Real Business Cycle models

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Introduction

- Key Words
  - Stationarity
  - Persistence
  - Spurious correlation
  - Stochastic trend
  - Deterministic trend
  - Calibration
  - Normalization
  - RBC (Real Business Cycle)

Introduction 2

- Reasons why Romer (2019) is not an ideal presentation of RBC models
  1. Section 5.1 – Unorthodox presentation of business cycle stylized facts
  2. Section 5.3 – Baseline RBC model presented without enough attention to the type of trend in technical progress
- In this part of the course, we will study the history of the development of real business cycle (RBC) models, and look in detail at a simplified model like the one in Kydland et Prescott (1982)
- I'll discuss (time permitting) the use of dynamic programming and the modelling of trends

My approach

- King and Rebelo (2000) with a few differences
  1. They emphasize one shortcoming of first-generation RBC models: the size of shocks necessary to reproduce the observed variability of output and other aggregates
  2. If one calibrates the size of technology shocks to reproduce this variability, negative technology shocks ("technical regress") become likely. This is implausible
  3. I emphasize a different shortcoming: first-generation RBC models fail completely to predict certain stylized facts of the labour market, notably the weak observed correlation between labour productivity (or the real wage) and employment
  4. King and Rebelo critique Cogley and Nason (1995) — see below — but don't relate this to the way of rendering variables stationary, and the fact that the Hodrick-Prescott and bandpass filters can introduce "spurious autocorrelations" in macro time series

The importance of trends in RBC models

- We're interested in **cyclical** properties of models and the data, so **stationarity** is very important, to avoid **spurious correlation** problems
- The way of measuring the cyclical components has important consequences for cyclical properties
- For example, using the Hodrick-Prescott filter versus calculating in growth rates can change measured **persistence** measured by **autocorrelations**, either contemporaneous or lagged

Evaluating RBC models

- The nature and origin of trends can affect the way we evaluate or test our business cycle models
- A simple example is from Romer, Section 4.8, who argues:
  - RBC models emphasize the importance of technology shocks
  - These shocks have an impact with a strong permanent component. Aggregate demand shocks (important in other approaches such as New Keynesian models), have a temporary impact
  - So, one way to evaluate RBC models is to measure the importance of the permanent component of GDP fluctuations

Evaluating RBC models 2

- We can see the problem with this reasoning with a counterexample
- Stadler (1990) builds an endogenous growth model in which technological progress depends on resource allocation (learning by doing — more later)
- The model also has nominal rigidities, which means monetary shocks have significant effects on aggregate demand (more later), and therefore on resource allocation, and therefore on growth
- So, in this model **all** shocks have permanent effects
- Testing the importance of the permanent component of fluctuations is therefore **not** a good way of discriminating between different approaches to the business cycle

Evaluating RBC Models 3

- The nature of trends is also important for some aspects of model building
- For example, in order to solve and simulate a model using techniques of dynamic programming, we must know **a priori** how to detrend the macro time series we are simulating
- This is necessary in order to use the theorems related to "stationary discounted dynamic programming"
- We'll see an example of this later when we look in detail at the typical RBC model

Two broad methodologies

1. Build a model of the cyclical components of macro time series. Generate predictions, most likely by numerical simulation. The series will be stationary by construction. Use a standard method to remove trends from the data series. Many methods will work. (H-P filters will give stationary cyclical components for series up to I(4) — see King and Rebelor (1990) for a proof). Compare the predictions with te equivalent statistiques in the data
2. Think seriously about the source of non-stationarity. For example, if R&D is subject to uncertainty, this leads inevitably to the idea that GDP follows a stochastic trend. This will restrict how we remove trends from the data. Build a theoretical model compatible with this hypothesis. Remove trends from the model time series and the data in the same way. Compare predictions to the data

Stylized facts

- We already looked at many of the stylized facts in the introductory lecture (introducs.pdf). For more details see:
  - volatilities: Tables 1.1, 5.1 in Cooley (1995), Phaneuf (1994), Ambler and Phaneuf (1994), Stock and Watson (1990), Fiorito and Kollintzas (1994)
  - contemporaneous correlations: same references;
  - autocorrelations, leading and lagging correlations: same references

