Monetary Policy

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Introduction

- Goals:
  1. Look at monetary policy in a unified framework
  2. Use a welfare function from the utility function of the representative consumer, adding the costs of price dispersion
  3. Understand why the central bank can neutralize demand shocks if it has perfect information on the source of disturbances
  4. Look at optimal policy under perfect information
  5. Look at optimal policy under perfect information with time lags
  6. Look at policy at the zero lower bound
  7. Look at optimal policy under rational expectations

Preliminaries

- The central bank follows either a fixed rule (like the Taylor rule) or uses its discretion
- Following a rule means it cannot react flexibly to all the information it has at its disposal
- Nevertheless, there can be advantages to following a rule. It can increase the **credibility** of monetary policy, especially when there is a possibility of **dynamic inconsistency**
- The benefits of credibility are that announcements of future policy have effects on individuals' expectations. In models with backward-looking expectations we can't model these benefits explicitly

Preliminaries (cont.)

- We use the following strategy
  1. For the moment, set aside AD
  2. Assume the central bank has perfect credibility so $\pi^e = \pi^* \Rightarrow \hat{\pi}^e = 0$
  3. Calculate the marginal social loss as a function of output, $\frac{\partial SL}{\partial \hat{y}}$
  4. Calculate the marginal social loss as a function of inflation, $\frac{\partial SL}{\partial \hat{\pi}}$
  5. This allows us to calculate the trade-off between output and inflation **along the AS curve**, whose slope is given by $\frac{\partial \hat{\pi}}{\partial \hat{y}}$

$$\frac{\partial SL}{\partial \hat{y}} + \frac{\partial SL}{\partial \hat{\pi}} \frac{\partial \hat{\pi}}{\partial \hat{y}} = 0$$

  6. So this is like a constrained optimization problem: the central bank chooses the optimal trade-off as long as it is a point on the AS curve
  7. Substitute the equation for the slope of AS in this equation to get the optimal trade-off
  8. Then we can use AD to find the policy rate compatible with this outcome

Welfare function

- The central bank minimizes the following social loss function:

$$SL = -a_d\left(\hat{y} - \hat{b}\right) + \frac{a_l}{2(1-\alpha)}\left(\hat{y} - \hat{b}\right)^2 + \frac{a_\pi}{2}\hat{\pi}^2. \tag{1}$$

- See Appendix A for a derivation of this equation
- The first-order term reflects the fact that the flexible-price equilibrium in the economy is suboptimal because of monopoly distortions and distortionary taxation (as we saw in Galí, Gertler and López-Salido)
- We can show (see appendix) that

$$a_d = 1 - \left(\frac{(1-\tau)}{\bar{m}^p\,\bar{m}^w}\right)$$

where τ is the tax rate on income, $\bar{m}^p$ is the equilibrium markup rate over marginal cost, and $\bar{m}^w$ is the equilibrium markup of wage over the marginal rate of substitution, which is zero without distortions

Optimal policy

- With perfect credibility ($\pi^e = \pi^*$), AS is given by

$$\hat{\pi} = \left(\frac{\alpha}{1-\alpha}\right)\hat{y} + \hat{m} - \frac{\hat{b}}{1-\alpha} \tag{2}$$

where $\hat{m}$ groups together the markup shocks

- From (1), the marginal social loss for a small change in $\hat{y}$ is

$$MSL_y = \frac{\partial SL}{\partial \hat{y}} = \frac{a_l}{(1-\alpha)}(\hat{y} - \hat{b}) - a_d$$

- and

$$MSL_\pi = \frac{\partial SL}{\partial \hat{\pi}} = a_\pi \hat{\pi}.$$

Optimal policy (cont.)

- Using $\frac{\partial \hat{\pi}}{\partial \hat{y}} = \frac{\alpha}{(1-\alpha)}$ along AS we have the FOC for the central bank to be

$$\frac{dSL}{d\hat{y}} = 0 \quad \Rightarrow \quad MSL_y + MSL_\pi \times \frac{\partial \hat{\pi}}{\partial \hat{y}} = 0$$

$$\Rightarrow \quad \frac{a_l}{1-\alpha}(\hat{y} - \hat{b}) - a_d + a_\pi \hat{\pi} \times \left(\frac{\alpha}{1-\alpha} \right) = 0,$$

so

$$\hat{y} = \hat{b} + (1-\alpha)\frac{a_d}{a_l} - \frac{\alpha a_\pi}{a_l}\hat{\pi} \tag{3}$$

- Solving for $\hat{\pi}$ we have

$$MPR : \hat{\pi} = \left(\frac{1-\alpha}{\alpha} \right) \frac{a_d}{a_\pi} + \frac{a_l}{\alpha a_\pi} \left(\hat{b} - \hat{y} \right) \tag{4}$$

