

Financial frictions and housing collateral constraints in a macro model with heuristics

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Abstract

The role of household debt in the real activity has attracted considerable attention recently mostly in the light of the observed increases in property prices and the increase of household indebtedness prior to the 2008 bust in many countries. The relevant literature on housing points to a number of the mechanism being likely to trigger or amplify real estate cycles (including bubbles). We focus on the interaction between banks and real estate developments, in particular assessing the implications of changing property prices on consumption decisions. We build on a previously described framework to introduce a real estate sector, accounting in itself for an explicit balance sheet dimension for consumers. The model thus results in an economy where - on the demand side - a collateral constraint limits households ability to borrow against the value of real estate, and - on the supply side - loan supply is constrained by bank capital. This allows studying the interactions of these two limits by drawing a stark distinction between the supply and demand for credit. While lending constraints are not a new feature of this framework, we take a step further and analyse the implications of lending constraints in a bounded rationality framework, proposing an extension of the model in De Grauwe and Macchiarelli (2015). Together with considering bounded rationality rules, the model features an endogenous mechanism for describing the probability distribution of housing bubbles.

Keywords: Animal spirits, financial frictions, housing collateral, credit cycle, loan-to-value ratio, capital requirement, interest rate spreads, endogenous housing bubble

JEL classifications: E03, E14, E44

1 Introduction

The role of household debt in the real activity has attracted considerable attention recently mostly in the light of the observed increases in property prices and the increase of household indebtedness prior to the 2008 bust in many countries. Practitioners and policy makers have hence rushed to incorporate a housing sector in their models mainly to understand the implications and interactions between monetary policy and financial stability. The relevant literature on housing points to a number of mechanisms being likely to trigger or amplify real estate cycles (including bubbles). Some of them are clearly related to nonfinancial characteristics, such as land supply, as in Carey (1990); construction lags and imperfect information (i.e. Herring and Wachter, 1999). In this paper, we focus on the interaction between banks and real estate developments through the interaction between asset prices (via banks' balance sheets) and borrowing constraints (Borio and Zhu, 2012; Brunnermeier and Pedersen, 2009; De Grauwe and Macchiarelli, 2015), in particular assessing the

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implications of collateral on consumption decisions.

We introduce a real estate sector, accounting in itself for an explicit balance sheet dimension for households/consumers; a dimension missing in De Grauwe and Macchiarelli (2015). In particular, the approach proposed herein mainly builds on the loans collateralized by real estate literature (i.e. Kiyotaki and Moore, 1997; then extended by Iacoviello, 2005; Iacoviello and Neri, 2010) where increasing real estate prices raise the market value of collateral on outstanding loans. In this literature, following real estate price changes, banks' risk will change, thereby making more (or less) difficult to obtain financing for households. As a result demand and prices will move pro-cyclically following real estate developments, suggesting that the use of real estate as a collateral will exacerbate credit cycles, especially having consequences on real estate developments themselves, hence having an impact on real activity. As underlined by Calza, Monacelli, and Stracca (2013), there is essentially one channel that is working in the housing to the macroeconomy direction: that is the extent to which a mortgage contract allows using housing as collateral into the credit availability for households. Credit can hence be used not only to finance new housing expenditure but also non-housing consumption. Of course, such a feedback loop between the housing sector and banks exists as long as raising real estate prices may encourage more lending to the housing and non-housing producing sector – i.e. the net worth of banks increase as banks' own holding of real estate rise in value. All arguments above suggest that the higher the exposure of banks to real estate, the more amplified the cycles.

In this paper, consistent with the model of De Grauwe and Macchiarelli (2015), loan supply is constrained by bank capital in our economy. Introducing this sort of (credit supply) constraint, in a model where - on the demand side - a collateral constraint limits households' ability to borrow against the value of real estate, as in the large literature spawned by Kiyotaki and Moore (1997), allows studying the interactions of these two limits by drawing a stark distinction between the supply and demand for credit. In the model, interactions between these two types of constraints may be non-trivial: on the credit supply side, a lending constraint can impede the flow of savings to the mortgage market. Likewise, a slackening of this constraint increases the funding available to borrowers, leading to lower mortgage rates and higher house prices, with no change in aggregate household leverage. On the contrary, an increase in the loan-to-value (LTV) ratio slackens the borrowing constraint and increases credit demand for given house prices, putting upward pressure on interest rates and leading to higher aggregate leverage (see also Justiniano, Primiceri, and Tambalotti, 2015). While lending constraints are a new feature of our framework, in this paper, we take a step further and analyse the implications of lending constraints in a bounded rationality framework, following De Grauwe (2012), De Grauwe and Macchiarelli (2015).

2 The Model

The following section presents the theoretical model, building on the macroeconomic model with financial frictions of De Grauwe and Macchiarelli (2015). In this new model, we introduce a housing sector producing new homes. As a result, aggregate demand in this economy is now made up of consumption, investment, and residential investment. Housing is expected to affect output both directly, via residential investment, and via consumption. Loan demanded by household is constrained by the value of housing collateral. For firms, loan demanded is constrained instead by their capital, consistent with De Grauwe and Macchiarelli (2015). Hence, banks accumulate households' and firms' collateral on their balance sheets and make loans to them. As usual, banks obtain funding only from households' deposits.

Overview

Our baseline behavioural macro model (BMM) consists of the usual Aggregate Demand (AD), Aggregate Supply (AS) and Taylor Rule (TR) equations (see De Grauwe, 2012; De Grauwe and Macchiarelli, 2015).

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1 - \hat{a}_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + \varepsilon_t \quad (a_2 < 0) \quad (1)$$

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \quad (2)$$

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 y_t + c_3 r_{t-1} + u_t \quad (3)$$

y_t is the output gap, r_t the nominal interest rate, π_t is inflation and ε_t is a white noise disturbance term. Note that $(r_t - \tilde{E}_t \pi_{t+1})$ is the real interest rate. $\tilde{E}_t (\cdot)$ is the expectation operator, which is different from the standard “rational expectation” term where the representative consumer is assumed to forecast using all available information. There is no direct derivation of the model at the micro level, albeit equations (1) to (3) are consistent with the log-linearized version of most New Keynesian models. The baseline model, based on Smets and Wouters (2007), shows for instance how the AD is equivalent to log-linearizing the Euler equation for consumer optimization, to which a smoothing term is added (see for instance Woodford 2003).

The AS in eq. (2) is the forward-looking New Keynesian version based on Calvo-pricing (see, for instance, Clarida et al. 1999; Woodford, 2003) when $b_1 = 1$ (Branch and McGaugh, 2009). As before, this equation can be obtained by log-linearization around the steady state. With $0 < b_1 < 1$ the relation incorporates an inertial term in the vein of Fuhrer and Moore (1995), Gali and Gertler (1999) and Woodford (2003).

The introduction of banks in De Grauwe and Macchiarelli (2015) happens through the decomposition of the AD equation into consumption and investment sides of the economy. Here, we refrain from reporting the full model and focus on highlighting the new features of the model. The full mode is detailed in a technical Annex.

Households

Household consumption and residential investment are financed by banks’ lending. Differently from firms, however, households face a borrowing constraint (see Calza, Monacelli, and Stracca, 2013; Gerali *et al.*, 2010).

Consistent with the literature on housing (i.e. Iacoviello and Neri, 2008), we shall distinguish between patient households (c), who largely save and make deposits, from impatient households (c'), who have negative savings and need to borrow.

For patient households, the consumption equation is:

$$c_t = \sigma d_1 y_t + \sigma d_2 \tilde{E}_t y_{t+1} + \sigma (1 - d_1 - d_2) y_{t-1} + d_3 (r_t - \tilde{E}_t \pi_{t+1}) + v_t^c \quad (4)$$

where σ is the fraction of income available to the unconstrained household. For impatient households it is true that $d'_1 > d_1$, corresponding to a lower subjective intertemporal discount factor, $\beta' < \beta$, in the standard Euler equation. The impatient household’s consumption is thus

$$c'_t = (1 - \sigma)d'_1 y_t + (1 - \sigma)d_2 \tilde{E}_t y_{t+1} + (1 - \sigma)(1 - d'_1 - d_2)y_{t-1} + d_3(\rho_{t,h} - \tilde{E}_t \pi_{t+1}) + v'_t \quad (5)$$

Here, $\rho_{t,h}$ is the borrowing rate impatient households face, in the light of the financial friction we shall introduce later on.

