taken as endogenous variable. In this case, the coherence and the dynamic correlation between unemployment and EPU index detects the intensity of the relation at each frequency and at each time. The phase and the gain instead, indicate how unemployment and EPU index are effectively related at each frequencies and time. **The same analysis is not feasible with the second specification of the TVP-VAR model, which takes the levels of both unemployment rate and CPI inflation as endogenous variables**. With this second specification, the evaluation of the relation between the long-run dynamics of unemployment and nominal volatility is only *qualitative*. In particular, the series of unconditional mean for unemployment and of normalized spectrum at low frequencies for the CPI inflation are recovered and compared to get some hints on the low frequency co-movement between the two measures.

4. The evidence for the G7 countries

This section shows the analysis of the TVP-VAR estimates along the lines introduced above. The same analysis is performed for all the G7 countries. This section is divided into three parts commenting the evidence on the long-run dynamics of unemployment and nominal volatility respectively, for United States, for the other Anglo-Saxon countries and Japan and for the Euro Area countries.

4.1 United States

For the first specification of the TVP-VAR model, the period covered by the estimates with US data spans from 1958:q3 to 2014:q3. Figure 1 compares both over time and over a scatter plot, the estimated medians of the time-varying unconditional means of unemployment and EPU index. Undeniably, the two series have a similar dynamics over all the sample period. This common dynamics emerges clearly from both the paths of the series in the panel above, and from the positive slope of the fitted line in the panel below. One might ask if such evidence is explained by transitory or permanent components of the original series. The plots of coherence and dynamic correlation in Figure 2 answer the question. When both measures of correlation are evaluated at high frequencies -that is when it is considered the mean of coherence and dynamic correlation for frequencies with cycle duration of less than 8 years-, the series of unemployment and EPU index are apparently uncorrelated. For these frequencies, estimated values of coherence and dynamic correlation are around zero over all the sample period. When instead, the means of coherence and dynamic correlation are considered for low frequencies -that is when the measures are averaging over the frequencies with cycle duration of at least 8 years-, the resulting values are positive and slightly increasing. Figure 2 still shows the credible intervals of coherence and dynamic correlation with respect to the iterations run in TVP-VAR estimation. The median of dynamic correlation at low frequency is quite stable around $0,5$ from the end of the 1960s on. Credible intervals of both coherence and dynamic correlation at low frequencies lay above the zero line for almost all the sample, meaning that the relation between unemployment and EPU index is positive and robust in the data. The evidence supports the findings of the model economy in Section 2, although the theory predicts a specific causality going from nominal volatility to unemployment. To tackle this issue, Figure 3 plots the phase of the cross-spectrum from EPU index to unemployment which, as illustrated in Appendix A.3, corresponds to the angle difference between the spectral representations of EPU index and unemployment. Negative values of a cross-spectrum phase so defined, implies that the second variable, i.e. unemployment, lags the first variable, i.e. EPU index. The fairly stable negative path of median cross-spectrum phase at low frequency in Figure 3, would then indicate that the long-run components of EPU index anticipate the ones of unemployment as the theory predicts. However, the credible interval for that phase spectrum is quite large and, at least until the half of the 1980s, is not fully below the zero line. Thus the indications of the phase spectrum about the lead-lag relation between unemployment and EPU index need to be taken with some cautions. Actually, still the analysis of the gains between the two variables is not definitive. Figure 4 plots in the panel above, the gains at high and low frequency of EPU index explaining the unemployment, while in the panel below the ones of unemployment explaining the EPU index. A first remark is that the median values for the low frequency gain in the panel below are much higher than in the panel above, suggesting that the absolute value of the regression coefficient for unemployment explaining EPU index is greater than the one for EPU index explaining unemployment. This is at odds with the causality relation found by the theory in equation (8). However, by considering how the gains behave at high frequencies, it results that only when EPU index explains unemployment the gain is negligible. It follows that according to the gains, only in the case of EPU index explaining unemployment, there is a clear separation between a null relation at high frequencies and a positive relation at low frequencies.

For the second specification of the TVP-VAR model, the US data used in the estimation cover a shorter period, namely from 1976:q1 to 2016:q2. This shorter interval allows to restrict the analysis to the sub-periods immediately before and after the onset of the Great Moderation. Figure 5 states graphically the break in the inflation volatility due to the Great Moderation. The scatter plot in Figure 5 shows how this break has drastically changed the relation with long-run unemployment. The median of unconditional mean for unemployment and the median of normalized spectrum at low frequency for CPI inflation co-move negatively for the first ten years of the sample, from 1976 to 1985, and positively from there on, that is from 1986 to 2016. This outcome is again in line with the theoretical result of a long-run relation between unemployment and nominal volatility, that prevails when nominal wages are really downwardly rigid. If the wages were flexible indeed, the expected long-run unemployment in the theoretical model would be equal to its natural level and not dependant upon the moments of price inflation. In this respect, one of the consequence of the drop of the inflation level during the Great Moderation was the lower pressure on nominal wages, that reasonably made them more affected by downward rigidities. If this really happened, the positive correlation at low frequency between unemployment and nominal volatility should be stronger during the Great Moderation. This is exactly what the US data say, when unconditional mean of unemployment and normalized spectrum at low frequency of CPI inflation are compared.

4.2 Canada, United Kingdom and Japan

There is some heterogeneity in the time intervals covered by the estimates of the two TVP-VAR specifications for Canada, United Kingdom and Japan. For the TVP-VAR specification with the EPU index, the intervals span respectively, from 1998:q3 to 2016:q1 with Canada data, from 1979:q1 to 2008:q1 with UK data, from 1998:q3 to 2016:q1 with Japan data. For the TVP-VAR specification with the CPI inflation, the intervals span respectively, from 1966:q3 to 2016:q1 with Canada data, from 1997:q3 to 2016:q1 with UK data, from 1971:q3 to 2016:q1 with Japan data.[23]

Figure 6 reports the same output of Figure 1, but using the data of Canada, United Kingdom and Japan. The fitted line in the scatter plots indicates that for Canada and United Kingdom, the unconditional means of unemployment and EPU index have a similar dynamics over all the sample period. For Japan, the paths of the unconditional means are not so close among each other, but the slope of the regression line in the scatter plot is however positive. A positive co-movement is more clear for Japan in Figure 7, when the unconditional mean of unemployment is plotted against the normalized spectrum at low frequency of CPI inflation. The same kind of correlation emerges for Canada, but not for the United Kingdom. Summing up, for Canada and Japan a direct relation between long-run measures of unemployment and nominal volatility is robust according to the data not only to different sample periods, but also to different proxies of volatility. For United Kingdom instead, the relation is not supported by the data when price inflation volatility is taken as a proxy of nominal volatility, although in this case, the sample period considered is sensibly shorter than when EPU index is taken as a proxy of nominal volatility.

[23] The shorter sample period for United Kingdom in the second specification is not due to the shortage of data, but by the constraint of selecting only stable draws in the estimation of the VAR measurement equation coefficients. Considering indeed longer sample intervals for UK did not guarantee the estimation of the TVP-VAR model in an acceptable time.

4.3 France, Germany and Italy

There is less heterogeneity across country in the time intervals covered by the estimates of the two TVP-VAR specifications for France, Germany and Italy. The shortage of data on EPU index implies shorter intervals for the first specification of the TVP-VAR model. In that case indeed, the sample periods span respectively, from 2003:q3 to 2016:q1 with France data, from 1998:q3 to 2008:q1 with Germany data, from 2002:q3 to 2016:q1 with Italy data. For the estimation of the TVP-VAR specification with the CPI inflation instead, the sample periods span respectively, from 1977:q1 to 2016:q1 with France data, from 1980:q5 to 2008:q1 with Germany data, from 1983:q3 to 2016:q1 with Italy data.

The plots in Figure 8 show clearly for all the Euro Area countries considered, a co-movement of the unconditional mean of unemployment with the unconditional mean of EPU index. This co-movement is positive for France and Italy, whereas is negative for Germany. Figure 9 plots the unconditional mean of unemployment against the normalized spectrum at low frequency of CPI inflation. In this case, the correlation sign is in line with the theory for all three countries. In particular, the regression lines in scatter plots of Figure 9 have a positive slope for the full sample period for France and Germany, whereas for the full period but with the exception of the very last part, for Italy. This last interval for Italy corresponds to the period that starts with the onset of the sovereign debt crisis.

