What can the theory of finance tell us about the effects of monetary policy on spending?

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Abstract

This paper uses the standard Intertemporal Capital Asset Pricing Model with Epstein-Zin (1989) preferences to develop a simple equation showing how monetary policy surprises may affect consumption and investment plans in a frictionless economy when agents can hedge against such surprises. I start with a standard equation showing how monetary policy affects spending through wealth and intertemporal substitution effects, but note that these will tend to offset each other since agents will optimally position their portfolios to hedge against the latter. I then use the portfolio investment conditions to derive a second equation that allows for this behavior. This equation is novel and shows that monetary policy surprises can affect spending if either (i) these risks are not spanned by the financial markets (ii), the Sharpe ratios make it costly to hedge spanned risks, or (iii) there is a preference for early resolution of uncertainty. I then use stylized assumptions based on the empirical finance literature to calibrate these effects.

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1 Introduction

The demise of the Bretton Woods fixed exchange rate system in the early 1970s gave national monetary authorities much more scope in the conduct of monetary policy. It also exposed households and firms to greater financial risk, particularly with respect to changes in inflation, interest and exchange rates. This spurred he development of derivatives and other financial instruments designed to mitigate these risks. At the same time, finance theorists developed mathematical tools for valuing these instruments and using them to manage risk.

These developments offer a new perspective on monetary policy. Traditional analysis of the monetary transmission mechanism distinguishes between wealth and intertemporal substitution effects. This distinction follows naturally from a macroeconomic specification in which consumption $C(t)$ is related to a scale variable such as wealth or permanent income, and an interest rate factor $R(t)$. However, if agents structure their portfolios to hedge against monetary policy and other surprises, these wealth and substitution effects will tend to offset each other. Indeed, it is not hard to find limiting cases in which hedging behavior would completely neutralize monetary policy. For example, Wachter (2003) shows that in the limit in which the Elasticity of Intertemporal Substitution (EIS) goes to zero and the Coefficient of Relative Risk Aversion (CRRA) goes to infinity, an agent hedges by investing exclusively in an annuity or consol with a maturity equal to their time horizon and just consumes the income this provides. Monetary policy and other shocks can affect the value of this instrument but do not affect consumption or investment decisions. I will show (section 2.3) that in a frictionless complete market setting, wealth and intertemporal substitution effects also offset each other perfectly due to hedging when utility is additive and liquidity and risk premia are negligible.

This paper uses the standard Intertemporal Capital Asset Pricing Model with Epstein and Zin (1989) preferences to develop a simple equation showing how monetary policy surprises can affect consumption and investment plans in a frictionless economy when agents hedge against such surprises. I first derive a standard equation showing how monetary policy affects spending through wealth and intertemporal substitution effects. I then use the investment optimality conditions to modify this and allow for hedging behavior. This modified equation shows that policy surprises can affect spending even in this frictionless world when either: markets are incomplete, making it impossible to hedge against unspanned shocks; risk premia make it costly to hedge or utility is non-additive. I use stylized assumptions based on theoretical restrictions and the empirical finance literature to assess these effect of conventional and unconventional monetary policy shocks.

In this framework, the effects of monetary policy depend critically upon the EIS and the CRRA. If the EIS equals the reciprocal of the CRRA, then utility is additive, but otherwise I show that monetary policy has classic wealth effects. Specifically, if the EIS is greater than the reciprocal of the CRRA, expansionary policies that induce windfall capital gains increase consumption, but they reduce it if the opposite holds. The first inequality implies that the agents have a preference for the early resolution of uncertainty, which seems more plausible than the opposite one. I then argue that monetary policy surprises are likely to be reflected in the yield curve and other asset returns. This means that agents can potentially hedge against policy surprises. If the EIS is less than unity, agents want to hedge against a fall in the general level of future returns by holding long term bonds and other assets that provide compensation in the form of capital appreciation in this situation. Assuming that risk premia make this hedging costly, an expansionary policy that lowers returns will increase an agent's current consumption relative to their previous plan, otherwise if the EIS is greater than unity their consumption will fall.

I show that the cost of hedging against changes in the slope of the yield curve also plays a critical role in the effectiveness of different types of monetary policy. Evidence drawn from studies of the US term structure of interest rates suggests that this enhances the effect of conventional short end operations but reduces the effect of unconventional monetary policies (UMPs) aimed at lowering long term yields (since these policies have opposite effects on the slope of the yield curve). UMPs work largely through wealth effects and thus depend critically upon the structure of the agent's portfolio and the preference for early resolution of uncertainty. Since the US household portfolio is weighted heavily towards equities and other real claims, the wealth effect of a cut in long rates depends critically upon the connection between their discount rates and long bond yields, which the empirical evidence suggests is surprisingly close for equities. On this view, UMPs provides a potentially effective way to influence spending.

2 The model

This paper is based on a literature due to Merton (1973), Breeden (1984), Cox, Ingersoll, and Ross (1985) and others who show that with additive utility and continuous tax and transactions-free trading in securities, a complete set of Arrow Debreu securities (or state-contingent futures contracts, Arrow (1964)) is not required for allocative efficiency. Under these assumptions, it suffices that the number of distinct securities is equal to the number of distinct stochastic processes affecting the economy. In this sense the market is complete, allowing agents to hedge all potential risks. Cox and Huang (1989) show that an agent's wealth may then be viewed as an asset that finances the consumption plan. Given the preference parameters, the consumption plan is determined by the Stochastic Discount Factor (SDF), and in this situation the effect of stochastic shocks on the SDF depend upon the associated hedging costs or prices of risk. If these prices are zero, it follows that the SDF and hence the consumption plan are no longer stochastic and the consumption simply evolves in line with plan, impervious to monetary policy and other surprises, as shown in section 2.3 below. However, this is a very special case. The model set out in this section is designed to relax these strong assumptions and analyse the effectiveness of monetary policy in continuous, friction-free markets.