Brief history 1: first generation

- See also De Vroey and Malgrange (2011), Duarte (2015)
- Kydland and Prescott (1982): closely resembles the basic model we study later on
- Some aspects are of secondary importance and aren't included in the basic model we study ("time to build" in capital accumulation and lagged consumption in utility ("habit persistence"))
- In evaluating their model, they emphasized the behaviour of aggregates related to the **goods market**: GDP and its components
- They concluded the model did remarkably well and could explain a large fraction of cyclical fluctuations

Brief history 2: relative variability of hours

- Hansen (1985): The Kydland and Prescott model **underestimmates** the relative variability of hours
- Hansen extends the model by adding a fixed cost of working a positive number of hours ('ìndivisible labor")
- He adds unemployment insurance which acts as a lottery to get an efficient allocation of resources
- Each period, a fraction of workers choose to work and they all choose to work the same number of hours
- Each worker must voluntarily participate in the lottery, so must be indifferent between working or not
- This gives an individual labor supply curve which is effectively infinitely elastic
- This gives an **aggregate** labor supply elasticity which is greater, and which increases the relative variability of hours

Brief history 3: the hours/productivity correlation

- Only one shock (technology shocks) in the basic RBC model
- The shock shifts the **labor demand curve** along a labor supply curve which is fixed
- Independently of labor supply elasticity, the **correlation** between hours and the real wage (and hence between hours and productivity) must be very high
- As we saw in the introductory lecture, a robust stylized fact is that this corrlation is quite weak. Its size **and** sign can change depending on the country and the historical period considered
- The next phase in the development of the RBC approach was to introduce different shocks to bring down this correlation
- Public spending shocks as in Aiyagari, Christiano and Eichenbaum (1992), "home production" as in Benhabib, Rogerson and Wright (1992), etc.
- The former introduces aggregate demand shocks, the latter labor supply shocks

Brief history 4: Cogley and Nason

- Cogley and Nason (1995) look at the effect of the H-P filter on the dynamic behaviour of macro time series. The filter exaggerates the size of autocorrelations and makes it look like there is more persistence
- See also Guay and Saint-Amant (2005)
- This means that, in most standard RBC models, **persistence** is a byproduct of the persistence of technology (and other) shocks
- It does not arise because of **endogenous propagation mechanisms**

Brief history 5: towards a new synthesis

- My own judgment is that it is difficult to respond to the Cogley-Nason critique without introducing some form of **nominal rigidity**, either prices, or wages, or both
- There also seems to be evidence of the importance of **monetary shocks**
- An increasing number of papers began to incorporate such rigidities
- This led to the New Neoclassical Synthesis — see Goodfriend and King (1998) — and a little later to the New Keynesian approach (more later) — see Woodford (2003) and Galí (2015)
- See Danthine (1998) for a good survey (in French)
- For examples applied to opitmal monetary policy, see Amato and Laubach (1999) and Erceg, Henderson and Levin (2000)
- This synthesis then led to the current generation of forecasting and policy analysis models of central banks

Brief history 6: possible problems

- Chari, Kehoe and McGrattan (2000) claim that the degree of nominal rigidity required to respond to the Cogley-Nason critique is not plausible. The length of nominal price rigidity which is necessary is too long to be compatible with the empirical evidence
- Huang and Liu (2002) show that models with a plausible degree of **wage** rigidity can answer the Cogley-Nason critique
- Ambler (2010) shows that wage rigidity can be justified as a Nash equilibrium with plausible size of fixed costs for adjusting wages, whereas it requires much larger fixed costs to justify price rigidity as a Nash equilibrium
- Ambler, Guay and Phaneuf (2011) show that a business cycle model with nominal wage rigidities alone can reproduce most of the business cycle stylized facts

Brief history 6: the return of econometrics

- In the basic model, there is a small number of parameters which have to be calibrated to do numerical simulations
- Relatively easy to calibrate using basic national accounting data or micro evidence (share of labour income, hours worked as a fraction of total time endowment, etc.). Leaves a large number of moments to evaluate the models either with formal tests or informal comparisons (eg. the real wage/hours correlation above)
- As the models have been extended, more and more free parameters have been introduced. This necessitates a more formal methodology for assessing the "distance" between predictions and the data
- Econometric estimation and testing of models has expanded greatly
- See Christiano and Eichenbaum (1992), Ambler, Guay and Phaneuf (2011), Ireland (2001, 2004)
- The modern generation of central bank models uses **Bayesian** techniques to estimate and evaluate DSGE (dynamic stochastic general equilibrium) models.