Conclusions

This paper aims to gain the evidence on the long-run relation between the level of unemployment and the volatility of the nominal side of the economy. The economic reasoning motivating this analysis is that whenever an economy is featured by some nominal rigidities that limit the downward adjustments of wages, the level of labor margins prevailing in the long-run can depend on the variability of the processes that ultimately lead the nominal wages. The more these wages are volatile, the more downward rigidities are binding and the more unemployment has to compensate upwardly to restore the labor market equilibrium. To stress on the point, the paper formalizes a stylized model admitting both downward nominal wage rigidities and an exogenous price growth process with a non-negligible volatility. The theoretical framework finds a positive dependence of expected long-run unemployment on the price inflation volatility. This kind of relation is tested among the G7 countries in an empirical analysis. The analysis starts with the estimation of a time-varying parameters VAR model for each country in the sample, in which the nominal volatility is measured alternatively by the EPU index and by the spectrum of the CPI inflation. With the EPU index, a direct proxy of nominal volatility is introduced as endogenous variable of the TVP-VAR model along with unemployment and nominal wage inflation. This set-up allows to evaluate quantitatively the strength of the co-movements at low frequency between unemployment and EPU index through some measures of spectral analysis. The spectral analysis is also used with the second TVP-VAR specification, that includes the CPI inflation instead of the EPU index as endogenous variable. In this case, a proxy of nominal volatility is recovered by the spectrum of CPI inflation, whose pattern at low frequency is compared with the one of unconditional mean of unemployment, to get a qualitative analysis of the common behavior. Estimation results are instructive for all G7 countries in both TVP-VAR specifications. For the United States, the unconditional mean of unemployment co-varies positively with both the unconditional mean of EPU index and the normalized low frequency spectrum of CPI inflation. In particular with the latter, the common behavior is true from the onset of Great Moderation on. This result confirms the underlying theoretical model, which prescribes a stronger long-run correlation between unemployment and nominal volatility when the trend of price inflation declines. When it occurs indeed, nominal wages are more affected on average by downward rigidities and labor margins have to absorb more often the negative shocks. However, the causality relation from volatility to unemployment dictated by the theory is not fully supported by the US data. Meanwhile, some simple indications in favor of a positive relation at low frequency between unemployment and nominal volatility, are confirmed in the data still for the other G7 countries. Although the sample periods considered are of different length due to different availability of the data, for all the countries the unconditional mean of unemployment co-moves with at least one of the measure considered for the nominal volatility at low frequency. For Canada, France, Italy and Japan, it happens with both the unconditional mean of EPU index and the normalized spectrum at low frequency of CPI inflation. For the United Kingdom instead, a positive co-movement is verified only with the unconditional mean of EPU index, whereas for Germany, only with the normalized spectrum at low frequency of CPI inflation.

Summing up, the paper unveils a common evidence for all the G7 countries in the last decades. For developed countries like them, in which the nominal rigidities have quite regularly affected the labor market dynamics, the long-run behavior of unemployment and nominal volatility seem to be particularly linked. But the cross-country heterogeneity on the sample intervals considered and on the estimated covariances in the two TVP-VAR specifications, suggests further steps in the empirical investigation. A similar analysis could be implemented for instance, with a different empirical framework. Instead of considering single TVP-VAR models for each of the G7 countries, one could consider a single model for all the G7 countries, that is a Panel TVP-VAR model. By collecting all the data available in a single econometric model, it should firstly help the estimation of the model when the data of some variables are short at country-level. Moreover, the estimation of a Panel TVP-VAR model allows to extrapolate from the data some specific components that are common to all countries or to only some of them. Following in particular, the shrinkage approach proposed by Canova and Ciccarelli (2009, 2013) for the estimation of Panel VAR models with Bayesian methods, it is possible to recover from the model estimates some components that are specific related to each endogenous variable, but are common between all countries in the panel. These components might be then interpreted as a sort of core measure of the variables. Considering then again the G7 countries data for unemployment and for a direct proxy of nominal volatility, the analysis of these core measures should be suggestive as well, on the empirical long-run relation between unemployment and nominal volatility in developed economies.

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Appendix

A.1 MCMC Algorithm

The empirical model illustrated in the section 3.1 is estimated with Bayesian methods using a Montecarlo Markov Chain (MCMC) algorithm, namely the Gibbs sampling. This algorithm allows to break the whole problem of estimating the joint model posterior into smaller problems of estimating separately the conditional posteriors of the states and of the hyperparameters. The procedure consists in drawing from the marginal density of one random variable at each time, conditional on the data and on the realizations of other random variables. The Markov Chain is constructed by iterating the drawing for all the unknown elements. Under standard conditions, the Markov Chain converges to an invariant distribution, which is the desired joint posterior.

To start with the algorithm, the model needs to be initialized. As an assumption, the prior distributions for the initial values of the states, θ_0 and h_0, are normal distributions, which are independent both from each other and from the distributions of the hyperparameters. These priors distributions can be written as,

$$\begin{aligned} \theta_0 &\sim N\left(\bar{\theta}_0, \bar{P}_0\right) , && (25) \\ \ln h_{i0} &\sim N\left(\ln \Sigma_0^{ii}, 10\right) . && (26) \end{aligned}$$

To calibrate (25) and (26), a standard fixed parameters VAR model is estimated using the data of a training sample. Training sample duration changes according to the data availability for each specification and each country.[24]. The coefficients $\bar{\theta}_0$ and $\bar{P}_0$ are the estimates respectively, of the parameter vector in the measurement equation and of the asymptotic covariance matrix for those parameters. The coefficient $\ln \Sigma_0^{ii}$ instead, is the logarithm of the estimated variance for the measurement equation residual of variable i. As regards the hyperparameters, the prior distribution for the covariance matrix Q of parameter residuals, is assumed to follow an inverse-Wishart distribution,

$$Q \sim IW\left(\bar{Q}_0^{-1}, T_0\right) , \tag{27}$$

with degrees of freedom T_0 -which is the time length of the training sample- and scale matrix $T_0\bar{Q}_0$, with $\bar{Q}_0 = 1.0 \times 10^{-4} \times \bar{P}_0$. The prior distribution for the vector α containing the free elements of matrix A, is postulated to be normal with zero mean and large covariance matrix, as follows

$$\alpha \sim N\left(0, 10000 \times I\right) , \tag{28}$$

[24]For the TVP-VAR specification with EPU index as endogenous variable, the training sample span 40 quarters for US, 30 quarters for UK, and 20 quarters for the rest of G7 countries. For the TVP-VAR specification with CPI inflation as endogenous variable, the training sample span 40 quarters for US, Canada, Germany, Japan, 30 quarters for France, and 20 quarters for Italy and UK. The training samples correspond to the intervals anticipating the sample periods indicated in Section 4 for all G7 countries.

Finally, the prior distribution for the variance of the innovations of the stochastic volatility h_i, is assumed to follow an inverse-Gamma distribution for all i,

$$\sigma_i^2 \sim IG\left(\frac{1.0 \times 10^{-4}}{2}, \frac{1}{2}\right) \ , \tag{29}$$

with degree of freedom parameter 1 and scale parameter 1.0×10^{-4}.

Once having defined for all variables i, the starting values for θ, P, Q, α, h_i and σ_i^2 as in Cogley and Sargent (2005), the Gibbs sampling algorithm can be implemented. Taking as known the entire sample history of the vector of endogenous variables Y^T, each iteration of the procedure consists in five steps:

1. sample the matrix Q from $f\left(Q|Y^T, \theta^T, H^T, \sigma, \alpha\right)$;
2. sample the vector σ from $f\left(\sigma|Y^T, \theta^T, H^T, Q, \alpha\right)$;
3. sample $\{h_{it}\}_{t=1}^T$ by applying the univariate algorithm of Jacquier et al. (1994);
4. sample the vector α from $f\left(\alpha|Y^T, \theta^T, H^T, Q, \sigma\right)$;
5. sample the matrix θ^T from $f\left(\theta^T|Y^T, H^T, Q, \sigma, \alpha\right)$ by applying the Carter and Kohn (1994) algorithm.

Matrix Q and vector σ are drawn respectively, from an inverse-Wishart and from inverse-Gamma posterior distributions. These posteriors are yielded from the combinations of the above priors for Q and σ with normal likelihoods.

To sample the stochastic volatilities $\{h_{it}\}_{t=1}^T$, it is applied a Metropolis algorithm that for each t simulates a proposal distribution mimicking the target conditional posterior $f\left(h_{i,t}|h_{i,t}, Y^T, \theta^T, \sigma_i\right)$, where $h_{i,t}$ indicates the rest of h vector at time t. With a certain probability a draw from the proposal distribution for $h_{i,t}$ is accepted. In that case the Metropolis chain is updated, otherwise it remains unchanged at the previous draw value.