2.1 Market structure

Cox, Ingersoll, and Ross (1985) provide a standard description of a securities market in a continuous time stochastic production economy in the absence of arbitrage. I adopt this structure, assuming that the markets are in continuous transactions-free equilibrium, but relaxing their assumption of additive utility. For convenience, the investment opportunity set includes an asset (possibly a contingent claim in zero net supply) with price P^0 and a safe instantaneous real return r that depends upon the state variables of the system:

$$\frac{dP^0(t)}{P^0(t)} = rdt. \tag{1}$$

The real values of the M risky assets are governed by the stochastic differential equation (SDE) system:

$$\frac{dP^i(t)}{P^i(t)} = \alpha_i(t)dt + \sum_{k=1}^{M} \Sigma_{ik}(t)dw^k; i = 1, ..., M. \tag{2}$$

where the drift and volatility coefficients are observable at time t. The dw^k's are independent Wiener processes. Ordering these prices, returns and processes consistently in the $M-$vectors $P(t), \alpha(t), dw$ allows (2) to be written as:

$$dP(t) = \hat{P}[\alpha(t)dt + dw] = \hat{P}[\alpha(t)dt + \Sigma(t)dw] \tag{3}$$

where $\hat{P} = diag(P(t))$ is an $M \times M$ matrix with the elements of $P(t)$ along the main diagonal and zeros elsewhere. $\Sigma(t) = (\Sigma_{ik}(t))_{1 \leq i,k \leq M}$ is a lower triangular $M \times M$ lower triangular Cholesky matrices. A prime ($'$) denotes a transpose in this paper.

If $\pi(t)$ is an $M-$ vector with the typical element $\pi_i(t)$ representing the $i-th$ portfolio share at time t, the dynamics of *real* wealth ($W(t)$) can be described by the SDE:

$$dW(t) = \{W(t)\pi'(t)(\alpha(t) - r(t)i) + W(t)r(t) - C(t)\}dt + W(t)\pi'(t)\Sigma(t)dw. \tag{4}$$

where i is the $M \times 1$ summation vector and $C(t)$ represents real consumption. Applying Ito's lemma to find $d\ln W$ and $d\ln P^i$ using (3) and (4), subtracting their drift and evaluating the expectation of their product gives the vector of covariances of asset returns with log wealth: $\sigma_{pw} = \Omega(t)\pi(t)$, where $\Omega(t) = \Sigma(t)\Sigma(t)'$. The risk premia $(\alpha(t) - r(t)i)$ are determined by the asset risk exposures $\Sigma(t)$ and the prices of risk $\lambda(t)$ by:

$$(\alpha(t) - r(t)i) = \Sigma(t)\lambda(t) \tag{5}$$

Finally, suppose that asset returns are driven by an $N-$vector of state variables. In Cox and Huang (1989), $N = M$ and the financial market is complete in the sense that the state space is spanned by the prices of tradeable assets. This means that unexpected changes in the state variables and hence future returns and consumption plans can be hedged perfectly. If $N \geq M$, the market is completed by a vector Y of

$N-M$ contingent claims. The shadow prices of risk on these claims are then varied so that they are not held in equilibrium. Modifying the Cox, Ingersoll, and Ross (1985) notation slightly, I assume that these prices follow:

$$dY(t) = [\mu(t)dt + \Phi(t)dw + \Theta(t)dv] \tag{6}$$

where dv is a vector of $N-M$ Wiener processes that is orthogonal to dw.

2.2 Preference structure

Given wealth and the state variables, agents choose consumption and portfolio holdings $\{C(t), \pi(t)\}$ to maximize the expected value of lifetime utility, derived from consumption $C(t)$ over the lifetime $[0,T]$ and terminal wealth $W(T)$. In the recursive utility framework this is:

$$J(t) = \max_{\{C(s),\pi(s)\}} E(t)[\int_t^T f[C(s), J(s)]ds \tag{7}$$

$$J(T) = \frac{W_T^{1-\gamma}}{1-\gamma} \tag{8}$$

where $f[.,.]$ is a normalized aggregator of current consumption and continuation utility of the form:

$$f[C(s), J(s)] = \left(\frac{\beta(1-\gamma)J(s)}{1-1/\psi}\right)\left(\left(\frac{C(s)}{((1-\gamma)J(s))^{\frac{1}{(1-\gamma)}}}\right)^{1-1/\psi} - 1\right) \tag{9}$$

where: γ is the CRRA, ψ the EIS and β the time preference factor. This is the continuous time limit of the Epstein and Zin (1989) discrete time specification (Duffie and Epstein (1992)). The time separable power utility model is a special case obtained when the coefficient of relative risk aversion $\gamma = 1/\psi$. The Epstein-Zin is specification relaxes that constraint and allows agents to have a preference for early resolution of uncertainty when $\psi > 1/\gamma$.[1]

