Endogenous Growth

Steve Ambler
Université Mohammed VI Polytechnique

Fall 2023

Introduction

- Objectives of this part of the course:
  1. Look at why the endogenous growth literature arose and took off in the 80s and 90s
  2. Look at the necessary conditions for growth to be self-sustaining and for policy to have an effect on economic growth in the long run
  3. Study a number of endogenous growth models, in which the growth rate depends on output, the savings rate, on the amount of resources devoted to research & development, etc.
  4. Understand the importance of increasing (or at least non-decreasing) returns to scale to **reproducible** factors of production for growth to be self-sustaining
  5. Look at the welfare implications of resources devoted to R&D (positive and negative externalities)
  6. Look at the predictions of endogenous growth models for economic **convergence** of growth rates and income levels across countries

Introduction

- Romer (1994):

  The phrase "endogenous growth" embraces a diverse body of theoretical and empirical work that emerged in the 1980s. This work distinguishes itself from neoclassical growth by emphasizing that economic growth is an endogenous outcome of an economic system, not the result of forces that impinge from outside.

Origins of endogenous growth theory

- From Romer (1994)

1. The "convergence controversy"
2. "Everyone agrees that a conventional neoclassical model with an exponent of about one-third on capital and about two-thirds on labor cannot fit the cross-country or cross-state data. Everyone agrees that the marginal product of investment cannot be orders of magnitudes smaller in rich countries than in poor countries."
    1. Convergence fails across a broad range of countries
    2. This is motivation for dropping the assumptions of the neoclassical growth model: technological change is exogenous and that the same technological opportunities are available in all countries of the world
    3. The convergence question continues to be controversial. See Romer (1994) for a good summary

Origins of endogenous growth theory (cont.)

3. Beyond perfect competition
   1. Discoveries differ from other inputs in the sense that many people can use them at the same time. Ordinary goods are rival but ideas are nonrival
   2. CRS production functions mean that doubling rival inputs doubles outputs. All firms get to use the same A where A is disembodied technology. But if payments to factors exhaust output (Euler's theorem), there is no income left to remunerate the discoveries that lead to changes in A. This means that growth **has to be exogenous**
   3. Technological advance comes from things that people do. "When more people start prospecting for gold or experimenting with bacteria, more valuable discoveries will be found. This will be true even if discoveries are accidental side effects of some other activity . . . or if market incentives play no role in encouraging the activity
   4. Many individuals and firms have market power and earn monopoly rents on discoveries. If a firm can control access to a discovery, it can charge a price that is higher than zero and earn monopoly profits because information has no opportunity cost

The importance of CRS to reproducible factors of production

- There are two factors of production in the neoclassical growth model: capital and labour
- Capital can be accumulated via investment. Labour grows at an **exogenous** rate n
- There are diminishing marginal returns to capital
- Can capital and output per capita grow faster than the exogenous rate of technological change g?
- No. There are diminishing marginal returns to the output of capital per efficiency unit of labour, therefore diminishing returns to realized savings if the saving rate s is fixed. On the other hand, **required investment** grows linearly with the capital stock and eventually there is no surplus saving for capital accumulation

CRS to reproducible factors (cont.)

- With CRS to reproducible factors, we don't necessarily have diminishing returns to realized investment
- We will see a simple example in the so-called AK model below

Sources of knowledge accumulation

1. Learning by doing: knowledge is simply a byproduct of production. This is a positive externality due to production
2. Pure spillover models (Romer 1986, Lucas 1988). "[T]he technology is endogenously provided as a side effect of private investment decisions. From the point of view of the users of technology, it is still treated as a pure public good, just as it is in the neoclassical model. As a result, firms can be treated as price takers and an equilibrium with many firms can exist"

Sources of knowledge accumulation (cont.)

3. Investment in knowledge accumulation
    - Basic R&D which improves productivity but offers no private return. Another positive externality
    - R&D in order to invent new products which convey monopoly power. There is an incentive for private firms to invest but also a positive externality. A negative externality as well if the new product subtracts demand from competing products
    - The Schumpeterian model has both positive externalities (spillover effects of new inventions) **and** negative externalities (new inventions give monopoly power, at least temporarily)
    - The importance of **positive versus negative externalities** is relevant for policy. Should investment in R&D be subsidized or taxed?

Simple AK model

- The simplest model in which growth can be self-sustaining (obviously unrealistic but a good starting point)
- In the simplest version there is a constant saving rate as in the Solow model
- Constant returns to capital:

$$Y = AK \tag{1}$$

- Rate of change of capital stock:

$$\dot{K} = I - \delta K = sY - \delta K = sAK - \delta K \tag{2}$$

- It follows immediately that the growth rate of capital is

$$\frac{\dot{K}}{K} = sA - \delta \tag{3}$$

Simple AK model (cont.)