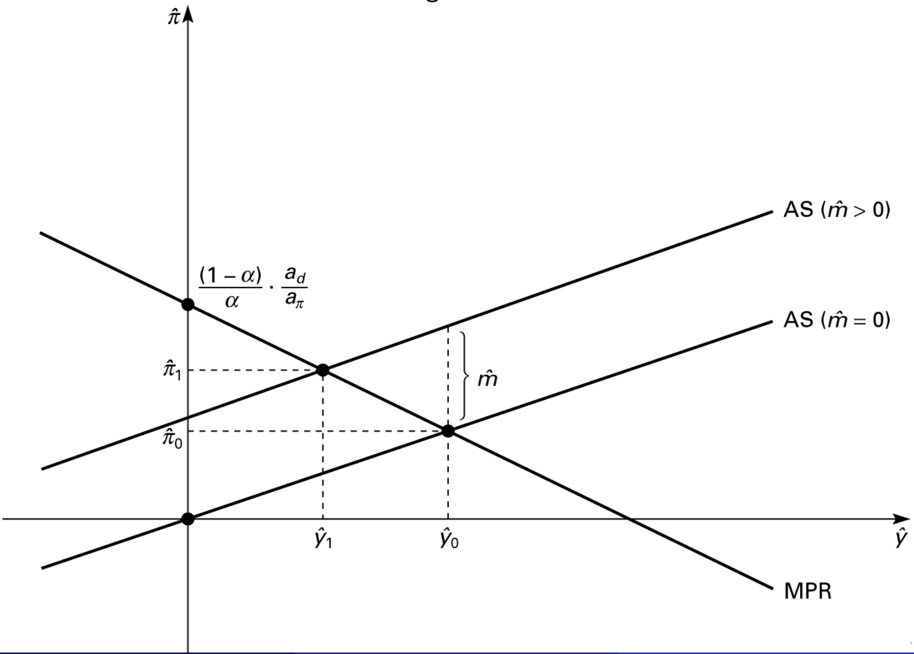
Optimal policy (cont.)

- This is the MPR curve in Figure 1 below
- There is a negative trade-off between inflation and output
- When $\hat{b} = \hat{y} = 0$, we have

$$\hat{\pi} = \frac{1-\alpha}{\alpha} \cdot \frac{a_d}{a_\pi}.$$

- This is the vertical intercept of the curve. It means that for the central bank, it is optimal for $\pi > \pi^*$. There is an **inflationary bias** because of distortions (remember that without them $a_d = 0$)

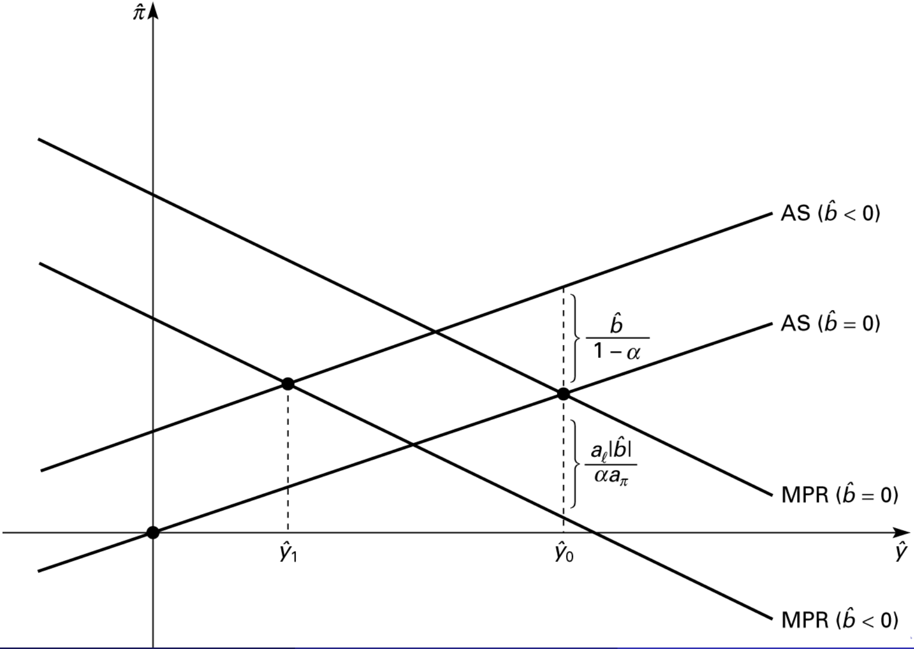
Figure 1


$\hat{\pi}$
$\frac{(1-\alpha)}{\alpha} \cdot \frac{a_d}{a_\pi}$
AS ($\hat{m} > 0$)
AS ($\hat{m} = 0$)
$\hat{m}$
$\hat{\pi}_1$
$\hat{\pi}_0$
$\hat{y}_1$
$\hat{y}_0$
$\hat{y}$
MPR

Optimal policy (cont.)

- Figure 1 also indicates the optimal monetary policy response to a positive markup shock, which shifts up AS by $\bar{m}$
- Rather than allowing the full impact to be felt on inflation, the optimal policy involves reducing the impact on inflation by allowing output to decrease
- Figure 2 below shows the response to a negative technology shock

Figure 2


$\hat{\pi}$
AS ($\hat{b} < 0$)
AS ($\hat{b} = 0$)
$\frac{\hat{b}}{1-\alpha}$
$\frac{a_\ell |\hat{b}|}{\alpha a_\pi}$
MPR ($\hat{b} = 0$)
MPR ($\hat{b} < 0$)
$\hat{y}_1$
$\hat{y}_0$
$\hat{y}$

Optimal policy (cont.)

- Note that a technology shock also shifts the MPR curve
- It is allowed to have a bigger effect on output because it shifts the economy's technological possibilities while a markup shift does not

Central bank policy rate

- To find the optimal $\hat{y}$, we use the equations for MPR and AS to eliminate $\hat{\pi}$
- Then, to find the central bank's policy rate, we use AD
- Substituting out $\hat{\pi}$ using (2) in (3)gives

$$\hat{y} = \hat{b} + (1-\alpha)\frac{a_d}{a_l} - \frac{\alpha a_\pi}{a_l}\left(\left(\frac{\alpha}{1-\alpha}\right)\hat{y} + \hat{m} - \frac{\hat{b}}{1-\alpha}\right)$$

$$\Rightarrow \left(1 + \frac{\alpha a_\pi}{a_l}\frac{\alpha}{1-\alpha}\right)\hat{y} =$$

$$(1-\alpha)\frac{a_d}{a_l} - \frac{\alpha a_\pi}{a_l}\hat{m} + \left(1 + \frac{\alpha a_\pi}{(1-\alpha)a_l}\right)\hat{b}$$