The Bellman optimality principle of dynamic programming implies:

$$0 = \max_{\{C(s),\pi(s)\}} f[C(t), J(t)] + E(t)dJ[W(t), P(t), Y(t), t]/dt$$

Under the usual regularity conditions, Ito's lemma with (3), (4) and (6) then implies:

$$\begin{aligned}
0 = &\max_{\{C(s),\pi(s)\}} f[C(t), J(t)] \qquad (10)\\
&+J_W(t)\{W(t)\pi'(t)(\alpha(t) - r(t)i) + W(t)r(t) - C(t)\} + J_P(t)\hat{P}\alpha(t) + J_Y(t)\mu(t) + J_t(t)\\
&+\frac{1}{2}\{J_{WW}(t)W(t)^2\pi'(t)\Omega(t)\pi(t) + Tr[\hat{P}\Omega(t)\hat{P}J_{PP}(t)] + Tr[\Phi(t)\Phi'(t)J_{YY}(t)]\\
&+Tr[\Theta(t)\Theta'(t)J_{YY}(t)]] + W(t)\pi'(t)\Omega(t)\hat{P}J_{PW}(t) + W(t)\pi'(t)\Sigma(t)\Phi'(t)J_{YW}(t)\\
&+Tr[\hat{P}\omega(t)\Phi'(t)J_{PY}(t)\};
\end{aligned}$$

with the terminal condition $J(W, Y, T) = \frac{W_T^{1-\gamma}}{1-\gamma}$. Algebraic subscripts denote partial derivatives in this paper[2].

2.3 Optimal consumption and investment behavior

The homothetic utility specification gives a solution for the value function of the separable form:

$$J[W(t), P(t), Y(t), t] = \frac{W^{1-\gamma}}{1-\gamma} H[P(t), Y(t), t]^{\frac{1-\gamma}{\psi-1}}. \qquad (11)$$

[1]Kreps and Porteus (1978) illustrate this using a couple of examples. For example, suppose you are about to leave to go on holiday and a letter arrives containing an examination or some other important result. Do you open it now, revealing a preferences for early resolution, or leave it until your return? Duffie and Epstein (1992) provide mathematical examples.

[2]In this paper, R_x is an $n \times 1$ vector of derivatives of the valuation ratio with respect to the state variables and R_{xx} is the associated $n \times n$ matrix of second derivatives. $Tr[M_1.M_2]$ denotes the trace or sum of the diagonal elements of the matrix product: $M_1.M_2$.

Substituting this into (10), the first order conditions [3]:

$$C(t) = W(t)/H(t) \tag{12}$$

$$\pi(t) = \underbrace{\frac{1}{\gamma}\Omega^{-1}(t)(\alpha(t) - r(t)i)}_{\text{myopic demand } (\pi^M(t))} + \underbrace{\frac{1}{\gamma}\left(\frac{1-\gamma}{\psi-1}\right)\left(\frac{\hat{P}H_P(t)}{H(t)} + \Sigma'(t)^{-1}\Phi'(t)\frac{H_Y(t)}{H(t)}\right)}_{\text{long-term hedging demand } (\pi^H(t))} \tag{13}$$

Taking logs of (**??**), applying Ito's lemma to $c(t) = \ln C(t)$ and subtracting its drift gives the analogue of Campbell, Lo, and MacKinlay (1996) eq(8.3.11):

$$dc(t) - E(t)dc(t) = \underbrace{\pi'(t)dw}_{\text{wealth effect}} \underbrace{- \left[\frac{H'_P(t)\hat{P}}{H(t)} + \frac{H'_Y(t)}{H(t)}\Phi(t)\Sigma(t)^{-1}\right]dw - \frac{H'_Y(t)}{H(t)}\Theta(t)dv.}_{\text{intertemporal substitution effect}} \tag{14}$$

This represents the covariances between asset returns and log consumption in terms of the portfolio holdings and the hedging terms:

$$\sigma_{pc} = \Omega(t)\left(\pi(t) - \frac{\hat{P}H_P(t)}{H(t)} - \Sigma'(t)^{-1}\Phi'(t)\frac{H_Y(t)}{H(t)}\right) \tag{15}$$

Equation (14) is interesting since it shows the effect of shocks to consumption, separating them neatly into wealth and intertemporal substitution effects. The homotheticity assumption means that shocks to wealth have an equi-proportionate effect on consumption. However, to be able to use this equation in practice we would need to solve the PDE (10) to determine the form of the valuation function $H(t)$ and its derivatives. More to the point, the wealth and substitution effects would offset to the extent that the agent was hedged. To investigate the implications of optimal hedging behavior, we now consider this equation alongside the first order conditions

[3] Equation (11) can be verified in the usual way by substituting (**??**) and (13) back into (10). Wealth cancels out, leaving a second order linear PDE that determines $H(t)$ given the initial conditions. Under some conditions (Kraft, Seifried, and Steffensen (2013)) this is a linear PDE with a regular closed form solution, but in general it is non-linear, requiring numerical solution methods.

for investment and the risk premia.

The risk premia can be determined by rearranging (13) using (15). This gives the standard equation due originally to Duffie and Epstein (1992):

$$\begin{aligned}(\alpha(t) - r(t)i) &= \gamma\Omega(t)\pi(t) - \left(\frac{1-\gamma}{\psi-1}\right)\left(\frac{\Omega(t)\hat{P}H_P(t)}{H(t)} + \Sigma(t)\Phi'(t)\frac{H_Y(t)}{H(t)}\right) \\ &= \gamma\sigma_{pw} + \left(\frac{1-\gamma}{\psi-1}\right)(\sigma_{pc} - \sigma_{pw}) \\ &= \underbrace{\left(\frac{\gamma\psi-1}{\psi-1}\right)\sigma_{pw}}_{CAPM} + \underbrace{\left(\frac{1-\gamma}{\psi-1}\right)\sigma_{pc}}_{C-CAPM}. \end{aligned} \tag{16}$$