- As long as $sA - \delta > 0$, the capital stock will continue to grow at the same rate
- This growth will continue even if the level of technology A remains static
- Technology growth will increase the rate of growth of the capital stock and of GDP, whether exogenous or endogenous

AK model with labour

- A slightly more realistic version of the model starts with a Cobb-Douglas production function

$$Y = AK^{\alpha}L^{(1-\alpha)} \tag{4}$$

- It adds the assumption that capital accumulation generates new knowledge about the production process:

$$A = BK^{(1-\alpha)} \tag{5}$$

- The choice of coefficient on capital in the last equation is not coincidental We get

$$Y = BKL^{(1-\alpha)} \tag{6}$$

AK model with labour (cont.)

- We get

$$\dot{K} = sY - \delta K = sBKL^{(1-\alpha)} - \delta K$$
$$\Rightarrow \frac{\dot{K}}{K} = sBL^{(1-\alpha)} - \delta$$

- If population and labour supply are growing, this would mean an **increasing** growth rate of the capital stock

AK model with utility maximization

- Ramsey version of AK model from Romer (1986). See Aghion and Howitt (1994, Chapter 2). There are N firms/workers
- Representative worker/firm maximizes

$$\int_0^\infty u(c_t)\, e^{-\rho t} dt \tag{7}$$

subject to $\dot{k} = \bar{A}k^\alpha - c$

- Here k is the capital of the individual firm, c_t is individual consumption, and $\bar{A}$ is aggregate productivity taken as given by each firm
- Aggregate productivity depends on the aggregate capital stock,

$$\bar{A} = A_0 K^\eta \tag{8}$$

AK model with utility maximization (cont.)

- Instantaneous utility is given by

$$u\left(c\right)=\frac{c^{1-\epsilon}-1}{1-\epsilon} \tag{9}$$

- We don't derive the Euler equation, but we get

$$\frac{\dot{c}}{c}=\frac{\alpha\bar{A}k^{\alpha-1}-\rho}{\epsilon} \tag{10}$$

- Consumption growth depends positively on the marginal product of capital (think the real interest rate) and negatively on impatience, in the usual way

AK model with utility maximization (cont.)

- All firms are identical, so will employ the same capital in equilibrium:

$$K = Nk$$

- The Euler equation becomes

$$\frac{\dot{c}}{c} = \frac{\alpha A_0 K^{\eta} k^{\alpha-1} - \rho}{\epsilon}$$

$$\Rightarrow \frac{\dot{c}}{c} = \frac{\alpha A_0 N^{\eta} k^{\alpha+\eta-1} - \rho}{\epsilon} \tag{11}$$

- Aggregate output is

$$Y = Ny = NA_0 K^{\eta} \left(\frac{K}{N}\right)^{\alpha} = \left(A_0 N^{1-\alpha}\right) K^{\eta+\alpha} \equiv AK^{\eta+\alpha} \tag{12}$$

Case 1: $\alpha + \eta < 1$

- Once again, growth will vanish asymptotically as in the neoclassical model without technological progress
- To see this, assume, on the contrary, that the growth rate is bounded above zero. The following argument shows that this assumption leads to a contradiction
  1. Positive growth implies that the capital stock k will converge to infnity over time
  2. This implies the RHS of (11) must converge to $-\rho/\epsilon$, since the exponent $(\alpha + \eta - 1)$ is negative
  3. This in turn implies that the growth rate $\dot{c}/c$ will become negative, which contradicts our assumption of positive growth

Case 2: $\alpha + \eta > 1$

- This time there will be explosive growth
- Consider again equation (11)
- If growth is positive in the long run, then *k* converges to infinity over time
- Since $\alpha + \eta - 1 > 1$, the RHS of (11) converges to infinity
- Therefore the **growth rate** of consumption converges to infinity

Case 3: $\alpha + \eta = 1$

- There are constant social returns to capital, so the economy will sustain a strictly positive but finite growth rate g
- Equation (11) implies

$$g = \frac{\dot{c}}{c} = \frac{\alpha A_0 N^\eta - \rho}{\epsilon} \tag{13}$$

- A higher discount rate ρ and a lower intertemporal elasticity of substitution $1/\epsilon$ implies a lower growth rate

Welfare

- Since we have introduced individual utility we can speak about welfare
- Because individuals and individual firms do not internalize the effect of individual capital accumulation on knowledge A when optimizing on c and k, equilibrium growth g must be **less than the socially optimal rate of growth**
- The social planner would maximize

$$u(c) = \frac{c^{1-\epsilon} - 1}{1 - \epsilon}$$

subject to

$$\dot{k} = A_0 (Nk)^{\eta} k^{\alpha}$$

- In other words, the planner **internalizes** the fact that $\bar{A} = A_0 (Nk)^{\eta}$ when choosing k

Welfare (cont.)