The specification above makes the total consumption, $c + c'$, comparable to the model in De Grauwe and Macchiarelli (2015). Overall consumption depends on current income, according to the parameter d_1 which represents the marginal propensity of consumption (MPC); on the expected future output gap, $\tilde{E}_t y_{t+1}$, and on past income (see also De Grauwe, 2008a; 2008b; 2012; Smets and Wouters, 2007). Finally, consumption depends negatively on the *ex ante* real interest rate, for $d_3 < 0$. The difference between patient and impatient households also depends on the real interest rate they face.

In equilibrium, patient households own the land. The land endowment is normalised to unity following a standard practice in the literature (e.g., Walentin, 2014).

The patient households maximise their utility subject to the budget constraint (Calza, Monacelli and Stracca 2013; Walentin, 2014; Justiniano, Primiceri, and Tambalotti, 2015):³

$$l_{t,h}^D - l_{t-1,h}^D + \sigma y_t = c_t + q_t[h_t - (1 - \delta_h)h_{t-1}] \quad (6)$$

where $l_{t,h}^D$ is the households' demand for loans, q_t is the real price of housing, h_t is the housing demand and δ_h is depreciation. Impatient households' borrowing and lending are related by a similar rule:

$$l_{t,h'}^D - l_{t-1,h'}^D + (1 - \sigma)y_t = c'_t + q_t[h'_t - (1 - \delta_h)h'_{t-1}] \quad (7)$$

The impatient households' problem is subject to a limited commitment problem: households cannot commit to repay more than the present value of the housing stock after depreciation (δ_h). This is represented by the collateral constraint below (see also Justiniano, Primiceri, and Tambalotti, 2015; Walentin, 2014):

$$\rho_t l_{t,h'}^D \leq (1 - m)(1 - \delta_h)h'_t \tilde{E}_t q_{t+1} \quad (8)$$

In equation (6), m is the fraction of housing that cannot be used as a collateral. Here, an expected future household appreciation contributes to the ability to borrow in the current period. As in Calza, Monacelli and Stracca (2013), one can think at the parameter m as the inverse of the loan-to-value (LTV) ratio, therefore representing a measure of flexibility of the mortgage market (Jappelli and Pagano, 1989, Calza, Monacelli and Stracca, 2013). q_t is the real price of housing.

Here, the expected future stream of income generated from the dwelling affects the household's current ability to borrow. Putting equations (7) and (8) together yields a clear interpretation of the impatient households' budget condition (Calza, Monacelli, and Stracca, 2013; Justiniano, Primiceri, and Tambalotti, 2015), i.e.

$$c'_t + q_t[h'_t - (1 - \delta_h)h'_{t-1}] - (1 - \sigma)y_t = (1 - m)(1 - \delta_h)W_t$$

where

³ This maximization of patient households is also subject to a collateral constraint analogous to the one for impatient households. In equilibrium, because of $\beta > \beta'$, the collateral constraint is never binding for patient households (see also Walentin, 2014).

$$W_t = \frac{h'_t \bar{E}_t q_{t+1}}{\rho_{t,h}} - h'_{t-1} \bar{q}$$

is the present value of *equity*. The latter is the difference between the present value of the household asset and the value of the current interests rate obligations on a loan contract purchased in the previous period. The discount factor here takes into account the spread households have to pay on top of the risk-free interest rate in the economy.

The borrowing rate for households is derived as a function of households' wealth. This has a similar interpretation that the *financial accelerator* parameter for banks (see De Grauwe and Macchiarelli, 2015), e.g. Justiniano, Primiceri, and Tambalotti (2015), whereby the economy is characterised by banks' asymmetry of information. In particular, we assume banks' evaluate the history of a household's net worth, as:

$$x_{t,h} = \theta W_{t-1} \quad (\theta < 0) \quad (9)$$

This households' spread term is defined as

$$\rho_{t,h} = r_t + x_{t,h} \quad (10)$$

making impatient households' consumption demand sensitive to a finance premium. This equation replaces the real interest rate in the impatient households' consumption equation (e.g., Walentin, 2014), as shown in equation (5).

Bank lending typically implies (stocks of) patient households' savings to flow into deposits as in De Grauwe and Macchiarelli (2015), i.e.

$$D_t - D_{t-1} = s_t \quad (11)$$

Firms

As in De Grauwe and Macchiarelli (2015), we assume intermediate goods are normalised to unity.⁴ Firms are nevertheless responsible for residential investment, which is generated as

$$IH_t = A_{h,t}(1 - \mu)y_t \quad (12)$$

Where, broadly consistent with the literature, $A_{h,t}$ is the productivity of the housing sector which is a driftless AR(1) process (see Annex). μ is the share of the housing sector in the total economy (see also Walentin, 2014).

Firms' loan demand (L^D) and investment are related by:

$$i_t = l_{t,f}^D - l_{t-1,f}^D \quad (13)$$

As in De Grauwe and Macchiarelli (2015), the spread between the borrowing and the deposit rate is derived using the *financial accelerator* approach of Bernanke, Gertler, and Gilchrist (1999). Defining n^f as firms' equity, we write:

⁴ For a relaxation of this normalization see De Grauwe and Gerba (2016), where an explicit production function is considered.

$$x_{t,f} = \rho_{t,f} - r_t = \varphi n_t^f \quad (\varphi < 0) \quad (14)$$

Thus, an increase in the firms' equity reduces the spread and *vice-versa*.⁵ The underlying *financial accelerator* theory is the following. Banks have imperfect knowledge of the credit risk they take when granting a loan to a firm. To cover this credit risk they charge a spread ($x_{t,f}$). When the value of equity of the firm increases this is interpreted by the bank as an improvement in the solvency of the firm. Thus banks will perceive the credit risk to have declined allowing them to reduce the spread. A decline in the value of the firms equity has the opposite effect. Such a decline is interpreted as reducing the solvency of the firm and increasing the credit risk. Banks react by raising the spread.

Given the existence of banks, investment depends now on the expected future output gap and, negatively, on firms' borrowing rate ($\rho_{t,f}$), owing to banks' incorporating the information asymmetry in the economy. In other words, when a firm borrows money from a bank, it must pay an interest which normally exceeds the interest rates that savers receive for deposits. Hence, the cost of a loan from banks, ρ_f , is usually equal to the rate savers receive (here equal to the *risk-free* rate set by the central bank, r) plus a spread, x_f

$$\rho_{t,f} = r_t + x_{t,f} \quad (15)$$

The investment demand is hence dependent on the cost of bank loans (see De Grauwe and Macchiarelli, 2015)

$$i_t = e_1 \tilde{E}_t y_{t+1} + e_2 (\rho_{t,f} - \tilde{E}_t \pi_{t+1}) + v_t^i \quad (e_2 < 0) \quad (16)$$

In this way, the spread is assumed to affect the overall investment decisions of firms directly. Firms' market equity, in particular, is equal to the number of shares (\bar{n}) multiplied by the current share price (S_t), i.e.

$$n_t^f = \bar{n} S_t \quad (17)$$

as in De Grauwe and Macchiarelli; where, to derive share prices, we use the standard Gordon discounted dividend model (see De Grauwe and Macchiarelli, 2015)

As we are interested in understanding the role of the housing market in the business cycle, we do not allow first to adjust the quantity of shares. Firms' leverage is negatively related to prices only, see De Grauwe and Macchiarelli (2016).⁶

$$\tau_t = \frac{l_{t,f}^D}{\bar{n} S_t} \quad (18)$$

To complete the model, we need to characterise the evolution of net worth. In De Grauwe and Macchiarelli (2015), it is shown that this must follow:

$$\bar{n}_t S_t = \frac{1}{\tau_t} \{ l_{t-1,f}^D + e_1 \tilde{E}_t y_{t+1} + e_2 (\rho_{t,f} - \tilde{E}_t \pi_{t+1}) + v_t^i \}$$

⁵ A further extension could be assuming the spread to depend on banks equity as well, in the vein of Gerali *et al.* (2010). When banks' equity decreases the leverage for a bank would increase and financing new investment opportunities would become riskier. The natural reaction will be for a bank to reduce its assets and stop lending, thus increasing the spread.

⁶ Differently from De Grauwe and Macchiarelli, 2015.

whereby firms' net worth is consistent with the inverse of the (time-varying) leverage ratio (see also Bernanke and Gertler, 2001).

Banks

The balance sheet of commercial banks accounts for loan supply (l^S), on the asset side, and deposits (D), on the liability side, where bank equity (n^b) is defined as the difference between assets and liabilities. A typical bank's balance sheet would take the form of the extended model of De Grauwe and Macchiarelli (2015), allowing asset prices to feedback on the bank's balance sheet. Here, in particular, it is assumed that banks own a fixed fraction, \bar{n} , of firms' shares, plus households' posted collateral.⁷ The latter represents the introduced effect of housing on banks' balance sheets.