Central bank policy rate (cont.)

$$\Rightarrow \left(a_l + \frac{\alpha^2 a_\pi}{1-\alpha}\right)\hat{y} = (1-\alpha)a_d - \alpha a_\pi \hat{m} + \left(a_l + \frac{\alpha a_\pi}{(1-\alpha)}\right)\hat{b}$$

$$\Rightarrow (a_l + \alpha\gamma a_\pi)\,\hat{y} = (1-\alpha)a_d - \gamma a_\pi(1-\alpha)\hat{m} + (a_l + \gamma a_\pi)\,\hat{b},$$

- This gives (finally)

$$\hat{y} = \left(\frac{1-\alpha}{a_l + \alpha\gamma a_\pi}\right)a_d - \left(\frac{\gamma a_\pi}{a_l + \alpha\gamma a_\pi}\right)(1-\alpha)\hat{m} + \left(\frac{a_l + \gamma a_\pi}{a_l + \alpha\gamma a_\pi}\right)\hat{b}, \quad (5)$$

where $\gamma = \frac{\alpha}{1-\alpha}$

- Use the AD curve (27) which is (see Appendix C)

$$\hat{y} = \alpha_1 \hat{g} - \alpha_2(r - \bar{r}) + v \quad (6)$$

Central bank policy rate (cont.)

- We get

$$r = \bar{r} + \frac{v + \alpha_1 \hat{g}}{\alpha_2} - \left(\frac{1-\alpha}{\alpha_2 (a_l + \alpha\gamma a_\pi)} \right) a_d$$

$$+ \left(\frac{\gamma a_\pi}{\alpha_2 (a_l + \alpha\gamma a_\pi)} \right) (1-\alpha)\hat{m} - \left(\frac{a_l + \gamma a_\pi}{\alpha_2 (a_l + \alpha\gamma a_\pi)} \right) \hat{b}$$

- Now use

$$r = i + \pi^e = i^p + \rho + \pi^e = \imath^p + \rho + \pi^*$$

where i^p is the **riskless** nominal rate of interest, ρ is a risk premium, and we use the perfect credibility assumption $\pi^e = \pi^*$

Central bank policy rate (cont.)

- We get

$$i^p = \pi^* + \bar{r} - \hat{\rho} + \frac{v + \alpha_1 \hat{g}}{\alpha_2} - \left(\frac{1-\alpha}{\alpha_2(a_l + \alpha\gamma a_\pi)} \right) a_d$$

$$+ \left(\frac{\gamma a_\pi}{\alpha_2(a_l + \alpha\gamma a_\pi)} \right) (1-\alpha)\hat{m} - \left(\frac{a_l + \gamma a_\pi}{\alpha_2(a_l + \alpha\gamma a_\pi)} \right) \hat{b} \qquad (7)$$

- The central bank responds to **shocks** rather than observable variables because it has perfect information
- The $\frac{v+\alpha_1\hat{g}}{\alpha_2}$ term means that the central bank can **perfectly neutralize** aggregate demand shocks. There is no output/inflation trade-off. This is an example of the so-called **divine coincidence**
- The negative coefficient on a_d is another way of seeing inflation bias. Even without shocks

Credibility

- We have seen that in the absence of shocks the central bank chooses an inflation rate above zero because of the first-order term in the social loss function
- This means that inflation **on average** will be above zero
- This means that our initial assumption that $\pi^e = \pi^*$ is questionable
- If π^e is systematically above π^*, individuals will eventually revise their expectations
- Let us turn to analyzing monetary policy when we drop the assumption of perfect information and credibility

Limited information

- Suppose the central bank just observes output and inflation but not the shocks, and refrains from attempting to increase output beyond its natural level ($a_d = 0$)
- The social loss function is now

$$E(SL) = \frac{a_l}{2(1-\alpha)}\left(\hat{y}\right)^2 + \frac{a_\pi}{2}\left(\hat{\pi}\right)^2 \tag{8}$$

- We also now assume static expectations

Limited information (cont.)

- With static expectations, we don't converge immediately to the long-run equilibrium
- The central bank takes into account **future expected** losses
- We assume all shocks are not serially correlated
- From (8) we have that the marginal social loss is

$$\frac{\partial E(SL)}{\partial \hat{y}} = \frac{a_l \hat{y}}{(1-\alpha)}.$$

- In the absence of shocks, the output gap returns towards its long-run equilibrium of zero at the rate β,

$$\hat{y}_{t+1} = \beta \hat{y}_t.$$

- Dropping time subscripts to simplify,

$$MSL_y^d = \frac{a_l}{(1-\alpha)} \left(\hat{y} + \frac{\beta}{1+\phi} \hat{y} + \left(\frac{\beta}{1+\phi} \right)^2 \hat{y} + \ldots \right),$$

where $1/(1+\phi)$ is the social discount rate of the central bank

Limited information (cont.)

- This gives

$$MSL_y^d == \frac{a_l \hat{y}}{(1-\alpha)} \left(\frac{1}{\left(1 - \frac{\beta}{1+\phi}\right)} \right)$$

$$= \frac{a_l \hat{y}}{(1-\alpha)} \left(\frac{1+\phi}{1+\phi-\beta} \right)$$

- Similarly for the marginal loss with respect to inflation we have

$$\frac{\partial E(SL)}{\partial \hat{\pi}} = a_\pi \hat{\pi}$$

- With $\hat{\pi}_{t+1} = \beta \hat{\pi}_t$ we get

$$MSL_\pi^d = a_\pi \hat{\pi} \left(\frac{1+\phi}{1+\phi-\beta} \right)$$

Limited information (cont.)