- We get the Euler equation

$$\frac{\dot{c}}{c} = \frac{(\alpha + \eta) A_0 N^{\eta} k^{\alpha+\eta-1} - \rho}{\epsilon} \tag{14}$$

- With constant returns to social capital $(\alpha + \eta = 1)$ we get

$$g^* = \frac{A_0 N^{\eta} - \rho}{\epsilon} > g = \frac{\alpha A_0 N^{\eta} - \rho}{\epsilon} \tag{15}$$

- This lets us measure the size of the externality: how much greater is growth when it is internalized?

$$\frac{\eta A_0 N^{\eta}}{\epsilon}$$

Conclusions on Basic AK models

- Growth has been endogenized, but it relies entirely on external (and therefore b) accumulation of knowledge
- Introducing rewards to technological progress adds a new dimension of complexity, since it necessitates moving away from a world of perfect competition into a world of imperfect competition large individual firms
- In the case where $\alpha + \eta = 1$, cross-country variations in parameters such as α and ρ and will result in **permanent differences** in growth rates, so the AK approach does not predict conditional convergence in income per capita
- The cross-section distribution of income should instead (perhaps) exhibit both absolute and conditional divergence

Open-economy AK with convergence

- Aghion and Howitt develop a simple model based on Acemoglu and Ventura (2002) capable of explaining convergence
- Three steps:
  1. closed economy with two sectors (intermediate good and final/capital good);
  2. open economy model with a foreign intermediate; and
  3. symmetric (except for possibly different saving rates) two-country model

Two-sector closed economy

- Production function

$$Y = K^{\alpha} X^{1-\alpha} \tag{16}$$

- Unit cost of X is just one unit of Y. The firm maximizes

$$\Pi = K^{\alpha} X^{1-\alpha} - X \tag{17}$$

- FOC is

$$(1-\alpha) K^{\alpha} X^{-\alpha} = 1 \Rightarrow X = (1-\alpha)^{1/\alpha} K \tag{18}$$

- So we get

$$Y = K^{\alpha} \left((1-\alpha)^{1/\alpha} K \right)^{1-\alpha} = (1-\alpha)^{(1-\alpha)/\alpha} K \tag{19}$$

Closed economy (cont.)

- Even though the production function has a diminishing marginal product of capital we still have an AK model, with $Y = AK$ and

$$A \equiv (1-\alpha)^{(1-\alpha)/\alpha}$$

- The reason for this is that the production technology for the final good has constant returns with respect to K and X which are both produced with K
- Assume a constant saving rate so that

$$\dot{K} = sY - \delta K \tag{20}$$

Small open economy

- Production now requires domestic intermediate good X and foreign intermediate good X_f with fixed relative price p_f. Capital is non-tradable:

$$Y = K^{\alpha} X^{\frac{(1-\alpha)}{2}} X_f^{\frac{(1-\alpha)}{2}} \tag{21}$$

- Domestic producers solve

$$\max_{X, X_f} \left(K^{\alpha} X^{\frac{(1-\alpha)}{2}} X_f^{\frac{(1-\alpha)}{2}} - X - p_f X_f \right) \tag{22}$$

- which gives demands

$$X = \frac{(1-\alpha)}{2} Y, \tag{23}$$

$$p_f X_f = \frac{(1-\alpha)}{2} Y \tag{24}$$

Small open economy (cont.)

- Substituting in the production function gives

$$Y = \left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} p_f^{-\frac{(1-\alpha)}{2\alpha}} K \tag{25}$$

- We have an AK model, with A depending on the price of the foreign intermediate or inversely on the terms of trade $1/p_f$:

$$A \equiv \left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} p_f^{-\frac{(1-\alpha)}{2\alpha}} \tag{26}$$

- Growth depends on the saving rate and the terms of trade

$$g = \frac{\dot{K}}{K} = sA - \delta = s\left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} p_f^{-\frac{(1-\alpha)}{2\alpha}} - \delta \tag{27}$$

Small open economy (cont.)