As they note, his may be viewed as a combination of the CAPM and Consumption-CAPM pricing models. Campbell, Lo, and MacKinlay (1996) eq(8.3.7) and Campbell and Viciera eq(2.51) provide an alternative derivation of (16)) in discrete time. This degenerates into the CAPM with $\gamma = 1$ and into the Consumption-CAPM with additive power utility ($\psi = 1/\gamma$). (16) can be inverted to give another representation of σ_{pc}: in terms of the portfolio holdings and the risk premia:

$$\sigma_{pc} = \left(\frac{\psi-1}{1-\gamma}\right)(\alpha(t) - r(t)i) + \left(\frac{1-\gamma\psi}{1-\gamma}\right)\sigma_{pw}. \tag{17}$$

We can now use (15) and (17) to modify the equation for consumption shocks (14) and allow for hedging. We first use premultiply them by $\Omega(t)^{-1}$, transpose and substitute $\pi(t) = \Omega(t)^{-1}\sigma_{pw}$ to show that:

$$\left(\pi'(t) - \frac{H'_P(t)\hat{P}}{H(t)} + \frac{H'_Y(t)}{H(t)}\Phi(t)\Sigma(t)^{-1}\right) = \left[\left(\frac{1-\gamma\psi}{1-\gamma}\right)\pi'(t) + \left(\frac{\psi-1}{1-\gamma}\right)(\alpha(t) - r(t)i)'\Omega(t)^{-1}\right]$$

Then substitute this into (14):

$$dc(t)-E(t)dc(t) = \left[\left(\frac{1-\gamma\psi}{1-\gamma}\right)\pi'(t) + \left(\frac{\psi-1}{1-\gamma}\right)(\alpha(t)-r(t)i)'\Omega(t)^{-1}\right]dw - \frac{H'_Y(t)}{H(t)}\Theta(t)dv. \tag{18}$$

In the case of additive logarithmic utility the coefficients on the terms in square brackets sum to unity. Also $\pi'(t) = \Omega(t)^{-1}(\alpha(t) - r(t)i)$ so they simplify to $\pi'(t)dw$, meaning that (absent unspanned shocks) the change in log consumption equals the change in log wealth (see table 1). A more useful expression for our analysis is obtained by substituting (5) into (18) to get:

$$dc(t)-E(t)dc(t) = \left[\underbrace{\left(\frac{1-\gamma\psi}{1-\gamma}\right)\pi'(t)}_{\text{wealth effect}} + \underbrace{\left(\frac{\psi-1}{1-\gamma}\right)\lambda'(t)\Sigma(t)^{-1}}_{\text{price of risk effect}}\right]dw - \underbrace{\frac{H'_Y(t)}{H(t)}\Theta(t)dv}_{\text{unspanned risk effect}} \tag{19}$$

This equation is central to the rest of the analysis. It shows how the optimal consumption plan is impacted by shocks that are (dw) and are not (dv) respectively spanned by market prices, when the investor is optimally hedged. If we are only interested in the former, there is no need to determine the form of the valuation function $H(t)$ and its derivatives.

2.4 Assessment

Equation (19) provides a novel decomposition of the effect of monetary policy surprises. The first term is a residual wealth effect, showing the effect of changes in wealth once hedging has been allowed for. It depends upon the portfolio structure, $\pi(t)$, which exposes wealth to shocks.. Its coefficient vanishes in the case of additive power utility. The second term depends upon the degree to which the optimal portfolio is left unhedged, which depends upon the cost of hedging, represented by the prices of risk $\lambda(t)$. If these are zero or if $\psi = 1$, this term also vanishes. If the market is complete so that all risks can be hedged, then the last term vanishes too. If all three conditions hold, then obviously the various shocks to consumption and investment net out, leaving them to evolve in line with the initial consumption plans, as noted earlier.

Equation (19) is a general relationship that holds for any agents with the preferences specified in section 2.2. It does not depend upon the time horizon of these agents, which could include myopic agents or long-lived families of overlapping generations. It does not depend upon the specification of monetary policy, which could be framed in terms of a Taylor rule, money supply or other targets. It assumes for simplicity that a real capital safe asset is provided by an inflation index-linked bond market but this assumption can be relaxed without difficulty. It is based upon delta-hedging and thus depends upon the assumption that asset returns are continuous, but is a close approximation when they are discrete. Apart from that, (19) would seem to handle any standard specification of asset returns and volatility.

There are several ways in which we could use this relationship. One would be to embed it in a structural model of the economy. However, in view of the emphasis this decomposition gives to prices of risk and other financial parameters, we remain agnostic about the macro and monetary policy specification in this paper and instead use empirical results drawn from the finance literature. This literature provides reduced form estimates of the financial parameters, which can be used to assess the impact of monetary surprises directly within this framework. Given these parameters, we can analyse the effect of these surprises on people with different preferences, informed by the work of (Vissin-Jorgensen (2002)) and others, who show how they differ across different types of investor. However, in this paper we use that literature to make stylized assumptions about a representative agent to inform the model and obtain estimates of the degree of leverage the Federal Reserve could have over aggregate consumption expenditure in the US.

2.5 Calibration

Table 1 reports some of the parameter values that have been employed in the finance literature. However, these represent simplifying assumptions rather than empirical evidence. For example, Campbell and Viciera (2002) assume a unit EIS in order to make their model tractable, but the empirical evidence tends to suggest that the EIS is well below unity (Hall (1988), Campbell and Mankiw (1989), Vissin-Jorgensen (2002)). However, this evidence, together with the intuition (and overwhelming empirical evidence) that the CRRA is greater than unity, [4] gives some useful bounds on the coefficients of the spanned effects in (19).