The free elements of matrix A are sampled starting from the following system of unrelated regressions,

$$A\varepsilon_t = \upsilon_t \tag{30}$$

with $E\left[\upsilon_t \upsilon_t'\right] = H_t$. The free elements of matrix A can be thus recovered as regression coefficients of the equations system (30). At each iteration, these elements are drawn from a normal posterior distribution.

The matrix θ_t of the measurement equation parameters are simulated by applying the Carter and Kohn algorithm. This algorithm requires firstly an initialization of the Kalman filter with some starting values for $\theta_{0|0}$ and $P_{0|0}$. Then, the Kalman filter is iterated forward up to the last period T, by delivering the conditional mean $\theta_{T|T}$ and variance $P_{T|T}$. The trajectory θ^T is recovered by backward recursion. Any matrix θ_t, for $t = 0, ..., T$, is indeed drawn by a normal distribution evaluated at mean $\theta_{t|t+1}$ and variance $P_{t|t+1}$, whose values are recovered by updating the conditional mean $\theta_{t|t}$ and variances $P_{t|t}$ with the information contained in $t + 1$.

Steps 1-5 are repeated for 100000 iterations. The first 80000 iterations are discarded as burn-in, the last 20000 are kept to provide an approximation to the marginal posterior distributions of the TVP-VAR model parameters.

A.2 Data

With the only exception of the EPU index, all other raw series are collected from the FRED database and the OECD database. The EPU index series are instead collected online for all G7 countries at www.PolicyUncertainty.com, which is the website of the project linked to Baker et al. (2015).

For US and UK, the EPU series downloaded are the Historical EPU indices. For other countries, the EPU series downloaded are the benchmark ones. Given that the raw series of EPU index are provided with monthly frequency, the quarterly series used in the estimations are obtained by averaging the original series for each quarter.

The rest of data used for US are collected from the FRED database, as seasonally adjusted quarterly series. The series considered are civilian unemployment rate (UNRATE), nominal compensation per hour in nonfarm business sector (COMPNFB_PC1), CPI index for all urban consumers for all items (CPIAUCSL_PC1). The series of nominal compensations and consumer prices are used in the TVP-VAR specifications in log-differences.

For the other G7 countries, the most of the data are extracted from OECD.stat, as seasonally adjusted quarterly series. For Canada, France, Germany and Japan, the unemployment rate is measured by the Short Term Labor Market Statistics series of unemployment rate for all persons, aged 15 years and over.[25] For Italy and UK, the unemployment rate is measured by the series of unemployment rate for all persons, aged respectively, 15 and 16 years and over. The series are provided by ISTAT for Italy and Office for National Statistics for UK. For the nominal worker compensations and consumer prices, the series considered are extracted by the OECD Main Economic Indicators database for all the countries, but US. The data used in the TVP-VAR specifications are the log-differences of seasonally adjusted indices for hourly earnings for manufacturing and consumer prices.

[25] The series provided by OECD.stat for France is not seasonally adjusted. The series is therefore adjusted by the author by using the X-13ARIMA-SEATS seasonal adjustment program provided by the US Census Bureau.

A.3 Lead-lag relation and phase spectrum

This appendix gives an insight in the interpretation of the cross-spectrum phase between two variables as an indicator of the lead-lag relation between the same variables. To do that, some elements of spectral theory need to be introduced.

Let $\mathbf{S}_t(\omega)$ be the spectrum matrix of a VAR model with vector $[X_t,\ Y_t,\ Z_t]'$ as endogenous variables,

$$\mathbf{S}(\omega) = \frac{1}{2\pi}\begin{bmatrix} \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{XX}(s)e^{-i\omega s} & \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{XY}(s)e^{-i\omega s} & \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{XZ}(s)e^{-i\omega s} \\ \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{YX}(s)e^{-i\omega s} & \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{YY}(s)e^{-i\omega s} & \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{YX}(s)e^{-i\omega s} \\ \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{ZX}(s)\, e^{-i\omega s} & \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{ZZ}(s)\, e^{-i\omega s} & \sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{ZZ}(s)\, e^{-i\omega s} \end{bmatrix}. \tag{31}$$

Following Hamilton (1994), the element at the 2^{th} row and 1^{st} column of matrix $\mathbf{S}_t(\omega)$ is the cross-spectrum from variable Y_t to variable X_τ, with $-\infty < t, \tau < +\infty$. It can be rewritten as,

$$\begin{aligned} s^{YX}(\omega) &= \frac{1}{2\pi}\sum_{s=-\infty}^{+\infty} \boldsymbol{\gamma}^{YX}(s)e^{-i\omega s} \\ &= \frac{1}{2\pi}\sum_{s=-\infty}^{+\infty} E_t\left[Y_t X_{t-s}\right] e^{-i\omega s} \\ &= \frac{1}{2\pi}\sum_{s=-\infty}^{+\infty} E_t\left[Y_t X_{t-s}\right] e^{-i\omega(t-(t-s))} \\ &= \frac{1}{2\pi}\sum_{t,\tau=-\infty}^{+\infty} E_t\left[Y_t X_\tau\right] e^{-i\omega(t-\tau)} \\ &= \frac{1}{2\pi}\sum_{t,\tau=-\infty}^{+\infty} E_t\left[Y_t\right] e^{-i\omega t} E_t\left[X_\tau\right] e^{i\omega\tau} \\ &= \frac{1}{2\pi}\sum_{t,\tau=-\infty}^{+\infty} E_t\left[Y_t\right]\left(\cos(\omega t) - \sin(\omega t)\right) E_t\left[X_\tau\right]\left(\cos(\omega\tau) + i\sin(\omega\tau)\right) \end{aligned} \tag{32}$$

Defining then the following quantities,

$$\frac{1}{2\pi}E_t\left[Y_t\right]\cos(\omega t) \equiv a_{t,Y}(\omega)\ , \tag{33}$$

$$\frac{1}{2\pi}E_t\left[Y_t\right]\sin(\omega t) \equiv b_{t,Y}(\omega)\ , \tag{34}$$

$$\frac{1}{2\pi}E_t\left[X_\tau\right]\cos(\omega\tau) \equiv a_{t,X}(\omega)\ , \tag{35}$$

$$\frac{1}{2\pi}E_t\left[X_\tau\right]\sin(\omega\tau) \equiv b_{t,X}(\omega)\ , \tag{36}$$

equation (32) can be rewritten as

$$\begin{aligned} s^{YX}(\omega) &= \frac{1}{2\pi}\sum_{s=-\infty}^{+\infty}\boldsymbol{\gamma}^{YX}(s)e^{-i\omega s} \\ &= \sum_{t,\tau=-\infty}^{+\infty}\left(a_{t,Y}(\omega)-ib_{t,Y}(\omega)\right)\left(a_{t,X}(\omega)+ib_{t,X}(\omega)\right) \\ &= \sum_{t,\tau=-\infty}^{+\infty} a_{t,Y}(\omega)\,a_{t,X}(\omega)+ia_{t,Y}(\omega)\,b_{t,X}(\omega)-ib_{t,Y}(\omega)\,a_{t,X}(\omega)+b_{t,Y}(\omega)\,b_{t,X}(\omega)\,, \end{aligned} \tag{37}$$

or equivalently,

$$\begin{aligned} s^{YX}(\omega) &= \sum_{t,\tau=-\infty}^{+\infty}\left(a_{t,Y}(\omega)\,a_{t,X}(\omega)+b_{t,Y}(\omega)\,b_{t,X}(\omega)\right)+ \\ &\quad +i\sum_{t,\tau=-\infty}^{+\infty}\left(a_{t,Y}(\omega)\,b_{t,X}(\omega)-b_{t,Y}(\omega)\,a_{t,X}(\omega)\right)\,, \end{aligned} \tag{38}$$

where the real and imaginary components of (38) are the following,

$$\begin{aligned} \mathrm{Re}\left(s^{YX}(\omega)\right) &= \mathrm{Re}\left(\sum_{s=-\infty}^{+\infty}\boldsymbol{\gamma}^{YX}(s)e^{-i\omega s}\right) \\ &= \sum_{t,\tau=-\infty}^{+\infty}\left(a_{t,Y}(\omega)\,a_{t,X}(\omega)+b_{t,Y}(\omega)\,b_{t,X}(\omega)\right)\,, \end{aligned} \tag{39}$$

$$\begin{aligned} \mathrm{Im}\left(s^{YX}(\omega)\right) &= \mathrm{Im}\left(\sum_{s=-\infty}^{+\infty}\boldsymbol{\gamma}^{YX}(s)e^{-i\omega s}\right) \\ &= \sum_{t,\tau=-\infty}^{+\infty}\left(a_{t,Y}(\omega)\,b_{t,X}(\omega)-b_{t,Y}(\omega)\,a_{t,X}(\omega)\right)\,. \end{aligned} \tag{40}$$