- Now assume that the domestic country can only export X in exchange for good X_f, and that initially the domestic growth rate exceeds the world growth rate
- Then the foreign demand for the country's exported good X will not grow as fast as the country's demand for X_f
- This will drive up the relative price p_f
- In turn this will drive the domestic growth rate down to the world growth rate
- This is convergence in **growth rates** but not in **levels**

Two-country model

- Foreign economy just like the domestic economy but with saving rate s_f
- Demand for domestic intermediate is

$$(1/p_f)\, F_X = \frac{(1-\alpha)}{2} Y_f, \tag{28}$$

the same as (24) except for foreign output and the inverse of the relative price

- By analogy with (25)

$$Y_f = \left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} p_f^{\,\frac{(1-\alpha)}{2\alpha}} K_f \tag{29}$$

Two-country model (cont.)

- From the last two equations,

$$F_X = \left(\frac{(1-\alpha)}{2}\right)^{\frac{1}{\alpha}} p_f^{\frac{(1+\alpha)}{2\alpha}} K_f \tag{30}$$

- But F_X is the foreign country's imports **and** the domestic country's exports
- If trade is balanced this in turn be equal to the value (in domestic goods) of the domestic country's imports, $p_f X_f$, so from equations (24) and (25) above we have

$$F_X = \left(\frac{(1-\alpha)}{2}\right)^{1/\alpha} p_f^{\frac{-(1-\alpha)}{2\alpha}} K \tag{31}$$

Two-country model (cont.)

- Equating the last two equations for F_X gives

$$p_f = \left(\frac{K}{K_f}\right)^{\alpha} \equiv {k_R}^{\alpha} \tag{32}$$

- If the domestic capital stock grows faster than the foreign stock, k_R will rise, so will the relative price p_f of foreign intermediates, and hence the domestic growth rate will fall, stabilizing k_R

Two-country model (cont.)

- More formally, from the domestic growth equation (27)

$$\frac{\dot{K}}{K} = sA - \delta = s\left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} p_f^{-\frac{(1-\alpha)}{2\alpha}} - \delta$$

$$\Rightarrow \frac{\dot{K}}{K} = s\left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} k_R^{-\frac{(1-\alpha)}{2}} - \delta \qquad (33)$$

- The analogous foreign growth equation is

$$\frac{\dot{K}_f}{K_f} = s_f\left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} k_R^{\frac{(1-\alpha)}{2}} - \delta \qquad (34)$$

Two-country model (cont.)

- The growth rate of the relative capital stock k_R is just the **difference** between the growth rate of the two capital stocks, so

$$\frac{\dot{k}_R}{k_R} = \left(\frac{(1-\alpha)}{2}\right)^{\frac{(1-\alpha)}{\alpha}} \left[s k_R^{-\frac{(1-\alpha)}{2}} - s_f k_R^{\frac{(1-\alpha)}{2}}\right] \tag{35}$$

- This is a stable ordinary differential equation. Setting $\dot{k}_R = 0$ gives

$$s k_R^{-\frac{(1-\alpha)}{2}} = s_f k_R^{\frac{(1-\alpha)}{2}} \tag{36}$$

$$\Rightarrow k_R^* = \left(\frac{s}{s_f}\right)^{\frac{1}{(1-\alpha)}} \tag{37}$$

- The growth rate of k_R will approach zero, implying the growth rates of K and K_f will approach each other — convergence in **growth rates** (but again not in levels)

Conclusions re two-country model

- Faster growth in the domestic economy increases the price of the imported intermediate good, resulting in a deterioration of the country's terms of trade, which in turn reduces the rate of capital accumulation
- Unfortunately, the prediction that growth reduces a country's terms of trade is counterfactual
- Although the AV model is an instructive extension of AK theory to the case of an open economy, in the end it too cannot account for the evidence on cross-country convergence

Schumpeterian growth models

- Basic assumptions:
  1. It is about growth generated by innovations;
  2. Innovations result from entrepreneurial investments that are themselves motivated by the prospects of monopoly rents; and
  3. New innovations replace old technologies: in other words, growth involves creative destruction
- Private firms invest in R&D to produce new products
- They create, but they also destroy demand for already-existing products (**creative destruction**)

Simplest Schumpeterian growth model

- Based on Aghion, Akcigit and Howitt (2013) and Wilde (2006)
- One final good is produced by competitive firms using a single intermediate good,

$$Y_t = {A_t}^{1-\alpha} {x_t}^{\alpha} \tag{38}$$

  where A_t is the productivity of the intermediate input
- Final output can be used for consumption, as an intermediate good in the production process, and for R&D:

$$Y_t = C_t + x_t + R_t \tag{39}$$

- If there is an innovation at time t, productivity improves by a factor $\gamma > 1$, so

$$A_t = \gamma A_{t-1}$$

  and if there is no innovation

$$A_t = A_{t-1}$$

Simplest Schumpeterian model (cont.)