First note that the coefficients in (19) sum to the EIS, and if this is not greater than unity then:

$$\left(\frac{1-\gamma\psi}{1-\gamma}\right)+\left(\frac{\psi-1}{1-\gamma}\right)=\psi\leq 1.$$

Similarly, the sum of the two coefficients in (16) equals the CRRA, and if this is not less than unity:

$$\left(\frac{\gamma\psi-1}{\psi-1}\right)+\left(\frac{1-\gamma}{\psi-1}\right)=\gamma\geq 1.$$

Together, $\psi \leq 1 \leq \gamma$ imply the price of risk effect in (19) $\left(\frac{\psi-1}{1-\gamma}\right) > 0$. A preference for the early resolution of uncertainty implies $\left(\frac{1-\gamma\psi}{1-\gamma}\right) > 0$. These conditions imply the bounds:

[4] This implies that (holding wealth fixed) the substitution effect of an fall in rates of return dominates the income effect, causing current consumption to increase. It also means that agents hedge against a fall in future returns by holding more long term assets (that rise in value in this eventuality) than a myopic agent with the same degree of risk aversion would. With $0 < \gamma < 1$ and $\psi > 1$ the opposite occurs: a fall in rates of return causes current consumption to decrease and agents 'reverse hedge' against this (Breeden (1984)).

$$0 \leq \left(\frac{1-\gamma\psi}{1-\gamma}\right), \left(\frac{\psi-1}{1-\gamma}\right) \leq \left(\frac{1-\gamma\psi}{1-\gamma}\right) + \left(\frac{\psi-1}{1-\gamma}\right) = \psi \leq 1.$$

The empirical evidence from asset pricing and consumption studies is in other respects very mixed. Early studies based upon the Euler equation with aggregate data (Hall (1988)) suggest an EIS close to 0.1 (i.e. a CRRA close to 10). However more recent studies bases on the Epstein-Zin formulation and employing micro data for different types of investor suggest values for the EIS in the range 0.3-0.4 for equity investors and 0.8-1.0 for bond investors (Vissin-Jorgensen (2002)). Splitting the distance between the upper and lower bounds respectively of these ranges would suggest a value of $\psi = 0.6$. To allow a preference for early resolution suppose that: $\gamma = 3/(2\psi) = 2.5$. This suggests suggests we use the values $\left(\frac{1-\gamma\psi}{1-\gamma}\right) = 1/3$; $\left(\frac{\psi-1}{1-\gamma}\right) = 0.26667$.

3 The effect of monetary surprises on consumption

We are now in a position to approach the questions raised in the introduction about the efficacy of monetary policy. We first need to ask whether these involve shocks that are spanned by the financial markets. This means that they have an immediate effect on asset prices, allowing agents potentially to structure their portfolios to insure against them. In contrast, unspanned risks only affect asset prices with a lag, ruling out this kind of insurance. For example, Joslin, Priebsch, and Singleton (2014) argue that shocks to macro variables like output growth only have a lagged effect on the bond market. However, the long term bond yields that are the focus of UMPs as well as the short term policy rate are spanned by the bond market and it is hard to think of policy effects that would only operate with a lag. This suggests that the last (unspanned risk) term in (19) is not likely to be very important in the case of monetary shocks, although it might leave agents exposed to productivity and other shocks. Thus I now focus on the first two terms in (19), showing price of risk (channel (i)) and wealth (channel (ii)) effects. Because this decomposition is novel, these effects, and in particular the first, need to be examined in detail.

3.1 Affine term structure models and the cost of hedging

The literature on the Affine Term Structure Model (ATMS) provides a reasonable starting point for this analysis given the role played by the bond market in the transmission mechanism of monetary policy. This was originally due to Vasicek (1977), who developed a single factor arbitrage-free model, showing how changes in policy rates ripple along the yield curve. However, this simple model is unrealistic since the single factor assumption means that all yields and bond prices are perfectly correlated. Since most of the assets in the portfolio are long term assets like equities and housing, this model would suggest a very small portfolio duration effect. Moreover, given the policy rate, the authorities cannot influence the shape of the yield curve, ruling out the effect of UMPs in this model.

To allow for differential maturity effects, the appendix examines the implications of a standard three factor version of the Vasicek model for the efficacy of channel (i). The model is specified in terms of short (6 month) medium (2 year) and long (10 year) yields. These are mapped into the 'level' factor (the average yield) the 'slope' factor (long minus short) and curvature factor (long plus short less twice the medium yield). Three models are considered. The first is the model of Hamilton and Wu (2012), henceforth HW, which is estimated using linear term structure regression methods and specifically designed to handle UMPs. The second model is calibrated using the returns data provided by Duffee (2010) and reproduced in Table 2, while the third is informed by the work on Return Forecasting Regressions of Cochrane and Piazzesi (2008). These models are used to simulate the effect of specific monetary policy initiatives on the term structure. Importantly, both the HW and Duffee estimates suggest that the price of yield level risk is negative and the price of slope risk is positive. The estimates of the price of curvature risk in the HW and Duffee versions of the model differ in sign but are small and are not critical to the analysis of monetary shocks which mainly involve the level and slope of the yield curve. Cochrane and Piazzesi (2008) agree that the price of level risk is negative but suggest that the price of the other two risks are effectively zero.

First consider the policy implications of the negative price of level risk. Since a bond price is inversely related to its yield, this means that investors require a positive premium to hold a bond portfolio with a value that is perfectly negatively correlated with the average level of yields. This persuades agents to hold more bonds than they need to hedge, exposing them to interest rate risk. So if there is a surprise fall in yields, bond prices rise and (19) says that consumption moves up relative to the previous plan.