Tot. Assets (A)	Liabilities (B)
Loans (l^S)	Deposits (D)
Risky assets ($\bar{n}S$) from firms	Equity (n^b)

In this way, the sum of total assets, A_t , to be considered in the banks' balance sheet identity is simply:

$$A_t = l_t^S + \bar{n}S_t \quad (20)$$

and the balance sheet constraint of the banks is given by the following,

$$A_t = n_t^b + D_t \quad (21)$$

For a commercial bank, the leverage is thus the ratio between its loans and equity. In this respect, banks are subject to an explicit capital-to-asset ratio, i.e.

$$\frac{n_t^b}{A_t} = \kappa \quad (22)$$

with κ being the equity ratio (the inverse of banks' leverage ratio).

Monetary policy

In the light of the housing sector, the Taylor rule is modified to include total output (Y_t), as in Walentin (2014), with the following equation substituting equation (3):

$$r_t = c_1(\pi_t - \pi_t^*) + c_2GDP_t + c_3r_{t-1} + u_t \quad (3')$$

Where GDP is the sum of the value added of the real and the housing sector at steady-state house prices (Walentin, 2014), consistent with the idea of the central bank targeting the gap i.e.

⁷ Banks hold only a fixed fraction of firms' shares, as making banks hold firms completely would otherwise erode incentives for banks to charge an excess premium over firms' lending.

$$GDP_t = y_t + \bar{q}IH_t$$

Market clearing

The household spread ($x_{t,h}$), together with the spread faced by firms, clear the lending market determining the total level of loanable funds to firms and households. Total loan demand will thus depend on investment and the present value of household equity, whereas loan supply will be a function of what banks accumulate: firms and households' equity minus deposits. As in De Grauwe and Macchiarelli (2015), the market is cleared via banks' balance sheet for

$$l_t^S = l_{t,f}^D + l_{t,h}^D$$

Market clearing for goods implies

$$c_t + c_t' + i_t = y_t$$

Which in turn implies the following cross-equation restrictions necessary to move from equation (1) to (1'), consistent with De Grauwe and Macchiarelli (2015), $a_1 = \frac{d_2 + e_1}{1 - \sigma d_1 - (1 - \sigma) d_1'}$, $\hat{a}_1 = \frac{d_2}{1 - \sigma d_1 - (1 - \sigma) d_1'}$, $a_2 = \frac{2d_3 + e_2}{1 - \sigma d_1 - (1 - \sigma) d_1'}$, $a_3 = \frac{d_3}{1 - \sigma d_1 - (1 - \sigma) d_1'}$, and $\frac{v_t^i + v_t^c + v_t^{c'}}{1 - \sigma d_1 - (1 - \sigma) d_1'} = \varepsilon_t$ where $0 < \sigma d_1 - (1 - \sigma) d_1' < 1$. As shown in the technical annex, the relationship of output is negative with respect to both spread, consistent with the counter-cyclical nature of lending conditions modelled in the literature (e.g., Gerali *et al.*, 2010).

Analogously, for houses

$$h_t + h_t' - (1 - \delta_h)(h_{t-1} + h_{t-1}') = IH_t$$

Total savings s is determined as the difference between income and consumption of patient households:

$$s_t = \sigma y_t - c_t$$

In order to characterise the equilibrium household supply, we shall assume that patient household's utility function implies a rigid demand for houses at the level \bar{h} (e.g., Justiniano, Primiceri, and Tambalotti, 2015):

$$h_t = \bar{h}$$

At this point, by putting equation (6) and (11) together one realises patient households have a negative demand for loans, which is essentially equal to their supply of deposits. The opposite holds true for impatient households, whose savings are negative.

Endogenous housing bubbles

In order to have a close-end solution, we have assumed a data generating process for the expected real price of housing $\tilde{E}_t q_{t+1}$. We assume a bubble materializes once the real

housing price departs from the its fundamental ratio, which we define as the house price-to-income ratio δ (see also Calza Monacelli and Stracca, 2013). Following Bernanke and Gertler (1999; 2001), the economy expects a housing bubble with a given probability $\alpha_{q,t}$, whereas the bubble bursts with probability $(1 - \alpha_{q,t})$. Thus, we define a high housing price expectation regime (H)

$$\tilde{E}_t^H q_{t+1} = q_t - \delta y_t = q_{t-1} - \delta y_{t-1}$$

and a low expectation regime (L)

$$\tilde{E}_t^L q_{t+1} = q_t - \delta y_t = 0$$

Differently from Bernanke and Gertler (1999; 2001), the probability of ‘house’ overpricing is endogenous in our model. Market forecasts of housing prices are then obtained in each period as a weighted average of each respective expectational regime.

$$\tilde{E}_t q_{t+1} = \alpha_{q,t} \tilde{E}_t^H q_{t+1} + (1 - \alpha_{q,t}) \tilde{E}_t^L q_{t+1}$$

This is a novel feature in the housing literature as it allows endogenizing the evolution of the bubble’s probability distribution, as the probability of high and low regimes is time-varying. We base the regime rule mechanism on a dynamic predictor selection, in line with discrete choice theory. This mechanism allows switching between the two regimes by computing the utility function conditional on each regime and increase (decrease) the relative weight of one rule against the other in each period. Under the formalisation that the utilities of the two alternative rules have a deterministic and a random component – and assuming the latter to be logistically distributed (see Manski and McFadden, 1981; Anderson, De Palma, and Thisse, 1992) – the probability can be defined based on each period profitability π^q (see also Leitch et al., 1991; Levich, 2001)

$$\alpha_{q,t} = \frac{\exp(\gamma \pi_{H,t}^q)}{\exp(\gamma \pi_{H,t}^q) + \exp(\gamma \pi_{L,t}^q)}$$

where

$$\pi_{j,t}^q = (q_t - \delta y_t) \text{signal}(\tilde{E}_t^j q_{t+1}) \text{ for } (j = H, L)$$

and

$$\text{signal}(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z = 0 \\ -1 & \text{if } z < 0 \end{cases}$$

For any generic asset, the $\text{signal}(\tilde{E}_t^j q_{t+1})$ function indicates the predicted direction of changes of housing price. This can either be decrease ($\tilde{E}_t^j q_{t+1} < 0$), increase ($\tilde{E}_t^j q_{t+1} > 0$) or no change ($\tilde{E}_t^j q_{t+1} = 0$).

In doing so, we construct buying and selling signals on the housing market. When markets forecast an increase in housing prices above the fundamental value (the market is “bubbly”) and such an increase is realised, market’s profit will equal the net observed increase in price. Vice versa when q_t declines, i.e. markets will realize a per-unit loss equal to the drop in the housing price.

The probability of a bubble α_t^q can then be understood as follows. When the rate of return of investing in the housing market increases, more and more agents step into the market, attracted by higher profits. This will push the probability of a bubble further up. Viceversa, when the realized profits are decreasing, less agents would be tempted to invest, leading to a bubble to burst. Such a mechanism governs the switch between two rules. In the weight function, the parameter γ measures the intensitivity with which agents switch between these two forecasting rules. In particular, for $\gamma \rightarrow \infty$, agents will choose the rule that is more profitable in every period. Hence, α_t^q will assume values which can be either zero or one; i.e. agents will always choose the "best" forecasting rule, among the ones availables, with probability one. Alternatively, when $\gamma = 0$ the weights associated to the two rules will become constant and equal 0.5, being equivalent to a random selection of forecasting rules.

Expectations dynamics

Under rational expectations, the forward-looking terms in the model, which are the expectations of the output gap and inflation gap at time $t + 1$ would be given by $\tilde{E}_t y_{t+1} = E_t y_{t+1}$ and $\tilde{E}_t \pi_{t+1} = E_t \pi_{t+1}$.

As in De Grauwe (2012) and De Grauwe and Macchiarelli (2015), we depart from rational expectations by considering instead a behavioural approach consistent with the idea that agents formulate different forecasts which are biased by definition (for a survey of the use of expectations in macroeconomic models, see Milani, 2012). The selection of the forecasting rules depends on the forecast performances of these rules given by a publicly available fitness measure, which is updated in every period. For each variable, we reduce the problem to a binary predictor choice (Brock and Hommes, 1997, 1998). After the equilibrium is revealed, predictors are evaluated *ex-post* and new fractions are being determined (see De Grauwe and Macchiarelli, 2015). Agents' rationality in this model thus consists in the agents' ability to choose the predictor that performs better based on its past performance.