- With static expectations, AS is given by

$$\hat{\pi} = \hat{\pi}_{-1} + \left(\frac{\alpha}{1-\alpha}\right)\hat{y} + s$$

$$\Rightarrow \frac{\partial \hat{\pi}}{\partial \hat{y}} = \frac{\alpha}{1-\alpha}.$$

- The optimality condition for the bank can now be written

$$MSL^d_y + MSL^d_\pi \frac{\partial \hat{\pi}}{\partial \hat{y}} = 0$$

$$\Rightarrow \frac{a_l \hat{y}}{(1-\alpha)}\left(\frac{1+\phi}{1+\phi-\beta}\right) + a_\pi \hat{\pi}\left(\frac{1+\phi}{1+\phi-\beta}\right)\frac{\alpha}{1-\alpha} = 0$$

$$\Rightarrow MPR : \hat{\pi} = -\left(\frac{a_l}{\alpha a_\pi}\right)\hat{y}$$

Limited information (cont.)

- This is an MPR which is qualitatively similar to the full-information case
- Because inflation and output return to equilibrium at the same rate, the infinite sums which collapse to $(1+\phi)/(1+\phi-\beta)$ cancel out. We would in fact get the same trade-off if the central bank ignored future deviations of output and inflation
- Note how the slope of the MPR depends on the ratio a_l/a_π
- When a_l/a_π is large, output fluctuations are relatively more costly and the MPR curve is steeper. Therefore inflation will fluctuate relatively more compared to output
- When a_l/a_π is small, inflation fluctuations are relatively more costly and the MPR curve is more shallow. Therefore inflation will fluctuate relatively less compared to output

Limited information (cont.)

- Now we need to eliminate $\hat{y}$ using AD. We have

$$\hat{y} = \alpha_1 \hat{g} + v - \alpha_2 \left(r - \bar{r} \right).$$

$$= \alpha_1 \hat{g} + v - \alpha_2 \left(i^p - \pi^e_{t+1} + \rho - \bar{r} \right)$$

$$= \alpha_1 \hat{g} + v - \alpha_2 \left(i^p - \pi^e_{t+1} + (\rho - \bar{\rho}) - (\bar{r} - \bar{\rho}) \right)$$

$$\Rightarrow \hat{y} = \alpha_1 \hat{g} + v - \alpha_2 \left(i^p - \pi^e_{t+1} + \hat{\rho} - \bar{r}^* \right),$$

  where $\bar{r}^* \equiv (\bar{r} - \bar{\rho})$
- Since the central bank cannot see shock, it assumes $\hat{g}$, v et $\hat{\rho}$ equal their expected values of zero. With static expectations we have $\pi^e_{t+1} = \pi$, so

$$\hat{y} = -\alpha_2 \left(i^p - \pi - \bar{r}^* \right)$$

Limited information (cont.)

- Substitute this expression for $\hat{y}$ in MPR to get

$$\hat{\pi} = \left(\frac{a_I}{\alpha a_\pi}\right) \alpha_2 \left(i^p - \pi - \bar{r}^*\right)$$

$$\Rightarrow i^p = \bar{r}^* + \pi + \frac{\alpha a_\pi}{\alpha_2 a_I} \hat{\pi}$$

$$\Rightarrow i^p = \bar{r}^* + \pi + h \left(\pi - \pi^*\right)$$

where $h \equiv \frac{\alpha a_\pi}{\alpha_2 a_I}$

- The optimal rule depends **only** on inflation

Limited information (cont.)

- Insert this in AD to get

$$\hat{y} = \alpha_1 \hat{g} + v - \alpha_2 \left(i^p - \pi^* + \hat{\rho} - \bar{r}^* \right),$$

nous obtenons

$$\hat{y} = \alpha_1 \hat{g} + v - \alpha_2 \left(\bar{r}^* + \pi + h \left(\pi - \pi^* \right) \right)$$

$$+\alpha_2 \left(\pi^* + \bar{r}^* - \hat{\rho} \right)$$

$$= v + \alpha_1 \hat{g} - \alpha_2 \hat{\rho} - \alpha_2 (1 + h) \hat{\pi}$$

$$\Rightarrow \alpha_2 (1 + h) \hat{\pi} = (z - \hat{y})$$

$$\Rightarrow \hat{\pi} = \frac{1}{\alpha_2 (1 + h)} \left(z - \hat{y} \right),$$