The positive price of slope risk means that a portfolio that mimics the slope factor by going long of short maturities and short of long maturities also requires a positive excess return. In other words, on balance agents prefer a fall in short yields to a fall in long yields. This is less easy to understand a priori, but it is consistent with the observation, reflected in Table 2, that empirically Sharpe ratios tend to fall with maturity. This could in turn be the result of long term investors paying a premium to hedge the risk of a fall in long term yields by going short of short bonds and holding long term bonds. Whatever the reason, they would like to hedge against a fall in the slope (i.e. a fall in long or rise in short yields). However the cost of hedging this leaves them exposed to a fall in slope. So a surprise fall in slope makes them worse off and (19) says this will cut consumption, while an increase in slope has the opposite effect. This slope effect enhances the effect of cuts in short and reduces the effect of cuts in long rates, except in the model of Cochrane and Piazzesi (2008), which suggests that slope risk is not priced.

3.2 Conventional monetary policy surprises

Consider the effect of conventional and unconventional monetary policies in this framework. The first shock is an unanticipated one point reduction in interest rates that is believed by the market to be temporary. For simplicity, assume this has no effect on the long yield or the curvature. Again, since most of the assets in the portfolio are long term assets, this would suggest a wealth effect that was small. However, the price of risk effect (channel (i)) *is* likely to be important. In this case the Appendix shows that the level factor (average yield) falls by up by a third of a percentage point and the slope factor (long minus short) increases by one point. Given the prices of risk are respectively negative and positive, both of these effects increase consumption through channel (i). The numerical estimates are reported in the appendix.

3.3 Unconventional monetary policy surprises

There is a literature, going back to Wallis (1981), that suggests that changes to central bank balance sheet and the timing of payments could be offset by private investors under Modigliani-Miller type assumptions, with no effect on activity or prices. However, the preferred habitat theory suggest that such operations could affect rates of return, and as HW note, this is ultimately an empirical question. They consider the effect of a purchase of $400 billion of long term Treasury bonds financed by the sale of very short maturity bills, characterizing the effect of the QE2 programme announced in September 2011[5]. They suggest that normally this 'twist'

[5] At that time $.y_t = [0.0028\ 0.0059\ 0.0355]'$ and using (20) and (22) $\lambda_t = [-0.1425\ 0.1650$ - $0.0601]$. However, to focus on the efficacy of different policies we use the December 2006 estimates to calibrate the model for this exercise.

would increase the short rate, but that this effect would be negligible at the lower bound. They argue that an outright purchase of $400 billion of long term bonds would have a similar effect for this reason. Their estimates suggest that such an operation would have a miniscule effect on the 2 year yield but reduce the 10 year yield by 0.14%. They conclude 'that QE2 as implemented had little potential to lower long-term interest rates via the mechanism explored in this paper'. However, the important question is: how would this affect consumption?

In this case, the wealth effect is likely to be much more important than the price of risk effect. Indeed, on our estimates, the latter is ambiguous. For ease of comparison with the short end operation, first consider an operation designed to reduce the long rate by one point at the ZLB, with no effect on the 6 month or 2 year rates. As in the previous simulation, the level factor falls by a third of a point. However in this case the slope $falls$ rather than increases by a point, since the prices of risk are respectively negative and positive, these effects pull in different directions, making the overall impact of the price of risk component on consumption ambiguous. The numerical estimates are reported in the appendix, with the HW estimates suggesting a small negative and the Duffee estimates a small positive effect on consumption. The caveat is that the Cochrane and Piazzesi (2008) model would suggest an unambiguously positive effect if slope risk is not priced, identical to that of a one point cut in the short rate.

However, now consider channel (ii). The wealth effect of a cut in long rates depends critically upon the effect this has on the discount rates in long term asset markets. In the case of the equity market, the empirical evidence suggests a strong effect. Indeed the well-known 'Fed' model used by practitioners would indicate a one-for-one effect Campbell and Vuolteenaho (2004), Bekaert and Engstrom (2010). This result is highly counter-intuitive, since equities are claims on real assets and their earnings, while bonds are monetary claims. Modigliani and Cohn (1979) originally put this effect down to money illusion on the part of investors, while Bekaert and Engstrom (2010) argue that this effect can be explained in terms of the impact of inflation on risk aversion and macroeconomic uncertainty. Whatever the reason for this, the link between dividend yields and long rates seems remarkably robust. Moreover, if the authorities were to succeed in changing the real long term interest rate, this would surely be expected to have a one-for-one effect on all long term real discount rates.

Looking at the US household balance sheet in mid-2012, we find that 33% was held in the form of real estate and other tangible assets; 40% in equities, pension and mutual funds. Just 11% was held in monetary assets, with the remaining 16% held in bonds and miscellaneous assets. For example, say we use a duration of 20 years for the household portfolio. This is defined here as its price elasticity with respect to a one point change in the 10 year yield.[6] With the estimate $\left(\frac{1-\gamma\psi}{1-\gamma}\right) = 1/3$, that would suggest that the QE2 program increased consumption by $1/3 \times 20 \times 0.14 = 0.933\%$ through channel (ii). Moreover, this is just an impression of the QE2 programme, which was part of a much larger expansion of the Fed balance sheet. Their weekly H41 report shows that their holdings on bonds with a maturity of 1 year or more increased by $1.9 trillions between August 2008 and February 2017, of which $850 billion of the increase had a maturity of over 5 years.

4 Conclusion

We identify two theoretical channels through which monetary policy surprises can influence spending even when agents optimally position their portfolios to hedge

[6] This compares with a duration of 33 for a perpetuity like an equity with a 3% discount rate. A portfolio that invested 60% of its funds in such an asset and the rest in monetary assets would thus have a duration of 20.

against such surprises. Although the difficulty of pinning down the relevant parameters makes it hard to come up with hard and fast estimates, some broad conclusions do to emerge from the numerical exercise. The price of slope risk plays the crucial role in respect of channel (i). There does not seem to be any evidence for a negative price of slope risk, and if this is positive, this reinforces the effect of cuts in short rates and reduces the effect of cuts in long rates. The sensitivity of discount rates in long term markets to the yield curve plays the crucial role in respect of channel (ii). These rates are likely to be much more sensitive to changes in long bond yields than they are to short term policy rates. Indeed, on the basis of the 'Fed model' it seems that the effect of UMPs could be economically meaningful, despite the Fed's relatively poor leverage over long term rates.