- Specifically, the probability that an innovation occurs during any period is assumed to be

$$\lambda \left(\frac{R_t}{A_t} \right)^{\sigma}, 0 < \sigma < 1 \tag{40}$$

- R_t is the amount of final good spent on R&D and λ measures the productivity of the R&D sector
- The probability of innovation depends inversely on A_t is that as technology advances it becomes more complex and thus harder to improve upon
- So what matters is the productivity-adjusted expenditure $(R_t/A_t) \equiv n_t$

Simple Schumpeterian model (cont.)

- The growth rate of A_t will equal $\frac{\gamma A_{t-1} - A_{t-1}}{A_{t-1}} = \gamma - 1$ with probability λn^{σ} and zero with probability $1 - \lambda {n_t}^{\sigma}$
- Therefore, the expected growth rate of A_t depends on productivity-adjusted R&D according to

$$g_t = \lambda {n_t}^{\sigma} (\gamma - 1) \tag{41}$$

- This means that to understand the determinants of growth we need to understand the determinants of investment in R&D

Simple Schumpeterian model (cont.)

- People are motivated to undertake R&D by the prospect of monopoly profits
- The innovator's monopoly profit π is determined as follows
- One unit of final good is needed as input for each unit of intermediate product, so total cost will be the quantity produced x_t
- The final sector is perfectly competitive, the price at which he can sell his intermediate good is its marginal product in that sector,

$$p(x_t) = \frac{\partial Y_t}{\partial x_t} = \alpha {A_t}^{1-\alpha} {x_t}^{\alpha-1} \tag{42}$$

- The innovator maximizes

$$\max_{x_t} \{p(x_t)x_t - x_t\} = \{\alpha {A_t}^{\alpha-1} {x_t}^{\alpha} - x_t\} \tag{43}$$

Simple Schumpeterian model (cont.)

- The FOC for a maximum is

$$\alpha^2 {A_t}^{\alpha-1} {x_t}^{\alpha-1} - 1 = 0 \tag{44}$$

$$\Rightarrow x_t = A_t \alpha^{\frac{2}{1-\alpha}} \tag{45}$$

- Substitute for x_t in the price equation (42) to get

$$p(x_t) = \alpha {A_t}^{1-\alpha} \left(A_t \alpha^{\frac{2}{1-\alpha}} \right)^{\alpha-1} = \frac{1}{\alpha} \tag{46}$$

- The parameter α is a measure of the degree of product-market competition
- Since an incumbent's marginal cost is one, $1/\alpha$ is the equilibrium markup of price over marginal cost (the "Lerner index of monopoly power")

Simple Schumpeterian model (cont.)

- Substituting for x_t and $p(x_t)$ from the last two equations gives

$$\pi_t = (p_t - 1)\, x_t = \left(\frac{1}{\alpha} - 1\right) A_t \alpha^{\frac{2}{1-\alpha}} \equiv A_t \delta \tag{47}$$

  for the appropriate definition of δ
- Equilibrium R&D is determined by the condition that the marginal expected gain from one more unit of R&D expenditure equals the marginal cost, which by definition is just unity
- The total expected gain to R&D is just the probability of an innovation times the profit earned by a successful innovator, or

$$\text{Expected gain} = \lambda \left(\frac{R_t}{A_t}\right)^{\sigma} \pi_t \tag{48}$$

Simple Schumpeterian model (cont.)

- The marginal expected gain is the derivative of this wrt R_t minus the marginal cost of R&D, which is one. Setting this equal to zero gives

$$\sigma\lambda\left(\frac{1}{A_t}\right)\left(\frac{R_t}{A_t}\right)^{\sigma-1}\pi_t - 1 = 0 \tag{49}$$

$$\Rightarrow \sigma^{\lambda}\left(\frac{1}{A_t}\right)\left(\frac{R_t}{A_t}\right)^{\sigma-1}A_t\delta = \sigma\lambda {n_t}^{\sigma-1}\delta = 1 \tag{50}$$

$$\Rightarrow n_t = (\sigma\lambda\delta)^{\frac{1}{1-\sigma}} \tag{51}$$

- Substituting (45) into (38) gives

$$Y_t = {A_t}^{1-\alpha}{x_t}^{\alpha} = {A_t}^{1-\alpha}\left(A_t\alpha^{\frac{2}{1-\alpha}}\right)^{\alpha} = \alpha^{\frac{2\alpha}{1-\alpha}}A_t \tag{52}$$

Simple Schumpeterian model (cont.)