where $z \equiv v + \alpha_1 \hat{g} - \alpha_2 \hat{\rho}$

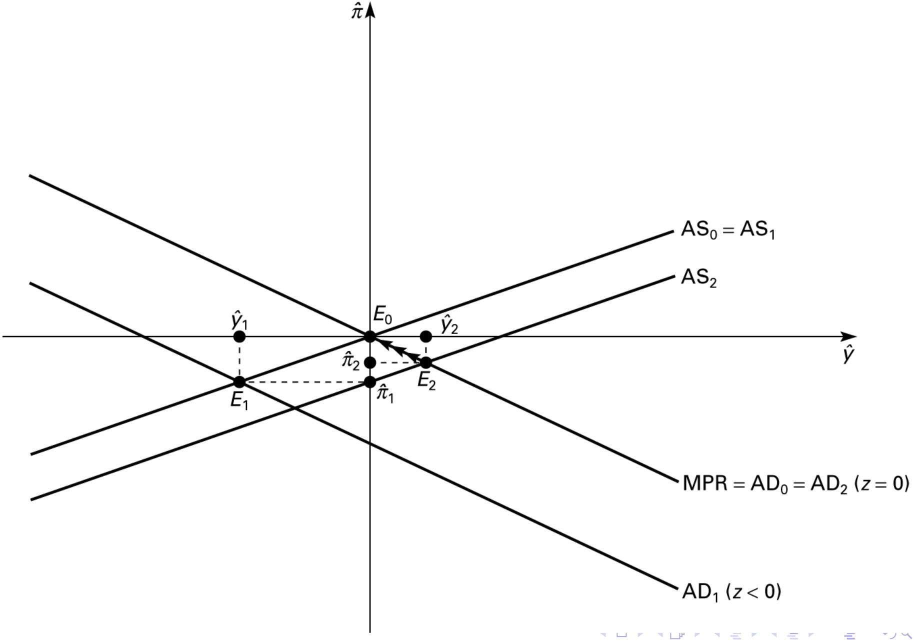
Limited information (cont.)

- This is an AD curve which embeds the optimal policy response of the central bank
- We see that when the central bank sets its interest rate based on observed inflation and output, we get a downward-sloping AD curve which depends on the demand shocks in z
- When h is high, the central bank responds strongly to changes in inflation and the AD curve is more shallow
- When the aggregate demand shock z takes its expected value of zero, the AD curve coincides with the MPR curve
- When an unanticipated demand shock hits, the AD curve will shift up or down, whereas the MPR curve will stay put
- The shift in the AD curve occurs because the central bank cannot perfectly control aggregate demand when it cannot immediately observe the current demand shocks

Limited information (cont.)

- Figure 3 below illustrates the response of the economy to a negative demand shock in period 1 under this form of inflation targeting
- Remember that the shock is temporary
- In response to the shock, the central bank decreases its interest rate to counter low inflation and low output
- When the shock goes back to zero in period 2, output actually overshoots. The output gap becomes positive and approaches its long-run equilibrium level from above as inflation gradually increases

Figure 3

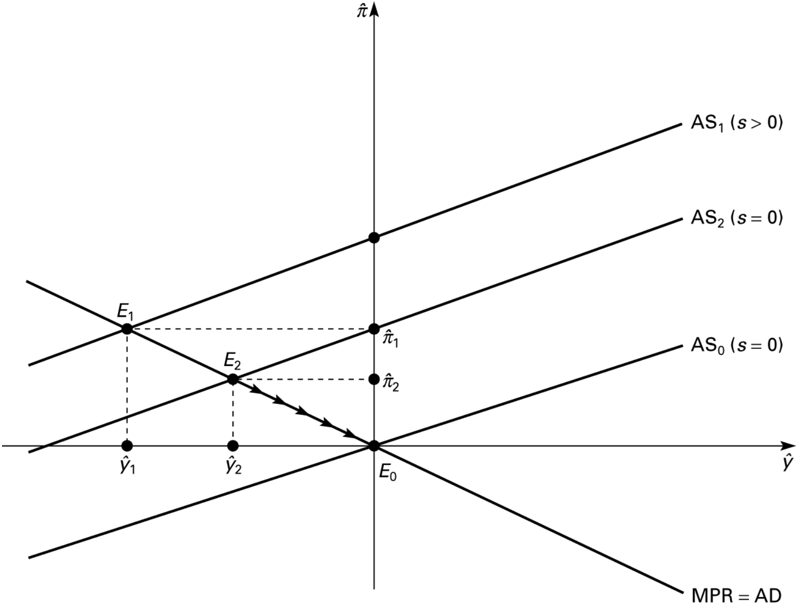


$\hat{\pi}$
$AS_0 = AS_1$
AS_2
E_0
$\hat{y}_1$
$\hat{y}_2$
$\hat{y}$
$\hat{\pi}_2$
E_2
E_1
$\hat{\pi}_1$
$MPR = AD_0 = AD_2$ $(z = 0)$
AD_1 $(z < 0)$

Limited information (cont.)

- Figure 4 below illustrates the response of the economy to a negative supply shock in period 1 under this form of inflation targeting
- The MPR curve does not shift because there is no demand shock
- When the shock goes back to zero in period 2, inflation is still above its long-run level and output is below its long-run level
- The economy approaches its long-run equilibrium from the north-west

Figure 4



$\hat{\pi}$
AS_1 $(s > 0)$
AS_2 $(s = 0)$
AS_0 $(s = 0)$
E_1
E_2
$\hat{\pi}_1$
$\hat{\pi}_2$
$\hat{y}_1$
$\hat{y}_2$
E_0
$\hat{y}$
MPR = AD

Rational expectations

- We look at the impact of monetary policy expressed as a **Taylor rule**, first when the central bank has no "informational advantage" and second when the central bank has an informational advantage over the private sector
- Expectations are rational
- The central bank must choose its policy before observing output or inflation (**no informational advantage**)
- The Taylor rule is

$$r_t = \bar{r} + h\left(\pi^e_{t,t-1} - \pi^*\right) + b\left(y^e_{t,t-1} - \bar{y}\right) \tag{9}$$

- We use the AD curve with $g = \bar{g}$,

$$y_t - \bar{y} = v_t - \alpha_2\left(r_t - \bar{r}\right) \tag{10}$$

Rational expectations (cont.)