Agents are supposed to forecast the output gap (y) and inflation (π) using two alternative forecasting rules. In particular, for $\tilde{E}_t y_{t+1}$, agents are assumed to use the steady-state value of the output gap (*fundamentalist rule*) - y^* , here normalized to zero - against a naïve forecast based on the gap's past observation (we call it *extrapolative rule*). Analogously for inflation, agents are assumed to switch between a fundamentalist and an extrapolative rule. Further, in an environment where the central bank explicitly announces its inflation target, inflation fundamentalists are assumed to base their expectations on the central bank's target, π_c^* . In contrast, inflation extrapolators behave exactly as output extrapolators do: by naively forecasting inflation based on inflation's last available observation (see technical Annex).

3 Solving the model

We obtain a model with 5 endogenous variables, inflation, the output gap, the two spreads, and savings. These are obtained by solving the following set of equations:

$$\begin{bmatrix} 1 & -b_2 & 0 & 0 & 0 \\ -a_2 c_1 & 1 - a_2 c_2 (1 + \bar{q} A_h (1 - \mu)) & -(a_2 - 2a_3) & -a_3 & 0 \\ -\varphi \tau^{-1} e_2 c_1 & -\varphi \tau^{-1} e_2 c_2 (1 + \bar{q} A_h (1 - \mu)) & (1 - \varphi \tau^{-1} e_2) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ d_3 c_1 & -(\sigma(1 - d_1) - d_3 c_2 (1 + \bar{q} A_h (1 - \mu))) & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ x_t^f \\ x_t^h \\ s_t \end{bmatrix} =$$

$$\begin{aligned}
&= \begin{bmatrix} b_1 & 0 & 0 & 0 & 0 \\ -a_2 & a_1 & 0 & 0 & 0 \\ -\varphi\tau^{-1}e_2 & \varphi\tau^{-1}e_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ d_3 & -\sigma d_2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{E}_t\pi_{t+1} \\ \tilde{E}_ty_{t+1} \\ \tilde{E}_tx_{t+1}^f \\ \tilde{E}_tx_{t+1}^h \\ \tilde{E}_ts_{t+1} \end{bmatrix} + \\
&+ \begin{bmatrix} 1-b_1 & 0 & 0 & 0 & 0 \\ 0 & 1-\hat{a}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\sigma(1-d_1-d_2) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ x_{t-1}^f \\ x_{t-1}^h \\ s_{t-1} \end{bmatrix} + \\
&+ \begin{bmatrix} 0 & 0 & 0 \\ d_2c_3 & 0 & 0 \\ \varphi\tau^{-1}e_2c_3 & \varphi\tau^{-1} & 0 \\ 0 & 0 & \theta \\ -d_3c_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_{t-1} \\ l_{t-1}^D \\ W_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 1 & 0 & 0 \\ 0 & \varphi\tau^{-1}e_2 & 0 & \varphi\tau^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -d_3 & 0 & 0 & -\sigma \end{bmatrix} \begin{bmatrix} \eta_t \\ u_t \\ \epsilon_t \\ v_t^i \\ v_t^c \end{bmatrix}
\end{aligned}$$

In compact form, this is written as $\mathbf{AZ}_t = \mathbf{B}\tilde{\mathbf{E}}_t\mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{DX}_{t-1} + \mathbf{E}v_t$. As in De Grauwe (2008a; 2008b; 2011; 2012), a solution for \mathbf{Z}_t is obtained as $\mathbf{Z}_t = \mathbf{A}^{-1}(\mathbf{B}\tilde{\mathbf{E}}_t\mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{DX}_{t-1} + \mathbf{E}v_t)$ with the only condition being \mathbf{A} to be non-singular. Some comments are warranted in the next section.

We then obtain the value of the interest rate recursively by substituting output gap and inflation into the Taylor rule. Investment (including residential investment), equity, deposits, and housing price are determined by the model solutions for inflation, total output, the spreads, and savings.⁸

4 Calibration

The parameters are calibrated consistently with De Grauwe and Macchiarelli (2015). In particular, the banking sector parameters are calibrated as follows: d_1 is computed to match a marginal propensity to consume (MPC) out of income equal to 0.2 for patient households and 1.1 for impatient households (see also Friedman, 1963; Iacoviello and Neri, 2010; for a survey see Carroll, 2012). The firms' leverage factor is set equal to $\tau = 1.43$, following Pesaran and Xu (2013) and $\varphi < 0$, as in De Grauwe and Macchiarelli (2015). Banks' equity ratio (the inverse of the leverage ratio, k) is set equal to 0.09, consistent with the target parametrization in Gerali *et al.* (2010).⁹ The new parameters for the housing collateral constraint are: a loan-to-value ratio $m = 0.4$ as in Calza Monacelli and Stracca (2013); the quarterly physical depreciation rate for households, $\delta_h = 0.003$ as in Justiniano, Primiceri and Tambalotti (2016), a catch-all variable housing sector share $\mu = 0.30$ as in Walentin (2014), and finally the price to income ratio, δ , is set equal to 0.27 consistent with the historical US norms (see Harvard Joint Center for Housing Studies; i.e. Calza Monacelli and Stracca (2013) report a house price to consumption price ratio equal to 24% for the US). The parameters that govern the selection mechanism across rules output and forecasting rules are

⁸ The spread and saving do not need to be forecasted. This does not affect the dynamics of the model (i.e. there is no structure of higher order beliefs as the LIE clearly does not hold in our framework; see Section 3.1).

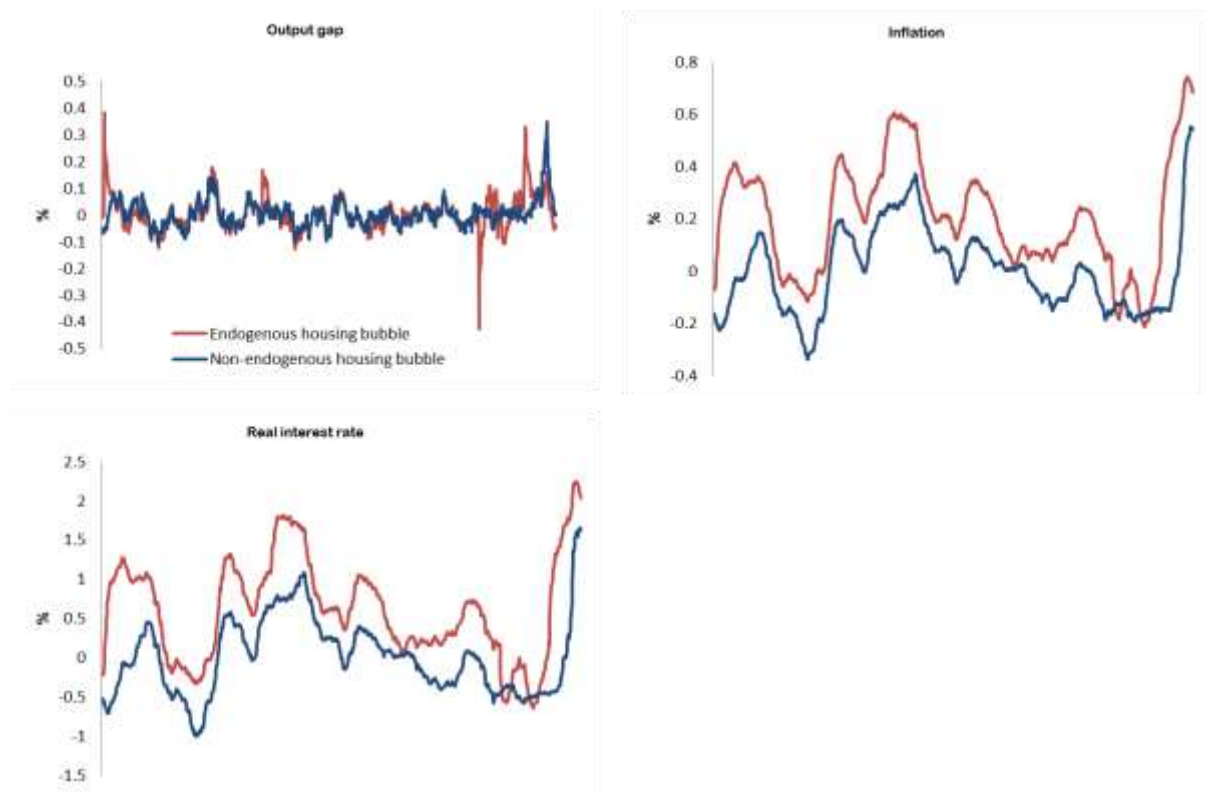
⁹ Note that the (risk weighted) capital ratio in Basel III, comprising common equity and a capital conservation buffer, is equal to 7%. Basel III also assumes the possibility of a countercyclical buffer, which is imposed within a range of 0 - 2.5%, bringing the common equity standard to an interval of 7 - 9.5% (Basel Committee on Banking Supervision, 2011). In section 6, we perform a sensitivity analysis by looking at the robustness of our results by varying banks' optimal equity ratio in the interval 0 - 18%.

calibrated as in De Grauwe (2011), De Grauwe and Macchiarelli (2015), which is $\gamma = 1$ and $p = 0.5$.