- So output is proportional to productivity
- The long-run average growth rate of output will be the same as the long-run average growth rate of A_t, i.e. g_t
- With constant population this is also the long-run average growth rate of per capita output

Non-drastic innovations

- What if the incumbent innovator is not automatically a monopolist
- Instead, a "competitive fringe" can produce the intermediate at a price $\chi > 1$, so the innovator cannot charge more than this, so

$$p(x_t) = \chi \tag{53}$$

- When $\chi > 1/\alpha$ this is not binding and corresponds to the "drastic innovation" case
- When $\chi < 1/\alpha$ this is a binding constraint
- The previous innovator can also potentially compete by producing the intermediate good with a cost of unity but with productivity of only A_t/γ. We can show that the previous innovator cannot compete if $\gamma > 1/\alpha^{\frac{\alpha}{1-\alpha}}$, i.e. if innovations are big enough

Non-drastic innovations (cont.)

- Combining (53) with (42) to get

$$\chi = \alpha {A_t}^{1-\alpha} {x_t}^{\alpha-1} \tag{54}$$

$$\Rightarrow x_t = A_t \left(\frac{\chi}{\alpha}\right)^{\frac{1}{\alpha-1}} \tag{55}$$

- Equilibrium profits are then

$$\pi_t = p_t x_t - x_t = (\chi - 1) A_t \left(\frac{\chi}{\alpha}\right)^{\frac{1}{\alpha-1}} = A_t \delta' \tag{56}$$

with

$$\delta' \equiv (\chi - 1) \left(\frac{\chi}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

Non-drastic innovations (cont.)

- This is increasing in χ_t
- Equilibrium investment is therefore

$$\Rightarrow n_t = \left(\sigma\lambda\delta'\right)^{\frac{1}{1-\sigma}} \tag{57}$$

- This is also increasing in χ_t as is equilibrium growth

$$g_t = \lambda {n_t}^{\sigma} \left(\gamma - 1\right)$$

- A higher χ may reflect stronger patent protection which increases the cost of imitating the current technology in any intermediate good sector. Or it may reflect a lower degree of competition in the intermediate good sectors
- In either case, it should lead to more intense R&D as it raises the expected rents that accrue to a successful innovator. This in turn should result in higher growth

Comparative statics

- To determine growth as a function of the underlying parameters of the system, use (47) to substitute out δ in (51) to get n_t and substitute in (41) to get

$$g_t = \lambda {n_t}^{\sigma} (\gamma - 1)$$

$$= \lambda \left((\sigma \lambda \delta)^{\frac{1}{1-\sigma}} \right)^{\sigma} (\gamma - 1)$$

$$= \lambda \left(\left(\sigma \lambda \left(\frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}} \right)^{\frac{1}{1-\sigma}} \right)^{\sigma} (\gamma - 1)$$

- We get the following comparative statics on the equilibrium growth rate

Comparative statics (cont.)

1. Growth is increasing in λ, the productivity of innovations. This points to the possible importance of education, particularly higher education
2. Growth increases with the size of innovations measured by γ. This points to the existence of a wedge between private and social innovation incentives. Given the choice between increasing the frequency λ or their size γ, the private individual will go for increasing frequency. Size increases the cost of innovation as well as the expected rents; the research arbitrage equation shows that these two effects cancel each other, leaving the equilibrium level of R&D independent of size
3. Growth is increasing in property right protection measured by χ
4. Growth is decreasing in the degree of product market competition (higher α or lower χ). At odds with some empirical work

Welfare

- Aghion, Akcigit and Howitt (2014) look at a simple social planning problem in the model we just looked at but with variable labour supply
- They show there is underinvestment in R&D iff γ is large enough compared to the size of the monopoly distortion which depends on α
- This result is intuitive. In this economy, there are two types inefficiencies: (i) monopoly distortion; and (ii) innovation externality
- The former is governed by α, which determines equilibrium markups The latter is governed by the size of innovations γ
- If the innovation externalities are above a threshold there is underinvestment in R&D and vice versa.
- To find the exact condition we have to look at the problem of a social planner

Social planning problem

- Let's consider a version of the social planning problem analyzed by Aghion, Akcigit and Howitt (2014)
- Aggregate consumption is given by

$$C_t = Y_t - x_t - R_t \tag{58}$$

- Final output is used for consumption, intermediate good production, and R&D
- Consider first the planning problem after R&D has already taken place, so it is a sunk cost. This will highlight the monopoly distortion

Social planning problem (cont.)