Given (39) and (40), the phase of the co-spectrum from Y to X is so defined,

$$\theta_{t,YX}(\omega)=\tan^{-1}\left(\frac{a_{t,Y}(\omega)\,b_{t,X}(\omega)-b_{t,Y}(\omega)\,a_{t,X}(\omega)}{a_{t,Y}(\omega)\,a_{t,X}(\omega)+b_{t,Y}(\omega)\,b_{t,X}(\omega)}\right)\,. \tag{41}$$

However, given the trigonometry identity $\tan^{-1}(\alpha)-\tan^{-1}(\beta)=\tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)$, the phase $\theta_{t,YX}(\omega)$ can be equally defined as,

$$\begin{aligned}
\theta_{t,YX}(\omega) &= \tan^{-1}\left(\frac{\frac{a_{t,Y}(\omega)b_{t,X}(\omega)}{a_{t,Y}(\omega)a_{t,X}(\omega)}-\frac{b_{t,Y}(\omega)a_{t,X}(\omega)}{a_{t,Y}(\omega)a_{t,X}(\omega)}}{1+\frac{b_{t,Y}(\omega)b_{t,X}(\omega)}{a_{t,Y}(\omega)a_{t,X}(\omega)}}\right) \\
&= \tan^{-1}\left(\frac{\frac{b_{t,X}(\omega)}{a_{t,X}(\omega)}-\frac{b_{t,Y}(\omega)}{a_{t,Y}(\omega)}}{1+\frac{b_{t,Y}(\omega)}{a_{t,Y}(\omega)}\frac{b_{t,X}(\omega)}{a_{t,X}(\omega)}}\right) \\
&= \tan^{-1}\left(\frac{b_{t,X}\left(\omega\right)}{a_{t,X}\left(\omega\right)}\right)-\tan^{-1}\left(\frac{b_{t,Y}\left(\omega\right)}{a_{t,Y}\left(\omega\right)}\right) \\
&= -\tan^{-1}\left(\frac{b_{t,Y}\left(\omega\right)}{a_{t,Y}\left(\omega\right)}\right)-\left(-\tan^{-1}\left(\frac{b_{t,X}\left(\omega\right)}{a_{t,X}\left(\omega\right)}\right)\right) \ . \qquad (42)
\end{aligned}$$

Equation (42) corresponds to the phase difference between two sinusoids. These sinusoids describe the behavior of variables X_t and Y_t at any frequency ω over the time domain. For $i = X$, Y, a sinusoid takes the following generic form,

$$a_{t,i}\left(\omega\right)\cos(\omega t)+b_{t,i}\left(\omega\right)\sin(\omega) \ , \qquad (43)$$

or equivalently,

$$A_{t,i}\left(\omega\right)\cos(\omega t+\theta_{t,i}) \ , \qquad (44)$$

with amplitude $A_{t,i}$ and phase $\theta_{t,i}$, which are defined respectively as,

$$\begin{aligned}
A_{t,i}\left(\omega\right) &= \left(\left(a_{t,i}\left(\omega\right)\right)^2+\left(b_{t,i}\left(\omega\right)\right)^2\right)^{1/2} \ , \qquad (45) \\
\theta_{t,i}\left(\omega\right) &= -\tan^{-1}\left(\frac{b_{t,i}\left(\omega\right)}{a_{t,i}\left(\omega\right)}\right) \ . \qquad (46)
\end{aligned}$$

Roughly speaking, the phase of a sinusoid (46) indicates when that sinusoid reaches a peak over the time domain. Given indeed that the cosine function has a local maximum at zero, independently from the amplitude -which is positive by definition-, the sinusoid (44) reaches a local peak when the cosine function is evaluated at zero. It occurs at $t = -\frac{\theta_{t,i}}{\omega}$. It follows that if the difference between $\theta_{t,Y}\left(\omega\right)$ and $\theta_{t,X}\left(\omega\right)$ in equation (42) is negative, the sinusoid describing variable Y_t reaches for frequency ω a local peak at a time t, let's say, that follows the corresponding time t for variable X_t. Equivalently, it means that variable Y_t follows or lags variable X_t at a certain frequency ω. Conversely, whether the difference between $\theta_{t,Y}\left(\omega\right)$ and $\theta_{t,X}\left(\omega\right)$ in equation (42) is positive, it means that variable Y_t anticipates or leads variable X_t at a frequency ω.

In the main text, this interpretation of the cross-spectrum phase is applied to the cross-spectrum phase from the EPU index to the unemployment. This specific cross-spectrum corresponds to the element $2,1$ of the spectrum matrix prevailing with the first specification of TVP-VAR model, namely the one with $Y_t^a = [EPU_t,\, u_t,\, \pi_t^W]'$ as vector of endogenous variables.[26] As shown in the main text, the spectrum matrix prevailing with this specification is approximated by matrix $\mathbf{S}_{t|T}(\omega)$, whose element $2,1$ approximates the cross-spectrum phase from EPU index to unemployment.

[26] Given the order of endogenous variables, EPU_t series correponds to variable X_t of this Appendix and u_t series to variable Y_t.

Figures

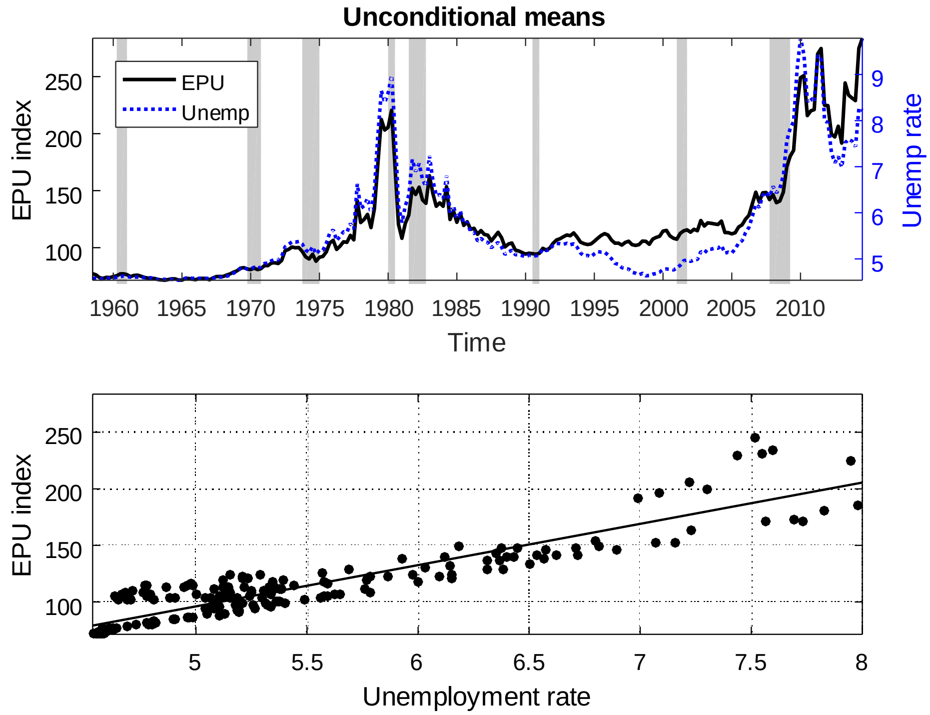

Figure 1. The panel above compares unconditional mean of EPU index (black solid line - left y-axis) and of unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility. NBER recessions in shaded columns. The panel below shows the scatter plot between unconditional mean of EPU index (y-axis) and unemployment rate (x-axis).