5 Results

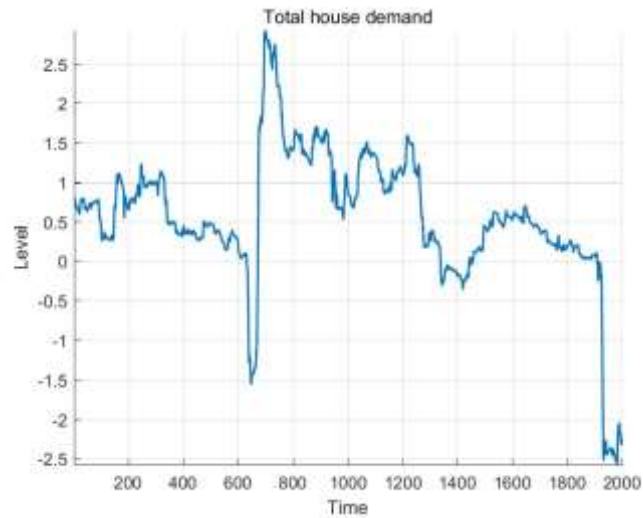
In this section, we present our main results. Together with the result of the simulation, we present some results with the housing price not varying endogenously, according to what described in the previous section, but rather being calibrated to the US historical norms based on the price to income ratio (with income being time-varying). Figures 1 shows the time pattern of output gap and animal spirits produced by the baseline model with banks and housing in a typical simulation run, where housing market is not generating the endogenous dynamics for prices. The results of the model with endogenous housing dynamics are presented in Figures 1, red line. In all cases, we observe a strong cyclical movement in the output gap.

Figure 1
Comparison of key series with and without endogenous housing bubble mechanism

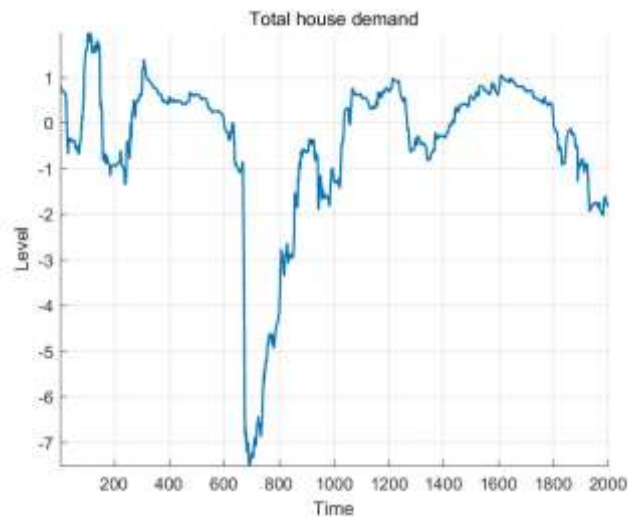


The source of these cyclical movements is seen to be the index of animal spirits in the market (Figure 2). As in De Grauwe and Macchiarelli (2015), we define animal spirits as the fraction of agents who forecast a positive output gap. When this fraction is 1 all agents forecast such a positive output gap, i.e. they are all optimistic. When the fraction is zero all agents forecast a negative output gap; they are all pessimistic. The model creates endogenous waves of optimism and pessimism. These are highly correlated with movements in the output gap (see also De Grauwe and Macchiarelli), with correlation between output and animal spirits being as high as 0.7. This reflects in the total house demand dynamics as well (Figure 2).

Figure 2
 (a) No endogenous “housing bubble”



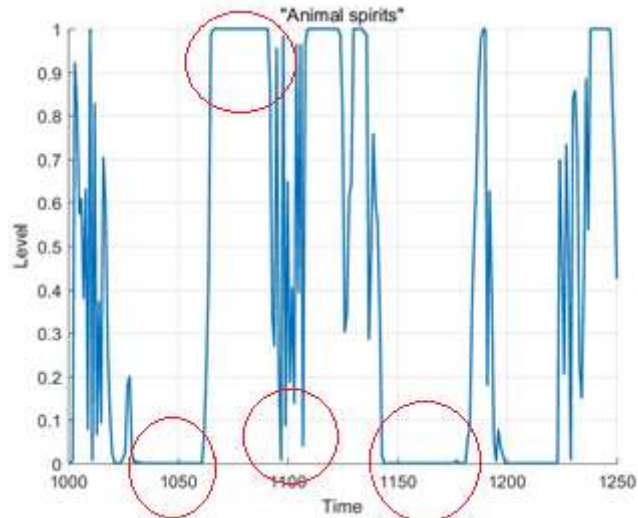
(b) Endogenous “housing bubble”



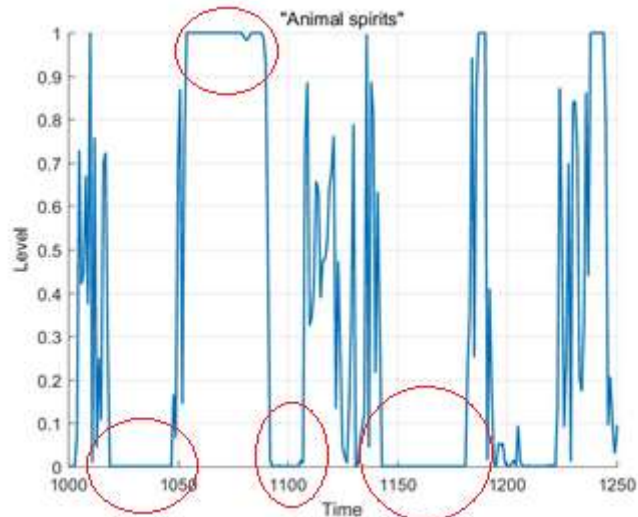
When the housing market is allowed to have endogenous dynamics, cyclical dynamics are accentuated in the output gap series, as well as in the total household demand, the latter generating longer boom and bust cycles. One of the reasons behind these renewed dynamics is that animal spirits are “strengthened” displaying somehow longer waves of optimism and pessimism (Figure 3).

The explanation is however that when agents are allowed to enter the housing market and the expected price is driven by profit-making, the “market frenzy” explained in De Grauwe and Macchiarelli (2015), typically describing large movements in real economic activity, is “exported” also to the financial housing cycle. This is in line with the recent literature pointing out how financial “super-cycles” may have an effect on the real economy and the business cycle, accentuating each other and become magnified, especially during coincident cyclical episodes in credit and housing markets. A key ingredient in the model of De Grauwe and Macchiarelli (2015) is a pro-cyclical boom-bust cycle in credit, following a tightening or loosening of credit conditions. We show that this interaction with the housing market leads to further amplifications, as an endogenous housing bubble mechanism increases correlations in beliefs making them more and more likely. Once again, this is only explained by the new endogenous housing bubble mechanism.

Figure 3
 (a) No endogenous “housing bubble”



(b) Endogenous “housing bubble”



The endogenous house-pricing mechanism, which we interpret as the probability of house over-pricing, is discussed below. This probability equal to 1 describes the fraction of people forecasting a housing price above its fundamental value. The results suggest that, on the overall simulation run, house market search for profits is hectic, with shifts between the two rules being particularly frequent. This overall leads to an amplification of the business cycle in the model.

In Figure 4 we present the probability distribution of house over-pricing, together with its representation in the frequency domain (Figure 5). What we gauge is that the model produces several twists and turns in the market alone, or as much polarisation, creating a complex dynamics. Polarisation is particularly present at masses $a_{q,t} = 0$, $a_{q,t} = 0.5$ and $a_{q,t} = 1$.