- The planner solves

$$\max_{x_t} \left\{ {A_t}^{1-\alpha} {x_t}^{\alpha} - x_t - R_t \right\} \tag{59}$$

  where R_t is taken as given

- The FOC is

$$\alpha {A_t}^{1-\alpha} {x_t}^{\alpha-1} - 1 = 0 \Rightarrow x_t = A_t \alpha^{\frac{1}{1-\alpha}} \tag{60}$$

- So socially optimal output is

$$\begin{aligned} Y_t^* &= {A_t}^{1-\alpha} \left(A_t \alpha^{\frac{1}{1-\alpha}} \right)^{\alpha} \\ &= A_t \alpha^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{61}$$

Social planning problem (cont.)

- So we have

$$Y_t^* = A_t \alpha^{\frac{\alpha}{1-\alpha}} > A_t \alpha^{\frac{2\alpha}{1-\alpha}} = Y_t \qquad (62)$$

- The decentralized economy under-produces due to monopoly distortions
- Socially optimal consumption is

$$C_t^* = A_t \alpha^{\frac{\alpha}{1-\alpha}} - A_t \alpha^{\frac{1}{1-\alpha}} - R_t$$

$$= A_t \alpha^{\frac{\alpha}{1-\alpha}} - A_t \alpha^{\frac{1-\alpha}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} - R_t$$

$$= A_t \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - R_t \qquad (63)$$

Social planning problem (cont.)

- Now we want to consider the socially optimal R&D expenditure decision
- Let's assume that the probability of an innovation occurring in any period depends on R&D expenditure **relative to** A_{t-1}

$$\lambda \left(\frac{R_t}{A_{t-1}} \right)^{\sigma}$$

- The planner maximizes

$$\max_{R_t} \left[\lambda \left(\frac{R_t}{A_{t-1}} \right)^{\sigma} \gamma A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) + \left(1 - \lambda \left(\frac{R_t}{A_{t-1}} \right)^{\sigma} \right) A_{t-1} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) - R_t \right]$$

Social planning problem (cont.)

- We can see that
- The FOC is

$$(\gamma - 1)\sigma\lambda \left(\frac{R_t}{A_{t-1}}\right)^{\sigma-1} \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha) - 1 = 0$$

$$\Rightarrow \left(\frac{R_t}{A_{t-1}}\right) = n_t^* = \left((\gamma - 1)\sigma\lambda\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\right)^{\frac{1}{1-\sigma}} \qquad (64)$$

- This is to be compared to the private optimum,

$$n_t = (\sigma\lambda\delta)^{\frac{1}{1-\sigma}}$$

with

$$\delta \equiv \left(\frac{1}{\alpha} - 1\right)\alpha^{\frac{2}{1-\alpha}} = (1-\alpha)\alpha^{-1}\alpha^{\frac{2}{1-\alpha}} = \alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)$$

Social planning problem (cont.)

- So we are comparing

$$n_t^* = \left((\gamma - 1)\sigma\lambda\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\right)^{\frac{1}{1-\sigma}}$$

  with

$$n_t = \left(\sigma\lambda\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\right)^{\frac{1}{1-\sigma}}$$

- So socially optimal investment is greater than private investment if and only if

$$(\gamma - 1) > \alpha^{\frac{1}{1-\alpha}} \qquad (65)$$

- Only the social planner takes the size of innovations γ into account

Industrial policy

- Aghion, Akcigit and Howitt go on to discuss how governments can intervene when the social optimum diverges from the private optimum
- We can imagine the following types of policies:
  1. reinforcing or weakening patent protection to encourage or discourage R&D spending;
  2. subsidizing or taxing R&D to encourage or discourage spending;
  3. subsidizing production of the intermediate good to reduce or eliminate the monopoly distortion;
  4. if lump-sum taxation is available (unlikely) it may be possible to achieve a first-best outcome, otherwise it will only be possible to reach a second-best outcome

Predictions of Schumpeterian growth models

- See Aghion, Akcigit and Howitt (2015)
  1. The rate of new inventions is positively correlated with growth
  2. The relationship between competition and growth follows an inverted U relationship
  3. More intense competition enhances innovation in "frontier" firms but may discourage innovations in "non-frontier" firms

Optimal growth-promotion policies

- The existence of positive spillover effects (**creation**) from investment in technological change means that there is a case to promote such investment
- The existence of negative spillover effects (**destruction**) means that promotion of investment in R&D can go too far

Growth and technnological change from 1 million BCE

- Let's look at the paper by Kremer (1993): one of the most ambitious economics papers ever in terms of what it tries to (and claims to be able to) explain
- We will look at the details if time permits
- Summary based on Chapter 3 in Romer
- The relationship between per capita income and population growth has been **Malthusian** for much of economic history
- To explain more recent developments, we need to assume that the relationship is non-monotonic. Beyond a certain level of development (per capita income, female labour force participation, average eductational attainment, particular among females), fertility rates turn down and can cross over mortality rates from above

1 million BCE (cont.)

