Dynare Tutorial Notes

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Introduction

- Collard, Juillard and Villemot (2009) is a bit laconic on how to actually derive the two main equations which go into the simulation model.
- I provide a few extra steps here.

Household's Lagrangian

- We can write the household's budget constraint for period t as

$$c_t = y_t - i_t = exp(a_t)k_t^{\alpha}h_t^{(1-\alpha)} - (k_{t+1} - (1-\delta)k_t) / \exp(b_t)$$

- We can write the household's maximization problem as

$$\max_{\lambda_{t+i}, c_{t+i}, k_{t+1+i}, h_{t+i}} E_t \Bigg\{ \ln(c_t -) - \theta \frac{h_t^{1+\psi}}{1+\psi}$$

$$+\lambda_t \left(exp(a_t)k_t^{\alpha}h_t^{(1-\alpha)} - (k_{t+1} - (1-\delta)k_t) / \exp(b_t) - c_t \right)$$

$$+ \ln(c_{t+1} -) - \theta \frac{h_{t+1}^{1+\psi}}{1+\psi}$$

$$+\lambda_{t+1} \left(exp(a_{t+1})k_{t+1}^{\alpha}h_{t+1}^{(1-\alpha)} - (k_{t+2} - (1-\delta)k_{t+1}) / \exp(b_{t+1}) - c_{t+1} \right)$$

$$+ \dots \Bigg\}$$

First Order Conditions

- The first order conditions with respect to c_t and h_t pretty simple. It's the FOC with respect to k_{t+1} which needs simplification.
- We have

$$c_t: \quad \frac{1}{c_t} - \lambda_t = 0;$$

$$h_t: \quad -\theta h_t^{1+\psi} + \lambda_t (1-\alpha) \exp(a_t) k_t^{\alpha} h_t^{-\alpha} = 0;$$

$$k_{t+1}: \quad -\frac{\lambda_t}{\exp(b_t)} + E_t \left[\beta \lambda_{t+1} \left(\alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + \frac{(1-\delta)}{\exp(b_{t+1})} \right) \right] = 0.$$

Simplify

- Simplify the FOC with respect to h_t using the first FOC and simple manipulations:

$$\theta c_t h_t^{1+\psi} = (1-\alpha)\, y_t.$$

- We've substituted out c_t and multiplied both sides of the equation by $c_t h_t$, and then replacing the aggregate procuction function with y_t.
- We could have left the production function as is, but the authors include an additional equation which is just the production function itself, adding an additional endogenous variable (y_t) to the system.

Euler Equation

- For the Euler equation, multiply both sides by $\exp(b_t)/\lambda_t$ to get

$$1 = \frac{\exp(b_t)}{\lambda_t} E_t \left[\beta \lambda_{t+1} \left(\alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + \frac{(1-\delta)}{\exp(b_{t+1})} \right) \right]$$

$$\Rightarrow 1 = \beta E_t \left[\frac{\exp(b_t) c_t}{\exp(b_{t+1}) c_{t+1}} \left(\alpha \exp(b_{t+1}) \frac{y_{t+1}}{k_{t+1}} + (1-\delta) \right) \right].$$

- This is just the equation in the paper.

Conclusion

- We have analyzed in detail how to derive the second equation of the equation system used to simulate the model.

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