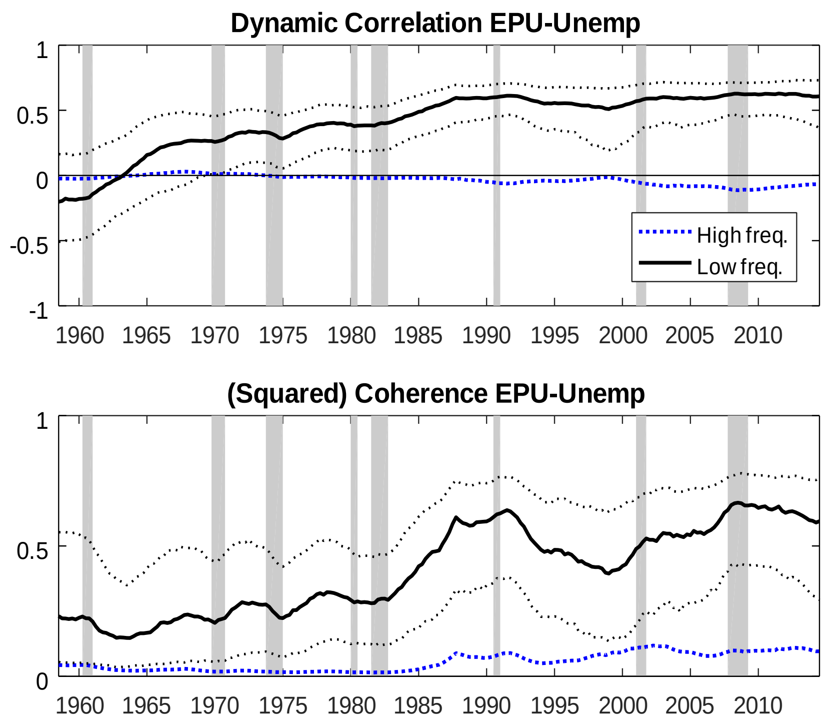

Figure 2. The panel above compares the posterior medians of the dynamic correlation index between EPU index and unemployment rate evaluated at high frequency (blue dotted line) and low frequency (black solid line). For the latter, there are also shown the 16^{th} and 84^{th} percentile of the estimated posterior distribution (black dotted lines). Panel below shows the same kind of objects considereing the coherence. The series at low frequency corresponds to the means of the same series at frequencies with 8 or more years per cycle. The series at high frequency corresponds to the means of the same series at frequencies with less than 8 years per cycle.

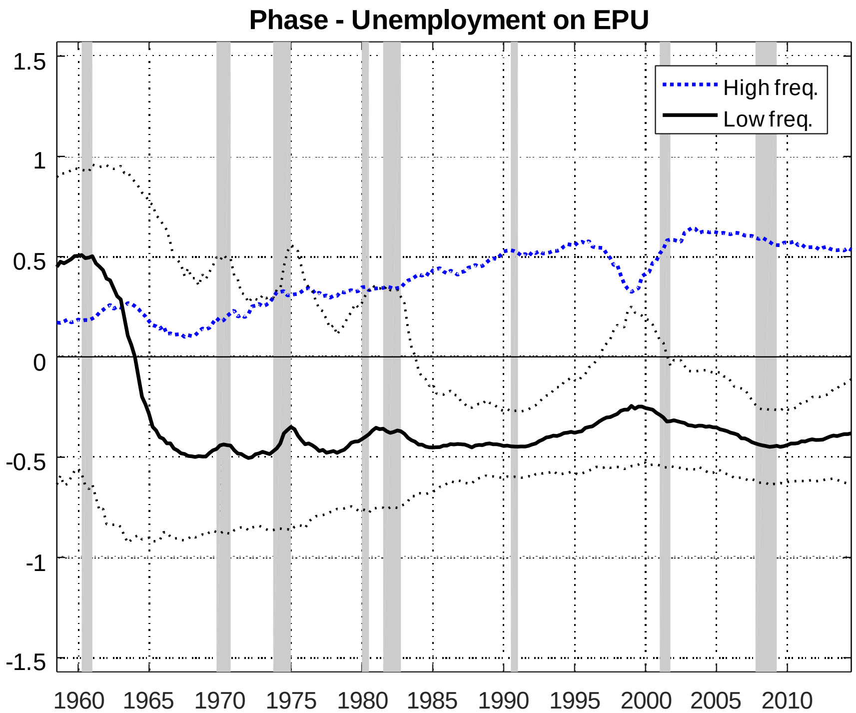

Figure 3. The panel above compares the posterior medians of the phase spectrum from the EPU index to the unemployment rate evaluated at high frequency (blue dotted line) and low frequency (black solid line). For the latter, there are also shown the 16^{th} and 84^{th} percentile of the estimated posterior distribution (black dotted lines). The series at low frequency corresponds to the means of the same series at frequencies with 8 or more years per cycle. The series at high frequency corresponds to the means of the same series at frequencies with less than 8 years per cycle.

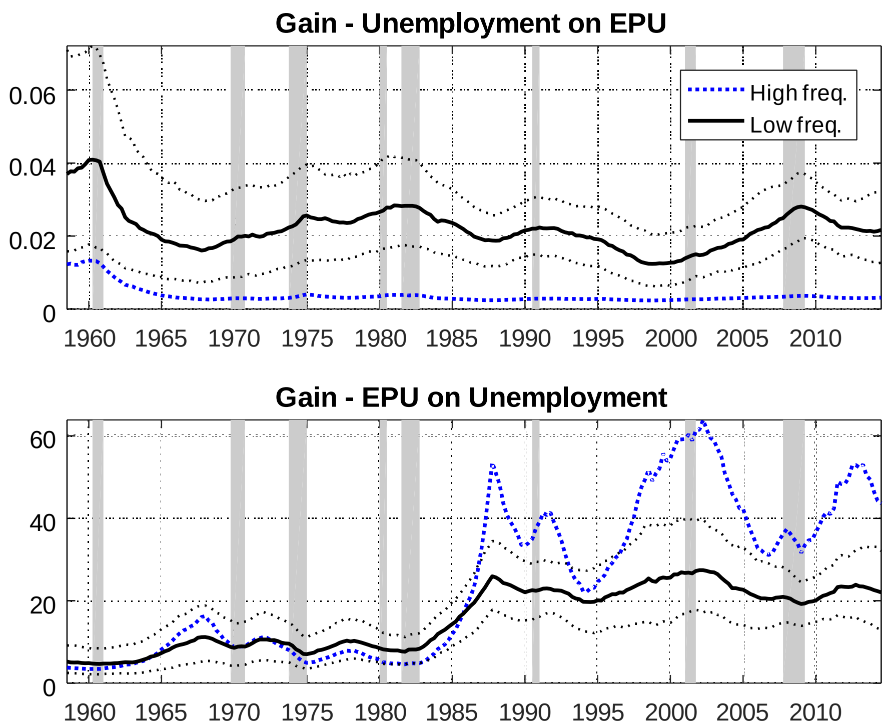

Figure 4. The panel above compares the posterior medians of the gain from EPU index to unemployment rate, evaluated at high frequency (blue dotted line) and low frequency (black solid line). For the latter, there are also shown the 16^{th} and 84^{th} percentile of the estimated posterior distribution (black dotted lines). Panel below shows the same kind of objects considering the gain from unemployment rate to EPU index. The series at low frequency corresponds to the means of the same series at frequencies with 8 or more years per cycle. The series at high frequency corresponds to the means of the same series at frequencies with less than 8 years per cycle.

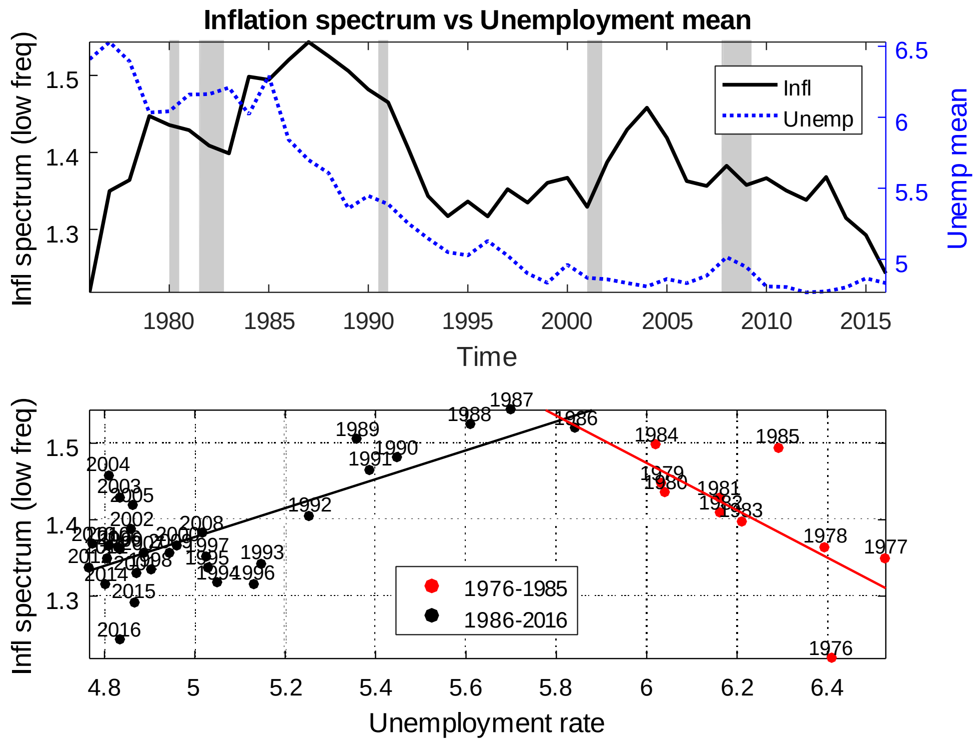

Figure 5. The panel above compares normalized spectrum of CPI inflation at low frequency (black solid line - left y-axis) and unconditional unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility. The series at low frequency corresponds to the means of the same series at frequencies with 8 or more years per cycle. NBER recessions in shaded columns. The panel below shows the scatter plot between normalized spectrum of CPI inflation at low frequency (y-axis) and unemployment rate (x-axis). Red points show the relation for the years 1976-1985 and black ones for the years 1986-2016.