Baseline RBC model: specification

- I'll follow Romer here, including his notation (ρ as the rate of time preference instead of β as the discount rate), and labelling the current time period as 0, so that summations run from $t = 0$ to ∞
- My old notes (cycleree.pdf — French only, sorry) develop a slightly different model and show how to solve it using dynamic programming. If you want a (fairly) gentle introduction to dynamic programming, start there. My notes on simulating linearized models (forbackd.pdf) contain yet another variation on the baseline RBC model
- The aggregate production function is Cobb-Douglas:

$$Y_t = {K_t}^{\alpha}(A_t L_t)^{(1-\alpha)}$$

- The capital stock evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t$$

- Using the basic national accounting identity ($Y_t = C_t + I_t + G_t$) gives

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t - G_t$$

Baseline RBC model: specification 2

- Labour and capital are paid their marginal products:

$$w_t = (1-\alpha) K_t{}^{\alpha} (A_t L_t)^{-\alpha} A_t = (1-\alpha) \left(\frac{K_t}{A_t L_t} \right)^{\alpha} A_t$$

$$r_t = \alpha K_t{}^{(\alpha-1)} (A_t L_t)^{(1-\alpha)} - \delta = \alpha \left(\frac{A_t L_t}{K_t} \right)^{(1-\alpha)} - \delta$$

- The second equation involves a small sleight of hand. If households rent capital to firms they get (per unit) the rental rate of capital R_t, and this has to be equal to the marginal product of capital. This has to cover interest costs plus depreciation, so that

$$R_t = r_t + \delta \Rightarrow r_t = R_t - \delta$$

The household's problem

- The representative household maximizes the expected value of

$$U = \sum_{t=0}^{\infty} e^{-\rho t} u\left(c_t, 1 - \ell_t\right) \frac{N_t}{H}$$

- c_t is per-member consumption, ℓ is per-member hours worked (as a fraction of total time endowment which is 1), N_t is population, and H is the number of households, so we are weighting by the number of members of the household
- If all households are the same, $c_t = C_t / N_t$ and $\ell_t = L_t / N_t$
- Population grows at rate n. The notation here is a little different from what we're used to:

$$\ln\left(N_t\right) = \bar{N} + nt \Rightarrow N_t = e^{\left(\bar{N} + nt\right)}$$

Shocks

- Technical progress A_t follows

$$\ln(A_t) = \bar{A} + gt + \bar{A}_t,$$

where $\bar{A}_t$ follows an AR(1) process

$$\bar{A}_t = \rho_A \bar{A}_{t-1} + \varepsilon_{A,t}$$

- Government spending G_t follows

$$\ln(G_t) = \bar{G} + (n+g)t + \bar{G}_t,$$

where $\bar{G}_t$ follows the AR(1) process

$$\bar{G}_t = \rho_G \bar{G}_{t-1} + \varepsilon_{G,t}$$

Household behaviour

- This RBC model differs from the Ramsey model because of two things: (1) there is a labour-leisure trade-off; and (2) there are stochastic shocks (to technology and government spending)
- We simplify the utility function to be logarithmic:

$$u_t = \ln(c_t) + b\ln(1 - \ell_t), b > 0$$

- Romer does not set up a formal Lagrangian. He merely argues what the optimality conditions have to be. We'll follow him
- The marginal utility of consumption per member is $e^{-\rho t}\left(N_t/H\right)\left(1/c_t\right)$. The next period the household has e^n times as many members. For a decrease in consumption (increase in savings) Δc_t, the increase in consumption per member in $t+1$ is $e^{-n}(1 + r_{t+1})\Delta c_t$