Figure 4

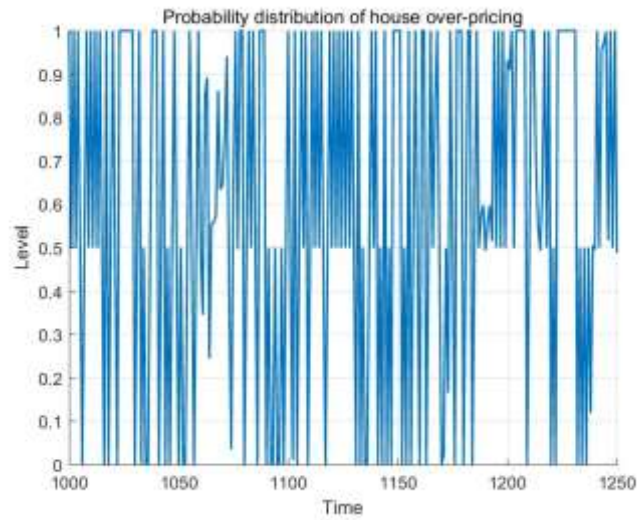
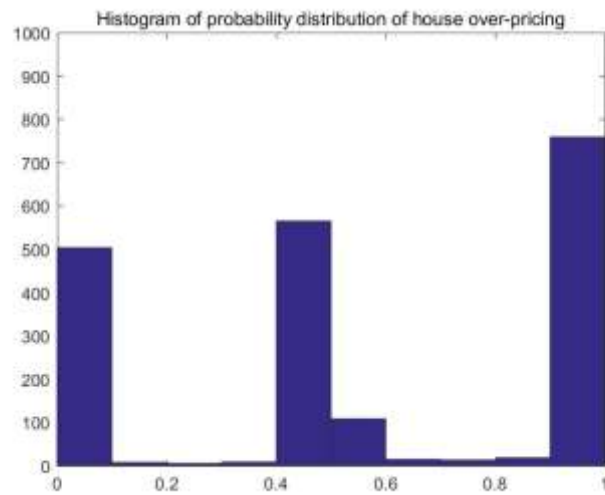
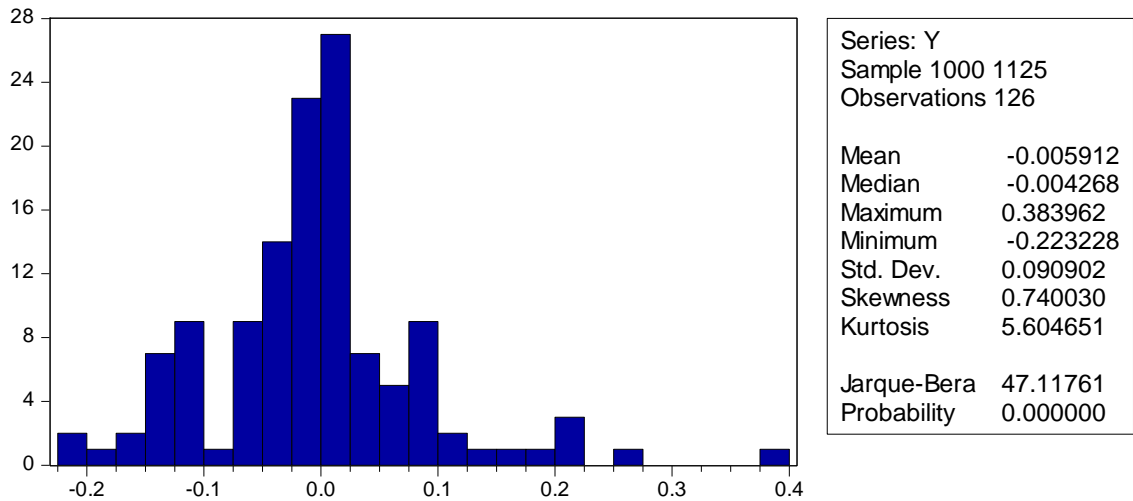


Figure 5



In Figure 6 we present the frequency distribution of the output gap. It turns out that this distribution is always not normal. It has fat tails: kurtosis is 5.6 on a 125 period (equal to 30 years) with the endogenous housing bubble, with normality being rejected in all cases based on the Jarque-Bera test.

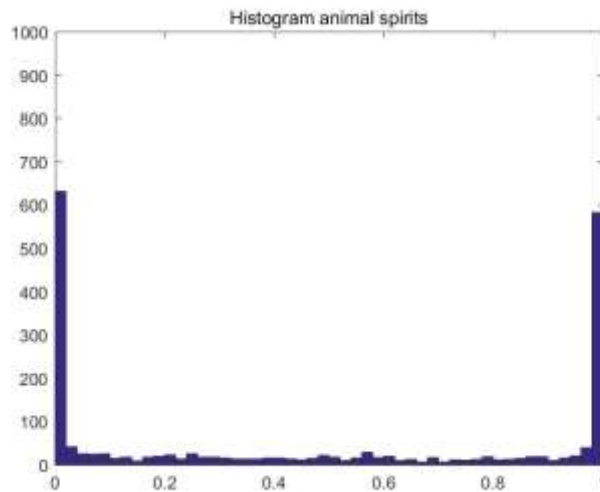
Figure 6



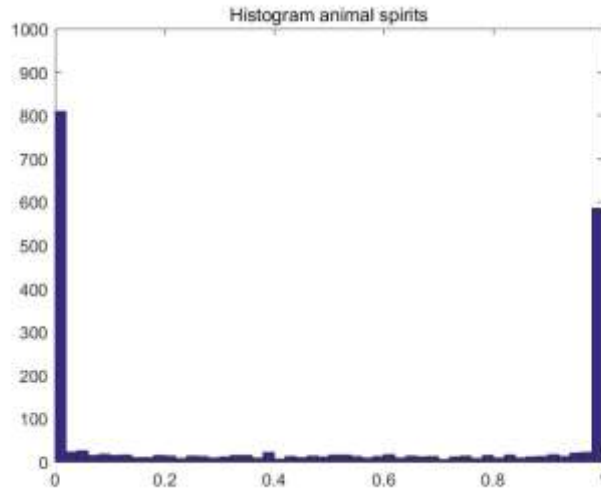
As in De Grauwe and Macchiarelli (2015), the stochastic shocks in the model are all *i.i.d.* thus the non-normality of the distribution of the output gap is generated by the model. The mechanism in our model that produces the non-normality is not coming from the endogenous housing market behaviour, as Figure 6(a) demonstrates, but rather the animal spirits in the model. When the housing bubble mechanism is “switched on”, this produced more non-normality in the time-series.

We obtain more insight into this mechanism by plotting the frequency distribution of animal spirits in Figure 7. The latter is just a representation of the same animal spirits of Figure 2 but in the frequency domain. We observe that there is a concentration of observations at the extreme values of +1 (everybody is an optimist) and 0 (everybody is a pessimist). Thus, the dynamics of animal spirits are characterised by movements in optimism and pessimism, leading to “market frenzy” and large movements in economic activity. This is accelerated when a housing market bubble mechanism is allowed in the model.

Figure 7
(a) No endogenous “housing bubble”



(b) Endogenous “housing bubble”

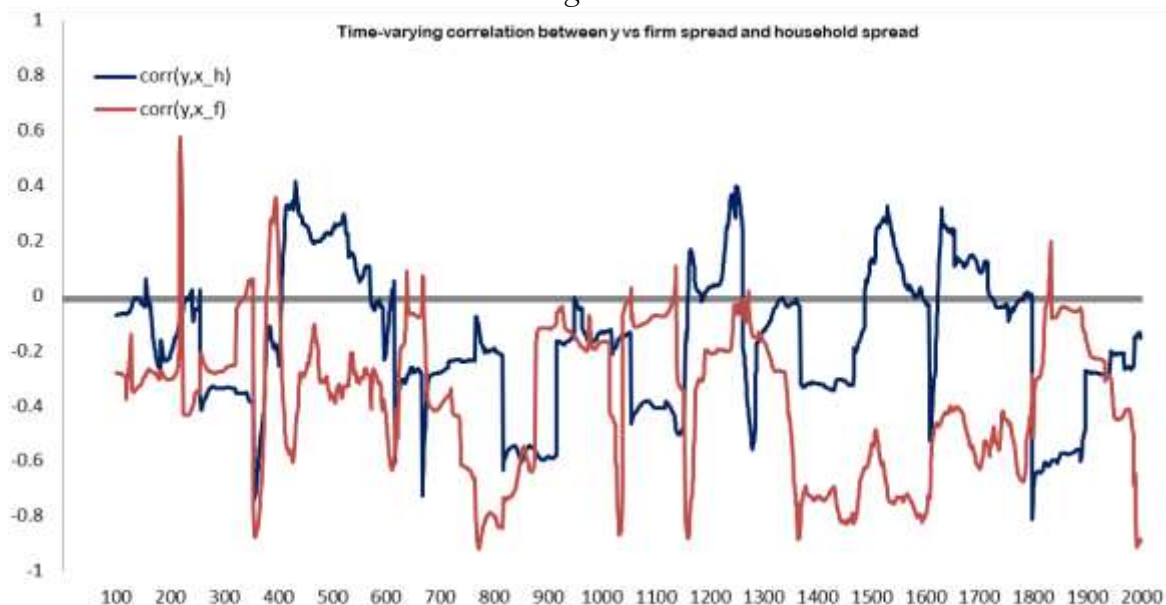


With respect to the evolution of the firms' and the households' spread we compute the time-varying correlation of each spread with respect to the real activity (Figure 8) as well as the expected house price (Figure 8), in the model where the house price is endogenous. Consistent with the financial accelerator literature, firms' spread display a pattern in which is negatively correlated with the real activity (-0.114) as well as with animal spirits (-0.082); consistent with De Grauwe and Macchiarelli (2015).