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Appendix

This appendix examines the implications of the ATSM models of Hamilton and Wu (2012) Duffee (2013) and Cochrane and Piazzesi (2008) for the efficacy of channel (i). The first model is a three factor econometric specification that defines the level, slope and curvature factors $f_t = [f_l \ f_s \ f_c]'$ in terms of short, medium and long term yields $y_t = [y_s \ y_m \ y_l]'$ (respectively 6 month, 2 year and 10 year yields) using the standard representation:

$$\begin{bmatrix} f_l \\ f_s \\ f_c \end{bmatrix}_t = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix} \begin{bmatrix} y_s \\ y_m \\ y_l \end{bmatrix}_t \tag{20}$$

or:

$$f_t = W y_t \Leftrightarrow y_t = W^{-1} f_t \tag{21}$$

where W is reported in (23):

Sharpe ratios

Conveniently, this 3 to 3 mapping is invertible. Moreover, we can mimic the behavior of these factors by portfolios of the original bonds that can be considered to be the assets of the system described by (19). The variance-covariance matrix for the factor portfolios $\Omega = \Sigma\Sigma'$ is fixed using the parameter estimates reported in (23) below. The prices of risk or Sharpe ratios λ_t associated with these factor portfolios are determined by the standard essentially affine specification, due originally to Duffee (2002).

$$\lambda_t = \lambda + \Lambda f_t \tag{22}$$

where:

$$\Sigma = \begin{pmatrix} 0.109 & 0 & 0 \\ 0.036 & 0.103 & 0 \\ 0.067 & 0.002 & 0.097 \end{pmatrix}; \Lambda = \begin{pmatrix} -0.0867 & -0.048 & -0.095 \\ -0.085 & -0.027 & 0.177 \\ -0.057 & 0.053 & -0.186 \end{pmatrix} \tag{23}$$

and $\lambda = [-0.138\ 0.160 - 0.0564]$. Using the yield structure in December 2006:.$y_t =$ [0.0491 0.0477 0.0475]$'$ this gives $f_t =$ [0.0481 -0.0016 0.0012] using (20). Substituting these values into (22) then gives the estimates of the prices of risk: λ_t =[-0.1420 0.1647 -0.0594].

These estimates need to be taken with a pinch of salt in view of the likely estimation biases. Duffee argues that ATSMs overestimate the degree of predictability in bond markets but shows that this bias is only a serious problem for models with more than the three factors used by HW. However, the parameters of (22) are given by the difference between the affine risk neutral world dynamics of the term structure and the time series dynamics of the factors. As Bauer, Rudebusch, and Wu (2012) note, VAR estimates of the latter suffer from small sample bias and are likely to cause the risk premia to be overestimated.

This makes it important to check these ATSM-based results against estimates of the ex post returns, variances and Sharpe ratios derived from samples of bond portfolio returns. Table 1 of Duffee (2010) provides a convenient set of bond portfolio returns for the sample period January 1952 to December 2008, reproduced in Table 2 of this paper. Note that these bond returns are negatively correlated with the associated yields. As we would expect, the Duffee estimates of the average returns and standard deviations increase with maturity while the Sharpe ratios fall with maturity.

To maintain invertibility, I use the returns for just 3 portfolios: the 1-6; 12-24; and 60-120 month maturity buckets in Table 2. These bond returns can then be converted into factor returns using (20). Specifically, the expected $factor$ excess returns $(\alpha(t)-r(t)i)^D$ and standard deviations $(D^D)^{1/2}$ are found by pre-multiplying the vector of mean bond returns and standard deviations by W. In the absence of any information on the cross-correlations, I base these on the HW estimates. The standard LDLT factorization of the HW Cholesky matrix gives: $\Sigma = L.D^{1/2}$. where shows the HW factor standard deviations. Replacing these by the Duffee estimates $(D^D)^{1/2}$ gives: $\Sigma^D = L.(D^D)^{1/2}$ and $\Omega^D = \Sigma^D \Sigma^{D\prime}$. Finally, the implied factor Sharpe ratios for factors $i = 1, 2, 3$ follow by taking the $i-$the element of

$(\alpha(t) - r(t)i)^D$ and dividing by the square root of the $i-$the diagonal element of Ω^D. This calculation gives: $\lambda_t^D = [-0.2473 \;\; 0.1085 \;\; 0.0556 \;]$.

Strictly speaking, because these estimates are based on unconditional sample averages, they are not directly comparable with the HW estimates. Nevertheless, they are reasonably close to those from the HW ATSM. The sign of the price of curvature risk changes, but this remains small, and as we will see in the next section, this factor plays a relatively minor role in the case of monetary shocks. The sign of the prices of level and slope risk, which play a major role, are the same, although they are respectively larger and smaller in absolute value than for HW. Once again, the estimate of the price of slope risk is positive. In this case the finding is due to the fact that in this sample, like most empirical samples, Sharpe ratios fall with maturity.