- With rational expectations AS becomes

$$\pi_t = \pi^e_{t,t-1} + \gamma \left(y_t - \bar{y} \right) + s_t \tag{11}$$

- Expectations of shocks are zero

$$s^e_{t,t-1} = v^e_{t,t-1} = 0$$

- From (11) we have

$$\begin{aligned} \pi^e_{t,t-1} &= \pi^e_{t,t-1} + \gamma \left(y^e_{t,t-1} - \bar{y} \right) \\ &\Rightarrow \gamma \left(y^e_{t,t-1} - \bar{y} \right) = 0 \\ &\Rightarrow y^e_{t,t-1} = \bar{y}. \end{aligned} \tag{12}$$

Rational expectations (cont.)

- From (9) and (10) we have

$$\begin{aligned} y_t - \bar{y} &= v_t - \alpha_2 \left(h \left(\pi^e_{t,t-1} - \pi^* \right) + b \left(y^e_{t,t-1} - \bar{y} \right) \right) \\ \Rightarrow y^e_{t,t-1} - \bar{y} &= -\alpha_2 \left(h \left(\pi^e_{t,t-1} - \pi^* \right) + b \left(y^e_{t,t-1} - \bar{y} \right) \right) \\ \Rightarrow -\alpha_2 h \left(\pi^e_{t,t-1} - \pi^* \right) &= 0 \\ \Rightarrow \pi^e_{t,t-1} &= \pi^* \end{aligned}$$

- This means from (9) we have

$$r_t = \bar{r}. \tag{13}$$

- Independent of the values of b and h, the central bank keeps the real interest rate constant

Rational expectations (cont.)

- Then, from (10) we have

$$y_t = \bar{y} + v_t, \tag{14}$$

and from (11) and making use of the solution for y_t,

$$\pi_t = \pi^* + \gamma v_t + s_t \tag{15}$$

Remarks

1. The solutions for output and inflation do not depend on b or h
2. For this reason, monetary policy is completely ineffective
3. Inflation and output fluctuations are not persitent. We have

   $$\hat{y}_t = v_t$$

   and

   $$\hat{\pi}_t = \gamma v_t + s_t.$$

   The deviations of these variables depend only on upredictable shocks
4. We can reestablish a role for activist monetary policy by supposing that the central bank has an informational advantage
5. However, we will still not be able to explain persistence

Informational advantage model

- Now suppose the central bank observes output and inflation before setting the interest rate
- The Taylor rule is now

$$r_t = \bar{r} + h\left(\pi_t - \pi^*\right) + b\left(y_t - \bar{y}\right) \tag{16}$$

- Substitute (16) into (10) to get

$$y_t - \bar{y} = v_t - \alpha_2\left(h\left(\pi_t - \pi^*\right) + b\left(y_t - \bar{y}\right)\right)$$

$$\Rightarrow \left(1 + \alpha_2 b\right)\left(y_t - \bar{y}\right) = v_t - \alpha_2 h\left(\pi_t - \pi^*\right)$$

$$\left(y_t - \bar{y}\right) = \frac{v_t - \alpha_2 h\left(\pi_t - \pi^*\right)}{\left(1 + \alpha_2 b\right)} \tag{17}$$

Informational advantage (cont.)

- From (11) we have

$$\left(\pi_t - \pi^*\right) = \left(\pi^e_{t,t-1} - \pi^*\right) + \gamma\left(y_t - \bar{y}\right) + s_t \tag{18}$$

- Taking expectations of (18) we get

$$\left(\pi^e_{t,t-1} - \pi^*\right) = \left(\pi^e_{t,t-1} - \pi^*\right) + \gamma\left(y^e_{t,t-1} - \bar{y}\right)$$
$$\Rightarrow \gamma\left(y^e_{t,t-1} - \bar{y}\right) = 0$$
$$\Rightarrow y^e_{t,t-1} = \bar{y}$$

- Using this result in (17) and taking expectations we get

$$\left(y^e_{t,t-1} - \bar{y}\right) = \frac{-\alpha_2 h\left(\pi^e_{t,t-1} - \pi^*\right)}{\left(1 + \alpha_2 b\right)} = 0$$

$$\Rightarrow \pi^e_{t,t-1} = \pi^*. \tag{19}$$

Informational advantage (cont.)

- Using this result in (18) we get

$$(\pi_t - \pi^*) = \gamma (y_t - \bar{y}) + s_t \tag{20}$$

- Equations (17) et (20) are two equations in two unknowns $(\pi_t - \pi^*)$ et $(y_t - \bar{y})$. Plugging (17) into (20) gives

$$(\pi_t - \pi^*) = \frac{\gamma v_t - \gamma \alpha_2 h (\pi_t - \pi^*)}{(1 + \alpha_2 b)} + s_t$$

$$\Rightarrow (1 + \alpha_2 b)(\pi_t - \pi^*) = \gamma v_t - \gamma \alpha_2 h (\pi_t - \pi^*) + (1 + \alpha_2 b) s_t$$

$$\Rightarrow (1 + \alpha_2 (b + \gamma h))(\pi_t - \pi^*) = \gamma v_t + (1 + \alpha_2 b) s_t$$

$$\Rightarrow (\pi_t - \pi^*) = \frac{\gamma v_t + (1 + \alpha_2 b) s_t}{(1 + \alpha_2 (b + \gamma h))} \tag{21}$$

Informational advantage (cont.)