The reason for the counter-cyclicality of the spread is that during an upturn of economic activity when agents are optimistic, the value of equity of firms increases, leading banks to lower the spread. The opposite occurs during a downturn.

In the model, we do also observe a negative and stronger correlation between the real activity and the housing spread. The households' spread is more negatively correlated with the real activity on average than the firms' spread itself (-0.31) and animal spirits (-0.084).

Figure 8



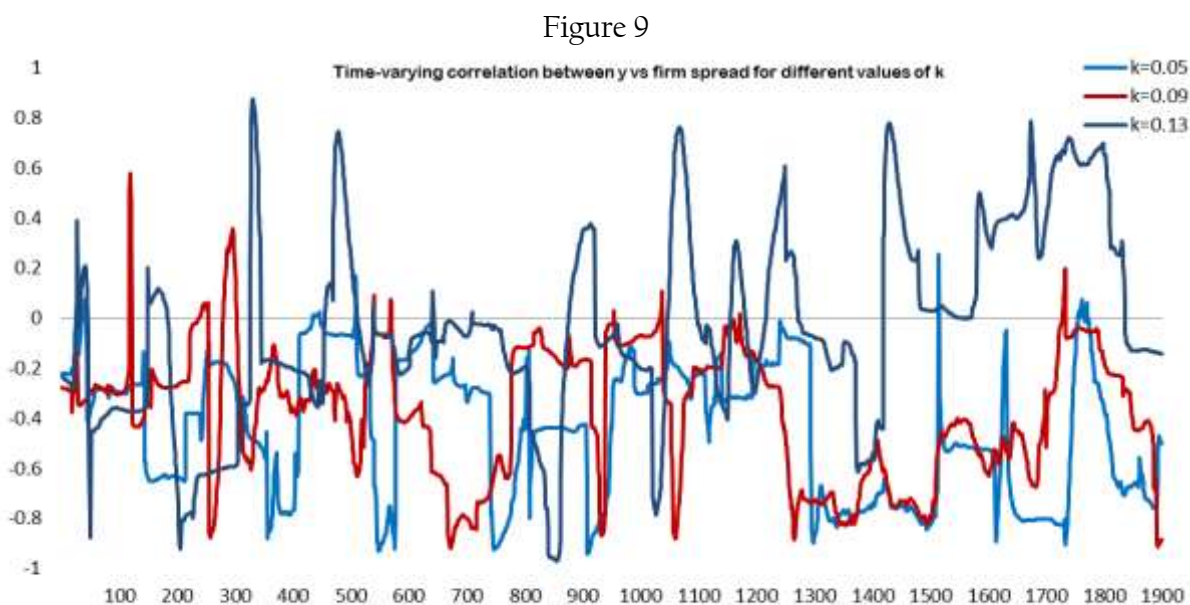
From Figure 8 one can also notice that there are phases in which the banking and the housing cycles are incidental. This is an interesting feature of this model and it is due to phases of negative correlation with the two spreads, consistent with the idea that house price over evaluations as well as share prices tend to relax collateral constraints, expanding the supply of credit to both households and firms in these periods.

5.1 Regulatory implications

The crisis has also led to a re-examination of some of the conventional policy lessons (see Blanchard 2011). In particular, it has led to an extensive debate about how monetary policies should take into account financial outcomes (see Cardarelli et. al. 2008) This debate has emphasised not just the link between inflation and business cycles, but also the importance of improving our understanding of financial cycles and their implications for business cycles. Our previous analysis seems to suggest that it is important to account for the interactions among cycles in different financial market segments when designing regulatory policies aimed at ensuring the overall health of the financial system, especially in terms of the design of macroprudential rules.

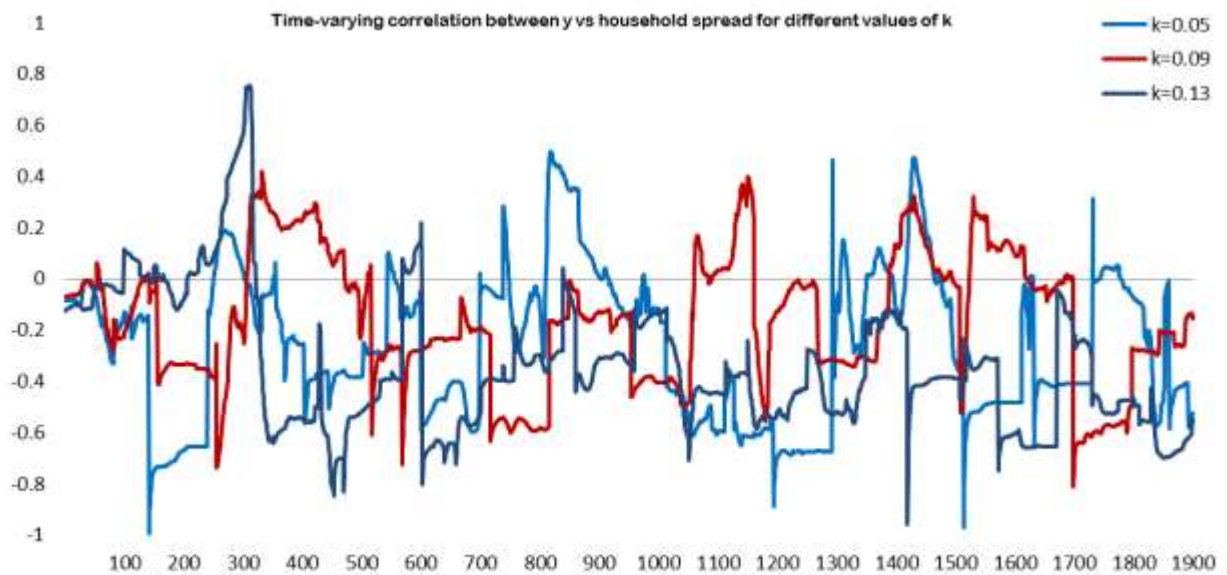
In what follows we particularly explore some of the implications of the model in terms of varying the following key parameters, such as the banks capital requirement, thus reducing the supply of credit.

Changing banks capital requirement, suggest indeed a pattern towards decreasing the correlation between the firms' spread and output. This relationship is weakened in particular as the inverse of the capital to asset ratio is tightened to 13%, as Figure 9 shows.



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Figure 10



6 Tentative conclusions

Our behavioural model produces self-fulfilling movements of optimism and pessimism (animal spirits), consistent with De Grauwe and Macchiarelli (2015). Banks intensifies these movements as credit availability (for both firms and households) works pro-cyclically by intensifying the volatility of output. Endogenous house price dynamics have equally a strong acceleration effect.

The model produces several phases when the financial and housing cycles are incidental. The prevailing view that excessive credit growth requires banks to hold excess regulatory capital. Credit growth is sometimes a good indicator of potential problems but note that this may be restricted to cases where excessive lending fuels a cycle of rising housing collateral which in turn propagates further credit growth. This transmission mechanism is under-studied and appears to be captured by our model.

Technical Annex

In what follows we detailed the full model. Equation numbering is the same as the main text.