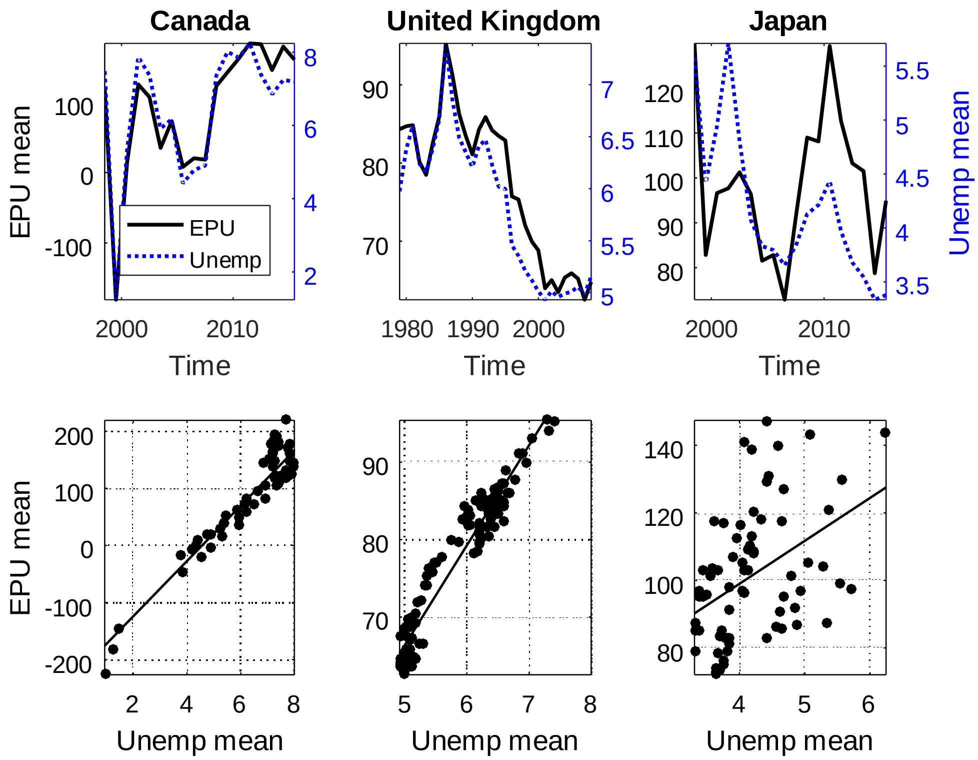

Figure 6. The three panels above compare unconditional mean of EPU index (black solid line - left y-axis) and of unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility using Canada, United Kingdom and Japan data. The three panel below show the scatter plot between unconditional mean of EPU index (y-axis) and unemployment rate (x-axis) for the same countries.

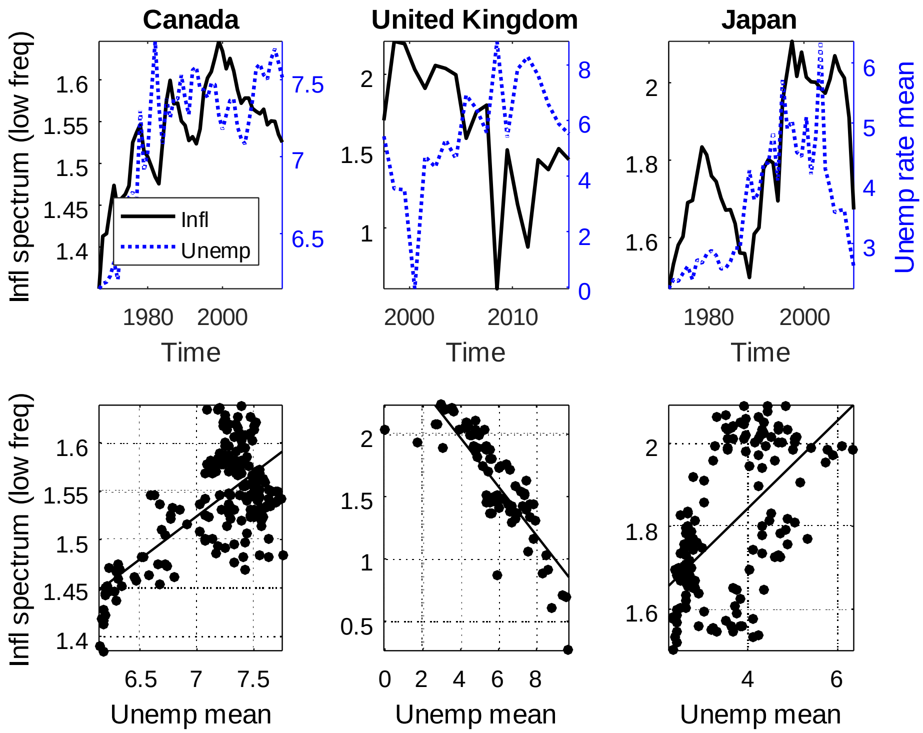

Figure 7. The three panels above compare normalized spectrum of CPI inflation at low frequency (black solid line - left y-axis) and unconditional unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility using Canada, United Kingdom and Japan data. The series at low frequency corresponds to the means of the same series at frequencies with 8 or more years per cycle. The three panels below show the scatter plot between normalized spectrum of CPI inflation at low frequency (y-axis) and unemployment rate (x-axis) for the same countries.

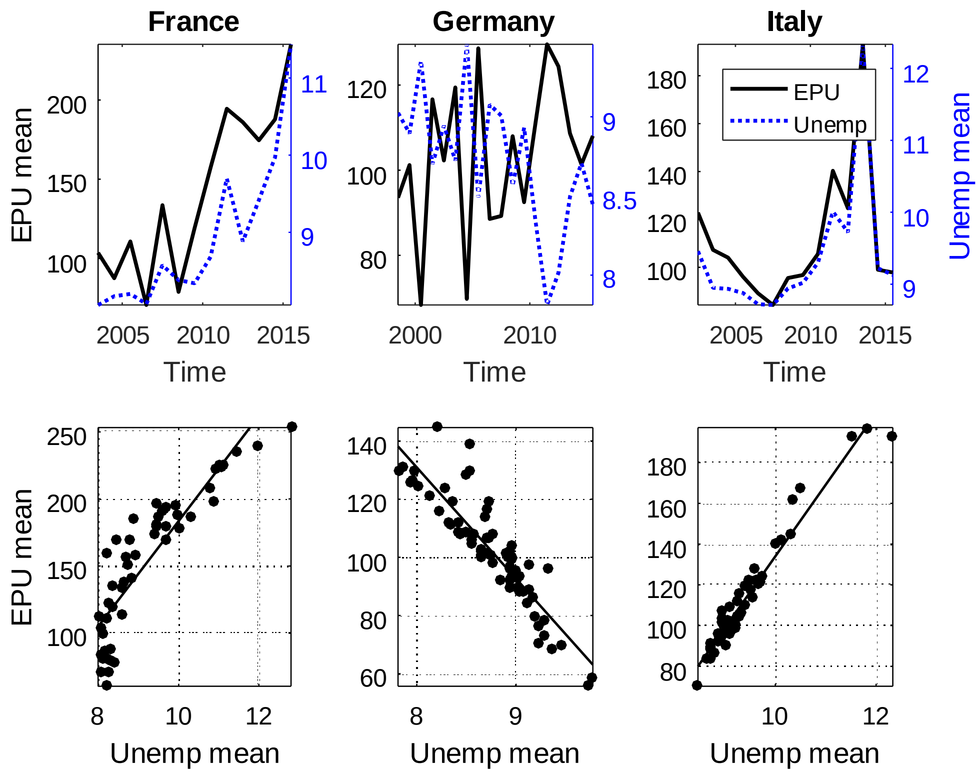

Figure 8. The three panels above compare unconditional mean of EPU index (black solid line - left y-axis) and of unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility using France, Germany and Italy data. The three panel below show the scatter plot between unconditional mean of EPU index (y-axis) and unemployment rate (x-axis) for the same countries.