- Plugging this result in (17) we get

$$(y_t - \bar{y}) = \frac{v_t}{(1 + \alpha_2 b)} - \frac{\alpha_2 h}{(1 + \alpha_2 b)} \frac{\gamma v_t + (1 + \alpha_2 b) s_t}{(1 + \alpha_2 (b + \gamma h))}$$

$$\Rightarrow (y_t - \bar{y}) =$$

$$\frac{v_t}{(1 + \alpha_2 b)} - \frac{\alpha_2 h}{(1 + \alpha_2 b)} \frac{\gamma v_t}{(1 + \alpha_2 (b + \gamma h))} - \frac{\alpha_2 h s_t}{(1 + \alpha_2 (b + \gamma h))}$$

$$\Rightarrow (y_t - \bar{y}) = \frac{v_t - \alpha_2 h s_t}{(1 + \alpha_2 (b + \gamma h))} \tag{22}$$

Remarks

1. As noted, there is still no persistence
2. The solutions now depend on b and h
3. The results on optimal policy are comparable to the ones when the central bank observes only output and inflation
4. We have the following expressions for the variance of output and inflation deviations:

$$\sigma_y^2 = \frac{\sigma_v^2 + (\alpha_2 h)^2 \sigma_s^2}{(1 + \alpha_2 (b + \gamma h))^2}, \tag{23}$$

$$\sigma_\pi^2 = \frac{\gamma^2 \sigma_v^2 + (1 + \alpha_2 b)^2 \sigma_s^2}{(1 + \alpha_2 (b + \gamma h))^2} \tag{24}$$

5. These solutions depend on the assumption $\text{Cov}(v_t, s_t) = 0$

Remarks (cont.)

6. If there are only demand shocks, large values of b and h will reduce the variability of **both** output and inflation deviations. This means an absence of trade-off between output and inflation in the face of demand shocks
7. If there are only supply shocks, a large value of h will reduce the variability of inflation deviations. If the weight on inflation in the social loss function is large, this will be optimal
8. With only supply shocks, a large value of b will reduce the variability of output deviations. This is optimal if the weight on inflation fluctuations in the social loss function is small
9. The is a trade-off between stable inflation and stable output in the face of supply shocks
10. In **quantitative** terms, if we calculate the values of b et h to minimize the expected value of the social loss function, the results under rational versus static expectations will be different

Appendix A: Social loss function

- The representative household has utility function

$$U(C, L) = \ln(C) - \frac{L^{1+\mu}}{1+\mu}$$

with $\mu > 0$

- We want to account for technology shocks, so we use

$$Y = BL^{(1-\alpha)} \quad \Rightarrow \quad \hat{y} = \hat{b} + (1-\alpha)\hat{l}$$

with the usual notation that a hat indicates a log deviation from trend of the long term value

- We will use a second-order expansion of log utility:

$$\ln(C) \approx \ln(\bar{C}) + \frac{1}{\bar{C}}(C - \bar{C}) - \frac{1}{2}\frac{1}{\bar{C}^2}(C - \bar{C})^2 .$$

Social loss function (cont.)

- The FOC for the choice of hours by the household gives

$$\frac{1}{C}\frac{W}{P}(1-\tau) - L^{\mu} = 0 \Rightarrow \frac{W}{P}(1-\tau) = CL^{\mu} \equiv MRS$$

  where MRS is the marginal rate of substitution. If the household has monopoly power,

$$\frac{W}{P}(1-\tau) = m^{w} MRS$$

- The firm maximizes profit by setting the marginal product of labour MPL to a markup over the real wage:

$$\frac{W}{P} = \frac{MPL}{m^{p}} = \frac{(1-\alpha)Y/L}{m^{p}}$$

- We define

$$\frac{MPL}{MRS} = \frac{m^{p} m^{w}}{(1-\tau)} \equiv \bar{\delta}.$$

Social loss function (cont.)

- Now take a second-order expansion of the utility function:

$$U(C, L) \approx U\left(\bar{C}, \bar{L}\right) + \frac{1}{\bar{C}}\left(C - \bar{C}\right) - \bar{L}^{\mu}\left(L - \bar{L}\right)$$

$$-\frac{1}{2}\frac{1}{\bar{C}^2}\left(C - \bar{C}\right)^2 - \frac{1}{2}\mu\bar{L}^{(\mu-1)}\left(L - \bar{L}\right)^2$$

- We just need to simplify

Social loss function (cont.)

- We have

$$U(C,L) - U\left(\bar{C},\bar{L}\right) \approx \left(\frac{\left(C-\bar{C}\right)}{\bar{C}}\right) - \frac{1}{2}\left(\frac{\left(C-\bar{C}\right)}{\bar{C}}\right)^2$$

$$-\bar{L}^{(1+\mu)}\left(\frac{\left(L-\bar{L}\right)}{\bar{L}}\right) - \frac{1}{2}\mu\bar{L}^{(1+\mu)}\left(\frac{\left(L-\bar{L}\right)}{\bar{L}}\right)^2$$

$$\Rightarrow \frac{\left(U(C,L) - U\left(\bar{C},\bar{L}\right)\right)/(1/\bar{C})}{\bar{C}} \equiv \frac{\Delta/\bar{U}_C}{\bar{C}} \approx$$

$$\hat{c} - \bar{L}^{1+\mu}\left(\hat{l} + \frac{\mu}{2}\hat{l}^2\right)$$

Social loss function (cont.)

- We use

$$\hat{c} \approx \left(\frac{(C - \bar{C})}{\bar{C}} \right) - \frac{1}{2} \left(\frac{(C - \bar{C})}{\bar{C}} \right)^2,$$

$$\hat{l} \approx \left(\frac{(L - \bar{L})}{\bar{L}} \right).$$

(second-order approximation for consumption, first-order for employment)

- So

$$\frac{\Delta / \bar{U}_C}{\bar{C}} \approx \hat{c} - \bar{L}^{1+\mu} \left(\hat{l} + \frac{\mu}{2} \hat{l}^2 \right) = \hat{c} - \bar{C} \bar{L}^{\mu} \bar{L} / \bar{C} \left(\hat{l} + \frac{\mu}{2} \hat{l}^2 \right)$$

$$= \hat{c} - \frac{1}{\bar{C}} \left\{ \bar{C} \bar{L}^{\mu} \right\} \left\{ \bar{L} \right\} \left(\hat{l} + \frac{\mu}{2} \hat{l}^2 \right).$$

Social loss function (cont.)