Monetary policy shocks

We now need to find the factor shocks $dw = \Sigma dw$ in (19) that simulate the effect of specific monetary policy initiatives on the term structure. Although the model is set up to handle continuous movements, we will use it to approximate the effect of discrete changes, denoted by $\Delta' s$. First, consider a one point reduction in interest rates that is thought to be temporary, having no effect on the long yield or curvature: $\Delta r_s = -1, \Delta r_l = 0, \Delta u_c = 0$. Substituting these values into (20) gives the solution: $\Delta r_m = -1/2$ and $\Delta u = [-1/2 \;\; +1 \;\; 0]'$.[7] Using the HW estimates gives the combined 'exposure' effect $\lambda_t' \Sigma^{-1} \Delta u = 2.7223$, which implies 2.7223%. The Duffee's portfolio estimates give a smaller exposure effect of $\lambda_t^{D\prime} \Sigma^{D-1} \Delta u = 0.7795$. Multiplying by the coefficient estimate: $\left(\frac{\psi-1}{1-\gamma}\right) = 0.2667$ suggests increases in consumption respectively of $0.2667 \times 2.7223 = 0.726\,\%$ and $0.2667 \times 0.7795 = 0.208\%$. Rescaling by the 5 point

[7] A monetary shock that cut the 6-month rate by 1% and had a half-life of 18 months would imply a very similar effect $\Delta r = [-1 \;\; -0.5 \;\; -0.08]'; \Delta u =$ [-0.5267 0.9200 -0.0800].

reduction in the Fed Funds rate from 5.25% in early September 2007 and 0.25% in February 2009 suggests increases in consumption respectively of 3.63% and 1.04%.

Now consider the effect of an 'operation twist' in November 2010[8], characterizing the lower bound period. For ease of comparison with the short end operation, first consider an operation designed to reduce the long rate by one point at the ZLB, which we simulate as: $\Delta r_l = -1, \Delta r_s = 0, \Delta u_l = 0$. Substituting these values into (20) gives the solution $\Delta u =$ [-0.5000 -1.0000 0]. Using the HW estimates gives the combined 'consumption exposure' effect $\lambda_t' \Sigma^{-1} \Delta u = -0.5134$, which that consumption falls. However, the Duffee's portfolio estimates give an exposure effect which is positive but small relative to the effect of the rate cut: $\lambda_t^{D\prime} \Sigma^{D-1} \Delta u = 0.3016$. Obviously, the effect of the purchase of $400 billion of long term bonds (reducing the long rate by 0.14% rather than 1%) would be smaller still.

These results hang critically on the positive price of slope risk. A reduction in long rates at the ZLB has the same effect on the level factor as a cut in short rates does in normal times, but has the effect of reducing rather than increasing the slope of the curve, tending to nullify the positive level effect. Although both the HW ATSM and Duffee portfolio approaches both suggest that investors need a premium to tolerate slope risk, Cochrane and Piazzesi (2008) conclude that only level risk is priced. That model would suggest that a reduction in long rates at the ZLB would have the same effect on the level factor and hence consumption as a cut in short rates does in normal times.

[8] At that time .$y_t =$ [0.0028 0.0059 0.0355]$'$ and using (20) and (22) $\lambda_t = [-0.1425\ 0.1650$ -0.0601]. However, to focus on the efficacy of different policies we use the December 2006 estimates to calibrate the model for this exercise.

Table 1: Consumption shocks: special cases of equation (<ref>dcc</ref>)

<table>
<tr><th>Model</th><th></th><th>Wealth effect $\left(\frac{1-\gamma\psi}{1-\gamma}\right)$</th><th>Price of risk effect $\left(\frac{\psi-1}{1-\gamma}\right)$</th></tr>
<tr><td>Permanent income hypothesis: Wachter (2003)</td><td>$\gamma \to \infty$</td><td>0</td><td>0</td></tr>
<tr><td>Additive power utility</td><td>$\psi = 1/\gamma$</td><td>0</td><td>$\psi = 1/\gamma$</td></tr>
<tr><td>Unit EIS: Campbell and Viceira (2002)</td><td>$\psi = 1$</td><td>1</td><td>0</td></tr>
<tr><td>Additive log utility</td><td>$\psi = \gamma = 1$</td><td colspan="2">1</td></tr>
</table>

The first term in equation (19) resembles a classic wealth effect, which depends on the way the portfolio exposes wealth to shocks: $\pi(t)$. Its coefficient vanishes in the case of additive power utility. The second term depends upon the degree to which the optimal portfolio is left unhedged, which depends upon the prices of risk $\lambda(t)$. Its coefficient vanishes in the case of a unit elasticity of intertemporal substitution ($\psi = 1$). With additive logarithmic utility, the two coefficients sum to unity. Also $\pi(t) = \Omega(t)^{-1}(\alpha(t) - r(t)i)$ and the terms in square brackets in (18) simplify to $\pi'(t)du$, meaning that the change in log consumption equals the change in log wealth.

Table 2: Bond portfolio return-based estimates of the price of risk

Portfolio n	Maturity bucket	Expected return $(\alpha - ri)_n$	Standard deviation σ_n	Sharpe ratio $\lambda_n = (\alpha - ri)_n/\sigma_n$
1	$0 < m \leq 6$	0.039	0.139	0.28
2	$6 < m \leq 12$	0.062	0.344	0.18
3	$12 < m \leq 24$	0.088	0.613	0.14
4	$24 < m \leq 36$	0.112	0.936	0.12
5	$36 < m \leq 48$	0.126	1.171	0.11
6	$48 < m \leq 60$	0.123	1.376	0.09
7	$60 < m \leq 120$	0.143	1.672	0.09
Memo:	Stock market	0.493	4.292	0.11

These are estimates of ex post returns, variances and Sharpe ratios derived from monthly portfolio returns for the sample period January 1952 to December 2008. They are reproduced from Table 1 of Duffee (2010).