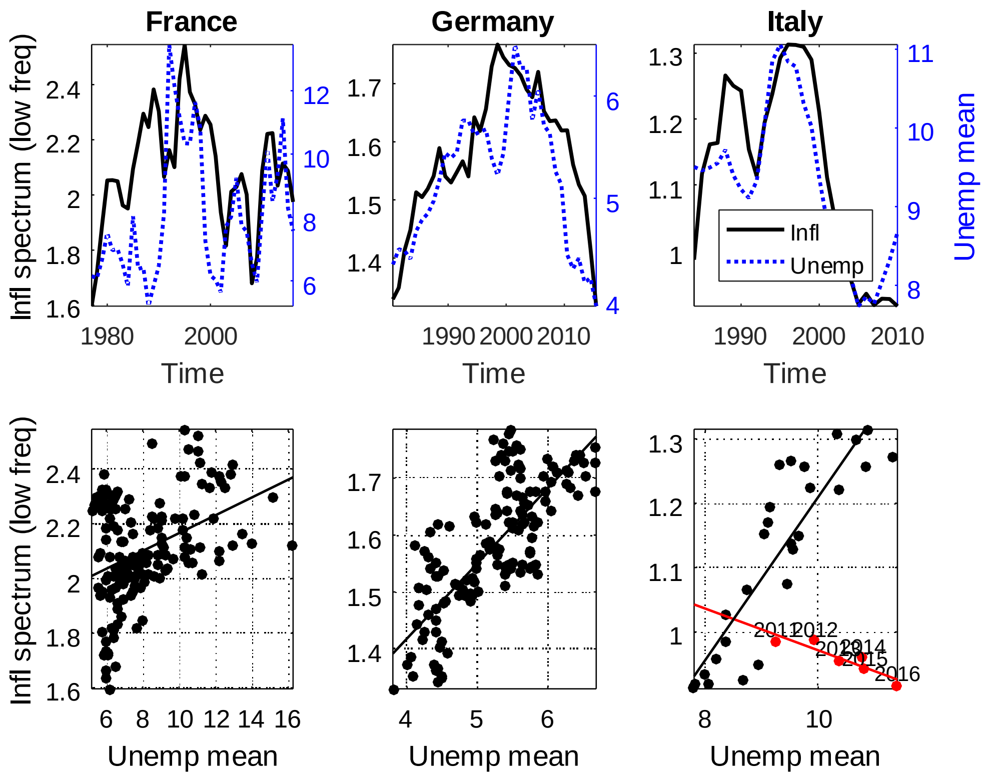

Figure 9. The three panels above compare normalized spectrum of CPI inflation at low frequency (black solid line - left y-axis) and unconditional unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility using France, Germany and Italy data. The series at low frequency corresponds to the means of the same series at frequencies with 8 or more years per cycle. The three panels below show the scatter plot between normalized spectrum of CPI inflation at low frequency (y-axis) and unemployment rate (x-axis) for the same countries.

Additional Figures (not for publication)

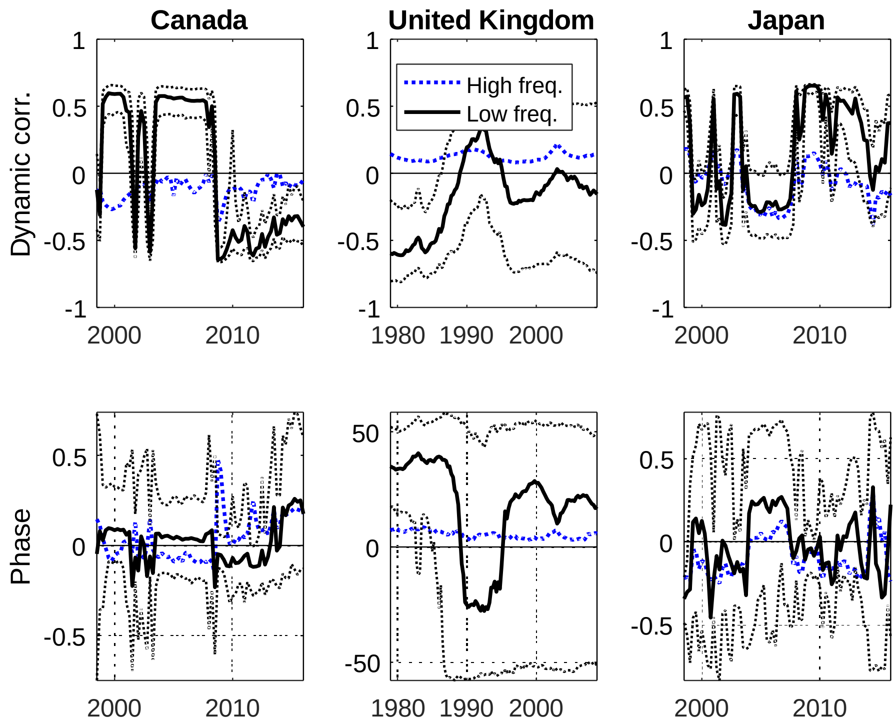

Figure. Using Canada, United Kingdom, Japan data, the panels above compare the posterior medians of the dynamic correlation index between EPU index and unemployment rate evaluated at high frequency (blue dotted line) and low frequency (black solid line). For the latter, there are also shown the 16^{th} and 84^{th} percentile of the estimated posterior distribution (black dotted lines). Panel below compare the posterior medians of the phase spectrum from the EPU index to the unemployment rate evaluated at high frequency (blue dotted line) and low frequency (black solid line).

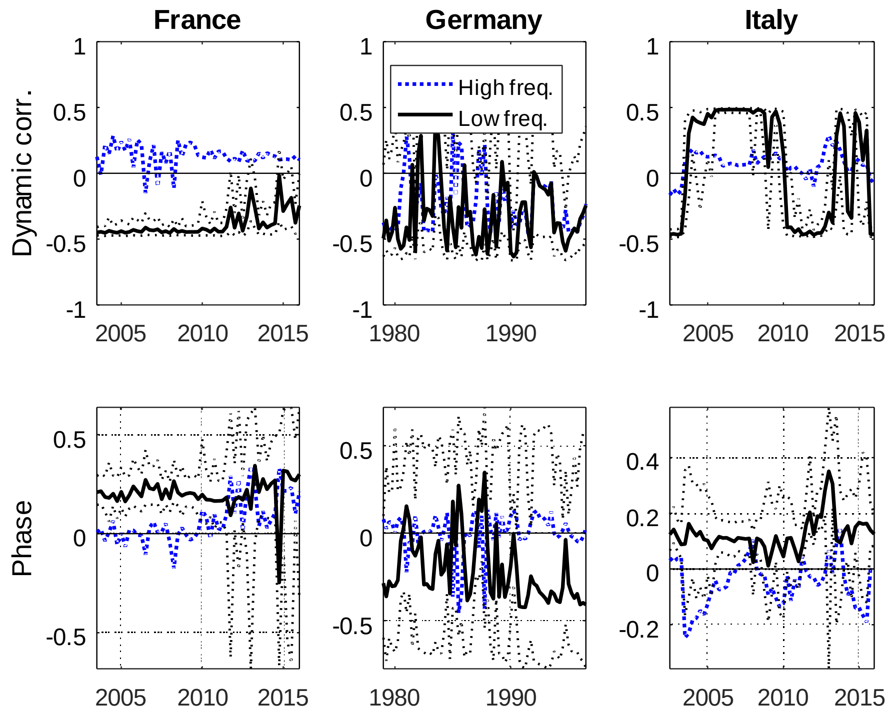

Figure. Using France, Germany, Italy data, the panels above compare the posterior medians of the dynamic correlation index between EPU index and unemployment rate evaluated at high frequency (blue dotted line) and low frequency (black solid line). For the latter, there are also shown the 16^{th} and 84^{th} percentile of the estimated posterior distribution (black dotted lines). Panel below compare the posterior medians of the phase spectrum from the EPU index to the unemployment rate evaluated at high frequency (blue dotted line) and low frequency (black solid line).