- Output depends on technology, labour and land:

$$Y_t = T^{\alpha} (A_t L_t)^{1-\alpha} \tag{66}$$

- Knowledge evolves according to

$$\dot{A}_t = B L_t {A_t}^{\theta} \tag{67}$$

- Population adjusts so output per person equals its subsistence level (the Malthusian assumption):

$$\frac{Y_t}{L_t} = \bar{y} \tag{68}$$

1 million BCE (cont.)

- First find the population which can be supported at a point in time. Substitute the production function into the Malthusian equation:

$$\frac{T^{\alpha}\left(A_t L_t\right)^{1-\alpha}}{L_t} = \bar{y} \tag{69}$$

- This gives

$$L_t = \left(\frac{1}{\bar{y}}\right)^{1/\alpha} A_t{}^{(1-\alpha)/\alpha} T \tag{70}$$

- The last equation implies the growth rate of L_t is $(1-\alpha)/\alpha$ times the growth rate of A_t:

$$\frac{\dot{L}_t}{L_t} = \frac{1-\alpha}{\alpha}\frac{\dot{A}_t}{A_t} \tag{71}$$

1 million BCE (cont.)

- With $\theta = 1$, population growth is proportional to its level
- In the general case, population growth is proportional to ${L_t}^{\psi}$ with

$$\psi = 1 - [(1 - \theta)\alpha/(1 - \alpha)]$$

- So population growth is increasing in L_t unless α is large or θ is much less than one
- Figure 3.9 in Romer is the scatter plot of population against population growth
- The fit is good but breaks down for more recent observations

Figure: Kremer (1993)

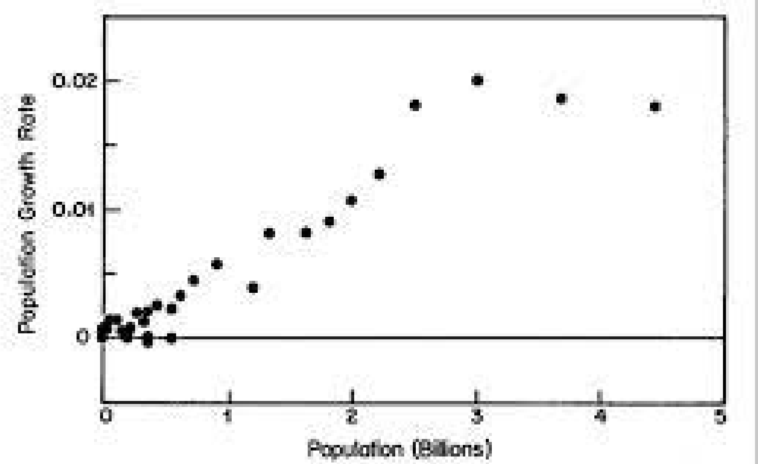

FIGURE 1
Population Growth Versus Population

1 million BCE (cont.)

- Instead of assuming $Y/L = \bar{y}$, we could assume that population growth n is a function of per capita income y, $n = n(y)$
- Furthermore, the relationship is non-monotonic, peaking at some level of y and then declining
- Therefore, an increasing fraction of the effect of technological progress falls on y rather than population
- If $n(y)$ becomes negative for some y sufficiently large, population itself will peak at some point

Empirical tests of growth models

- Section 3.6 in Romer
- Romer discusses the tests by Jones (1995, reference in Romer)
- Is growth stationary?
- Does it matter?
    1. Tests can lack power
    2. Growth can be non-stationary for other reasons, for example non-stationary population growth
- Romer makes the following general recommendation: "*always focus on confidence intervals and their economic interpretation, never on t statstics*"
- Jones notes that since WWII the number of scientists engaged in R&D and R&D spending have gone up by a factor of five, while growth rates of income per capita have not
- It seems that there are decreasing returns to R&D

Predictions for business cycles

- See Stadler (1990), which we don't have time to look at in detail
- If there is learning by doing **and** monetary policy has strong real effects, monetary policy can have **permanent** effects on output and productivity
- This is a way of reintroducing **persistent effects** of monetary policy, so there are implications for business cycles as well
- Stadler: "There has been some debate as to whether models that emphasize aggregate demand innovations, or models that rely on real shocks as an impulse mechanism, provide a better characterization of output fluctuations. However, if technology is endogenous, real and monetary business cycle models exhibit very similar properties, as a comparison of equations (19) and (25) illustrates. Both classes of models can account for the persistence of innovations in output, and both can account for the rise in the growth rates of real output observed by Romer."

This version: **27/11/2023**