Household behaviour 2

- Equating marginal cost with marginal benefit gives a type of Euler equation (we take expectations since consumption in $t+1$ is uncertain):

$$e^{-\rho t}\frac{N_t}{H}\frac{\Delta c_t}{c_t} = E_t\left[e^{-\rho(t+1)}\frac{N_{t+1}}{H}e^{-n}\frac{1}{c_{t+1}}(1+r_{t+1})\right]\Delta c_t$$

- We can take anything which is known outside of the expectations operator. Simplifying,

$$\frac{1}{c_t} = e^{-\rho}E_t\left[\frac{1}{c_{t+1}}(1+r_{t+1})\right]$$

- Note: we are taking the expectation of the **product** of future marginal utility and the rate of return. The interaction is important

Labour supply

- The household which increases its labour supply by $\Delta \ell_t$ has disutility $e^{-\rho t} (N_t/H) [b/(1-\ell_t)] \Delta \ell_t$. The per-member increase in the marginal utility of consumption is $e^{-\rho t} \frac{1}{c_t} w_t \Delta \ell_t$
- Weighting by the number of households and equating marginal cost and marginal benefit gives

$$e^{-\rho t} \frac{N_t}{H} \frac{b}{1-\ell_t} \Delta \ell_t = e^{-\rho t} \frac{N_t}{H} \frac{1}{c_t} w_t \Delta \ell_t$$

$$\Rightarrow \frac{c_t}{1-\ell_t} = \frac{w_t}{b}$$

Linearization

- Romer goes on to consider (Section 5.4) a simplified version of the model which has an analytical solution. The model presented in Section 5.3 of the book doesn't, because of the mixture of linear elements (depreciation) and log-linear elements (production and preferences)
- I will jump straight to Section 5.8 to discuss a calibrated version of the model
- For the nitty gritty of linearization, see my notes on simulating lineared models: `https://www.steveambler.uqam.ca/511/oldnotes/forbackd.pdf`

Calibration

- Romer runs through some of the advantages of calibration
  1. We can appeal to evidence from outside the model (for example micro data)

Calibration 2

- No government: we only have to calibrate technology shocks

Param.	Value	Explanation
α	0.36	capital share in GDP
δ	0.025	micro evidence on depreciation
ρ	0.01	pins down real interest rate in steady state
b	2.0	division of time between work and non-work
ρ_A	0.95	based on behaviour of Solow residuals
$\sigma(\varepsilon_A)$	0.011	based on behaviour of Solow residuals

Simulation results

Moment	U.S. data	Model
σ_Y	1.92	1.30
σ_C/σ_Y	0.45	0.31
σ_I/σ_Y	2.78	3.15
σ_L/σ_Y	0.96	0.49
$Corr\,(L, Y/L)$	-0.14	0.93

- This reveals the 2 fundamental weaknesses we talked about:
  1. relative volatility of hours too low; and
  2. correlation between hours and productivity **much** too high

Monetary shocks

- Romer discusses the fact that money is **neutral** in the basic RBC model
- Its only effect is to change nominal prices
- So if money **does** have strong real effects empirically, the RBC approach is in trouble
- Reduced-form regressions of output (or other real variables) don't tell us much because of endogeneity and reverse causation problems
- Romer and Romer (1989) find evidence of strong effects of money on output by looking at **natural experiments**, instances of big changes in monetary policy which (it can be argued) took place **independently** of developments on the real side of the economy
- The consensus (for what it's worth) view is that money does have strong real effects

Monetary shocks 2

- Cooley and Hansen (1989) introduce money via a cash-in-advance constraint. It has real effects because **anticipated** inflation acts like a distortionary **tax** (**neutrality** versus **super neutrality**)

  "In this paper we study the quantitative importance of money in a real business cycle model where money is introduced in a way that emphasizes the influence on real variables of anticipated inflation operating through the inflation tax."

- Main result:

  "Although we have shown that anticipated inflation can have significant effects on the long-run values of real variables, our model economy predicts that the business cycle will be the same in a high inflation economy as in a low inflation economy."

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