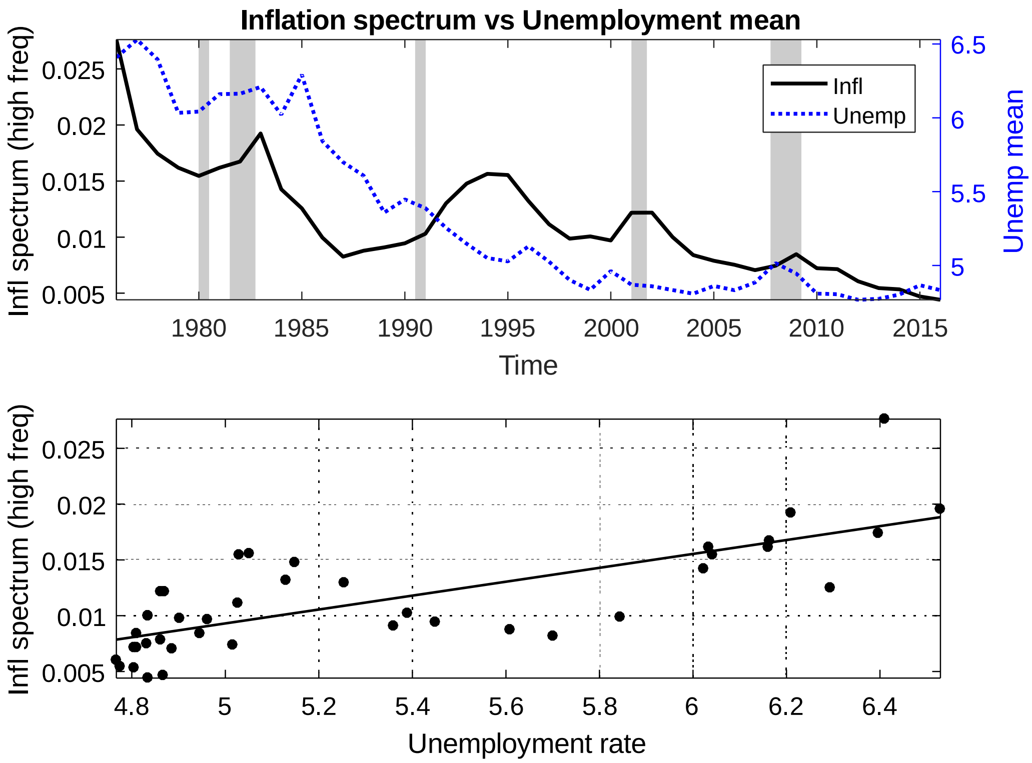

Figure . Using US data, the panel above compares normalized spectrum of CPI inflation at high frequency (black solid line - left y-axis) and unconditional unemployment rate (blu dotted line - right y-axis) obtained by estimating the TVP-VAR model with stochastic volatility. The series at low frequency corresponds to the means of the same series at frequencies with less than 8 years per cycle. NBER recessions in shaded columns. The panel below shows the scatter plot between normalized spectrum of CPI inflation at high frequency (y-axis) and unemployment rate (x-axis).

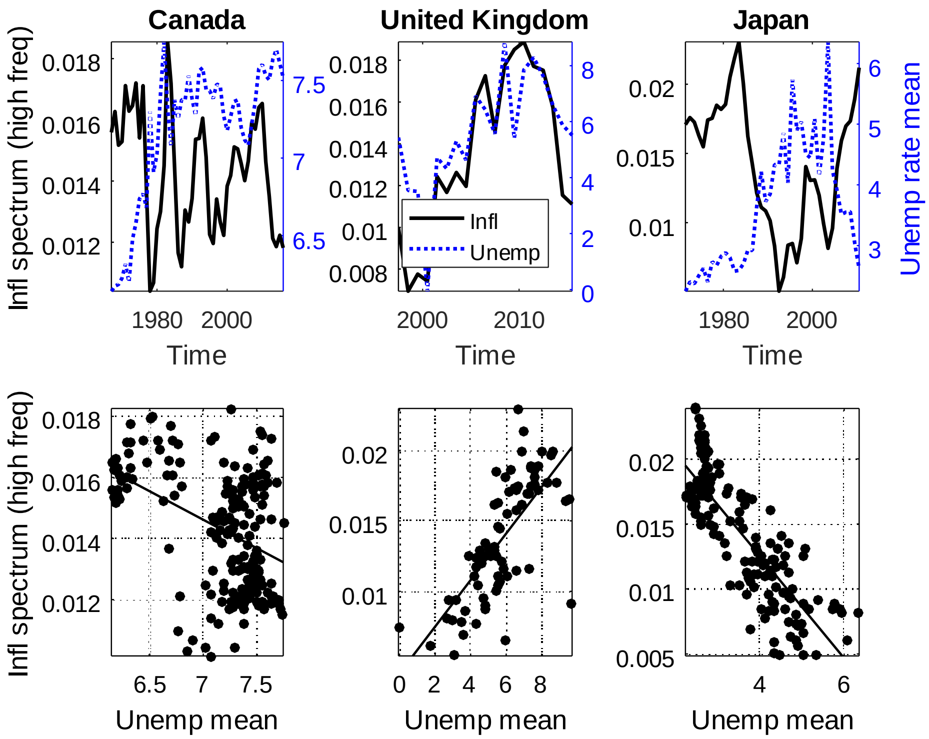

Figure . Using Canada, United Kingdom, Japan data, the panels above compare normalized spectrum of CPI inflation at high frequency (black solid line - left y-axis) and unconditional unemployment rate (blu dotted line - right y-axis). The series at high frequency corresponds to the means of the same series at frequencies with less than 8 years per cycle. The three panels below show the scatter plot between normalized spectrum of CPI inflation at high frequency (y-axis) and unemployment rate (x-axis) for the same countries.

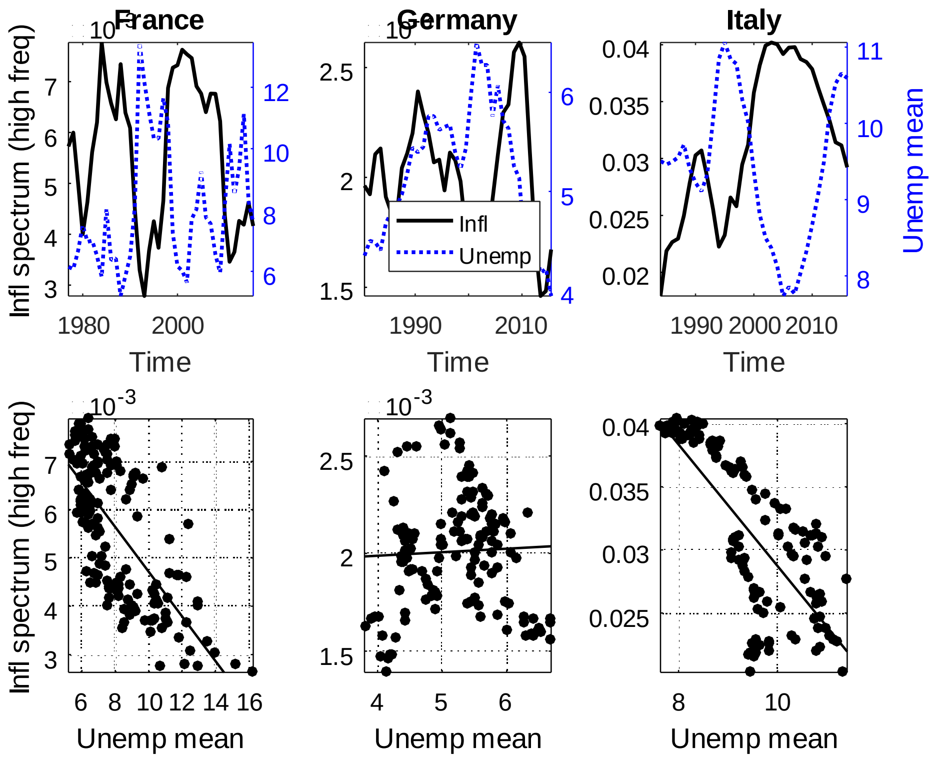

Figure . Using France, Germany, Italy data, the panels above compare normalized spectrum of CPI inflation at high frequency (black solid line - left y-axis) and unconditional unemployment rate (blu dotted line - right y-axis). The series at low frequency corresponds to the means of the same series at frequencies with less than 8 years per cycle. The three panels below show the scatter plot between normalized spectrum of CPI inflation at high frequency (y-axis) and unemployment rate (x-axis) for the same countries.