The Aggregate Demand (AD)

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1 - \hat{a}_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + (a_2 - 2a_3) x_{t,f} + a_3 x_{t,h} + \varepsilon_t \quad (1)$$

($a_2, a_3 < 0$)

The Aggregate Supply (AS)

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \quad (2)$$

Households

Patient households

$$c_t = \sigma d_1 y_t + \sigma d_2 \tilde{E}_t y_{t+1} + \sigma (1 - d_1 - d_2) y_{t-1} + d_3 (r_t - \tilde{E}_t \pi_{t+1}) + v_t^c$$

Budget constraint

$$l_{t,h}^D - l_{t-1,h}^D + \sigma y_t = c_t + q_t [h_t - (1 - \delta_h) h_{t-1}] \quad (6)$$

Impatient households

$$c'_t = (1 - \sigma) d'_1 y_t + (1 - \sigma) d_2 \tilde{E}_t y_{t+1} + (1 - \sigma) (1 - d'_1 - d_2) y_{t-1} + d_3 (\rho_{t,h} - \tilde{E}_t \pi_{t+1}) + v_t^{c'} \quad (4)$$

Budget and collateral constraints

$$l_{t,h'}^D - l_{t-1,h'}^D + (1 - \sigma) y_t = c'_t + q_t [h'_t - (1 - \delta_h) h'_{t-1}] \quad (7)$$

$$\rho_t l_{t,h'}^D \leq (1 - m) (1 - \delta_h) h'_t \tilde{E}_t q_{t+1} \quad (8)$$

Banks' friction for impatient households:

$$x_{t,h} = \theta W_{t-1} \quad (9)$$

($\theta < 0$)

The households' spread over the risk-free interest rate

$$\rho_{t,h} = r_t + x_{t,h} \quad (10)$$

Patient households deposits:

$$D_t - D_{t-1} = s_t \quad (11)$$

Firms

Firms' residential investment

$$IH_t = A_{h,t} (1 - \mu) y_t \quad (12)$$

where $A_{h,t} = \rho_A A_{h,t-1} + \epsilon_{A,t}$

Firms' loan demand (L^D) and investment

$$i_t = l_{t,f}^D - l_{t-1,f}^D \quad (13)$$

Banks' friction for firms:

$$x_{t,f} = \rho_{t,f} - r_t = \varphi n_t^f \quad (\varphi < 0) \quad (14)$$

The firms' spread over the risk-free interest rate

$$\rho_{t,f} = r_t + x_{t,f} \quad (15)$$

The investment demand

$$i_t = e_1 \tilde{E}_t y_{t+1} + e_2 (\rho_{t,f} - \tilde{E}_t \pi_{t+1}) + v_t^i \quad (e_2 < 0) \quad (16)$$

Firms' equity

$$n_t^f = \bar{n} S_t \quad (17)$$

Standard Gordon discounted dividend model for deriving stock prices (De Grauwe and Macchiarelli, 2015)

$$S_t = \frac{E_t \bar{\Lambda}_{t+1}}{R_t^S}$$

Discount rate

$$R_t^S = r_t + \xi$$

Firms' leverage

$$\tau_t = \frac{l_{t,f}^D}{\bar{n} S_t} \quad (18)$$

Banks

Banks' assets' side

$$A_t = l_t^S + \bar{n} S_t \quad (20)$$

Balance sheet constraint

$$A_t = n_t^b + D_t \quad (21)$$

Capital-to-asset ratio, i.e.

$$\frac{n_t^b}{A_t} = \kappa \quad (22)$$

Monetary policy

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 GDP_t + c_3 r_{t-1} + u_t \quad (3')$$

Housing augmented GDP

$$GDP_t = y_t + \bar{q} IH_t$$

Market clearing

Banks' balance sheet

$$l_t^S = l_{t,f}^D + l_{t,h}^D$$

Market clearing for goods

$$c_t + c_t' + i_t = y_t$$

Market clearing for houses

$$h_t + h'_t - (1 - \delta_h)(h_{t-1} + h'_{t-1}) = IH_t$$

Total savings

$$s_t = \sigma y_t - c_t$$

Patient households' rigid demand for houses

$$h_t = \bar{h}$$

Endogenous housing bubbles

High housing price expectation regime (H)

$$\tilde{E}_t^H q_{t+1} = q_t - \delta y_t = q_{t-1} - \delta y_{t-1}$$

Low housing price expectation regime (L)

$$\tilde{E}_t^L q_{t+1} = q_t - \delta y_t = 0$$

Housing price overall expectations

$$\tilde{E}_t q_{t+1} = \alpha_{q,t} \tilde{E}_t^H q_{t+1} + (1 - \alpha_{q,t}) \tilde{E}_t^L q_{t+1}$$

Probability of switching to H regime

$$a_{q,t} = \frac{\exp(\gamma \pi_{H,t}^q)}{\exp(\gamma \pi_{H,t}^q) + \exp(\gamma \pi_{L,t}^q)}$$

Utility of switching to regime H

$$\pi_{H,t}^q = (q_t - \delta y_t) \text{signal}(\tilde{E}_t^H q_{t+1})$$

$$\text{signal}(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z = 0 \\ -1 & \text{if } z < 0 \end{cases}$$

Probability of switching to L regime

$$1 - a_{q,t} = \frac{\exp(\gamma \pi_{L,t}^q)}{\exp(\gamma \pi_{L,t}^q) + \exp(\gamma \pi_{H,t}^q)}$$

Utility of switching to regime L

$$\pi_{L,t}^q = (q_t - \delta y_t) \text{signal}(\tilde{E}_t^L q_{t+1})$$

$$\text{signal}(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z = 0 \\ -1 & \text{if } z < 0 \end{cases}$$

Heterogeneous expectations

Output overall market forecast

$$\tilde{E}_t y_{t+1} = \alpha_{f,t} \tilde{E}_t^f y_{t+1} + \alpha_{e,t} \tilde{E}_t^e y_{t+1}$$

Output fundamentalist rule

$$\tilde{E}_t^f y_{t+1} = y^*$$

Output extrapolative rule

$$\tilde{E}_t^e y_{t+1} = \theta y_{t-1}$$

(with $\theta = 1$)

Probability of using output fundamentalist rule

$$\alpha_{f,t}^y = \frac{\exp(\gamma U_{f,t}^y)}{\exp(\gamma U_{f,t}^y) + \exp(\gamma U_{e,t}^y)}$$

Probability of using output extrapolative rule

$$\alpha_{e,t}^y = 1 - \alpha_{f,t}^y = \frac{\exp(\gamma U_{e,t}^y)}{\exp(\gamma U_{f,t}^y) + \exp(\gamma U_{e,t}^y)}$$

Utility of using output fundamentalist rule

$$U_{f,t}^y = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}_{t-k-2}^f y_{t-k-1}]^2,$$

$$w_k = (\rho^k(1 - \rho)) \text{ (with } 1 \leq \rho \leq 0)$$

Utility of using output extrapolative rule

$$U_{e,t}^y = -\sum_{k=0}^{\infty} w_k [y_{t-k-1} - \tilde{E}_{t-k-2}^e y_{t-k-1}]^2$$

$$w_k = (\rho^k(1 - \rho)) \text{ (with } 1 \leq \rho \leq 0)$$

Inflation overall market forecast

$$\tilde{E}_t \pi_{t+1} = \alpha_{f,t} \tilde{E}_t^f \pi_{t+1} + \alpha_{e,t} \tilde{E}_t^e \pi_{t+1}$$

Inflation fundamentalist rule

$$\tilde{E}_t^f \pi_{t+1} = \pi^*$$

Inflation extrapolative rule

$$\tilde{E}_t^e \pi_{t+1} = \theta \pi_{t-1}$$

(with $\theta = 1$)

Probability of using inflation fundamentalist rule

$$\alpha_{f,t}^\pi = \frac{\exp(\gamma U_{f,t}^\pi)}{\exp(\gamma U_{f,t}^\pi) + \exp(\gamma U_{e,t}^\pi)}$$

Probability of using inflation extrapolative rule

$$\alpha_{e,t}^\pi = 1 - \alpha_{f,t}^\pi = \frac{\exp(\gamma U_{e,t}^\pi)}{\exp(\gamma U_{f,t}^\pi) + \exp(\gamma U_{e,t}^\pi)}$$

Utility of using inflation fundamentalist rule

$$U_{f,t}^\pi = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}_{t-k-2}^f \pi_{t-k-1}]^2$$

$$w_k = (\rho^k(1 - \rho)) \text{ (with } 1 \leq \rho \leq 0)$$

Utility of using inflation extrapolative rule

$$U_{e,t}^\pi = -\sum_{k=0}^{\infty} w_k [\pi_{t-k-1} - \tilde{E}_{t-k-2}^e \pi_{t-k-1}]^2$$

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