- The household's FOC and firm's optimality conditions give

$$\bar{C}\bar{L}^{\mu} = \frac{\bar{W}}{\bar{P}}\frac{(1-\tau)}{\bar{m}^{w}},$$

$$\bar{L} = \frac{(1-\alpha)\bar{Y}}{\bar{m}^{p}\frac{\bar{W}}{\bar{P}}}$$

- So

$$\frac{\Delta/\bar{U}_C}{\bar{C}} \approx \hat{c} - \frac{1}{\bar{C}}\frac{\bar{W}}{\bar{P}}\frac{(1-\tau)}{m^{w}}\frac{(1-\alpha)\bar{Y}}{m^{p}\frac{\bar{W}}{\bar{P}}}\left(\hat{l} + \frac{\mu}{2}\hat{l}^{2}\right)$$

$$= \hat{c} - \frac{(1-\tau)}{\bar{m}^{p}\bar{m}^{w}}\frac{(1-\alpha)\bar{Y}}{\bar{C}}\left(\hat{l} + \frac{\mu}{2}\hat{l}^{2}\right) = \hat{c} - \frac{(1-\alpha)}{\bar{\delta}}\left(\hat{l} + \frac{\mu}{2}\hat{l}^{2}\right)$$

since $\bar{C} = \bar{Y}$

Social loss function (cont.)

- Now use $\hat{c} = \hat{y}$ and $\hat{l} = \frac{1}{(1-\alpha)}\left(\hat{y} - \hat{b}\right)$ to get

$$\frac{\Delta/\bar{U}_C}{\bar{C}} \approx \hat{y} - \frac{(1-\alpha)}{\bar{\delta}}\left(\frac{1}{(1-\alpha)}\left(\hat{y} - \hat{b}\right) + \frac{\mu}{2}\frac{1}{(1-\alpha)^2}\left(\hat{y} - \hat{b}\right)^2\right)$$

$$= \hat{y} - \frac{1}{\bar{\delta}}\hat{y} + \frac{1}{\bar{\delta}}\hat{b} - \frac{\mu}{2(1-\alpha)\bar{\delta}}\left(\hat{y} - \hat{b}\right)^2$$

$$= \frac{(\bar{\delta} - 1)}{\bar{\delta}}\left(\hat{y} - \hat{b}\right) + \hat{b} - \frac{\mu}{2(1-\alpha)\bar{\delta}}\left(\hat{y} - \hat{b}\right)^2$$

- Add the welfare loss from inflation to get

$$SL = -\hat{b} - a_d\left(\hat{y} - \hat{b}\right) + \frac{a_l}{2(1-\alpha)}\left(\hat{y} - \hat{b}\right)^2 + \frac{a_\pi}{2}\hat{\pi}^2 \qquad (25)$$

with $a_d \equiv \frac{(\bar{\delta}-1)}{\bar{\delta}} > 0$, $a_l > 0$ and $a_\pi > 0$. We can drop the first term, which in any case is exogenous to the central bank's policies

Appendix B: AS

- We analyze labour market equilibrium and then use the production function to get output. Here, I ignore questions of aggregation across individual firms (price setters) and unions (wage setters) (see my notes offre.pdf for a more detailed analysis)
- The firm sets its price as a markup over the marginal cost of labour. Recall that the production function is

$$Y = BL^{(1-\alpha)}$$

so

$$P = m^p \frac{W}{(1-\alpha)BL^{-\alpha}} \Rightarrow \frac{W}{P} = \frac{(1-\alpha)BL^{-\alpha}}{m^p}$$

- The markup is inversely related to the elasticity of demand for individual goods, as we have seen before

AS (cont.)

- There is a union which sets the real wage as a markup over the opportunity cost (reservation wage) variable b, before observing the realized price level:

$$\frac{W}{P^e} = m^w b \Rightarrow \frac{W}{P} = \frac{P^e}{P} m^w b$$

- Equating the two expressions for the real wage gives

$$\frac{P^e}{P} m^w b = \frac{(1-\alpha)BL^{-\alpha}}{m^p}$$

$$\Rightarrow L = \left(\frac{P^e}{P} \frac{m^w m^p b}{(1-\alpha)B} \right)^{-1/\alpha}$$

AS (cont.)

- In the long run, $P^e = P$ so we have (with bars over variables indicating their long-run values)

$$\bar{L} = \left(\frac{\bar{m}^w \bar{m}^p b}{(1-\alpha)\bar{B}} \right)^{-1/\alpha}$$

- Take the ratio of short-term and long-term employment to get

$$\frac{L}{\bar{L}} = \left(\frac{P^e}{P} \frac{m^w}{\bar{m}^w} \frac{m^p}{\bar{m}^p} \frac{\bar{B}}{B} \right)^{-1/\alpha}$$

- Take logs to get

$$(l - \bar{l}) = -\frac{1}{\alpha} \left((p^e - p) + \ln\left(m^w / \bar{m}^w\right) + \ln\left(m^p / \bar{m}^p\right) - \ln\left(B / \bar{B}\right) \right)$$

AS (cont.)

- From the production function we have

$$\left(l-\bar{l}\right)=\frac{1}{(1-\alpha)}\left(y-\bar{y}\right)-\frac{1}{(1-\alpha)}\ln\left(B/\bar{B}\right)$$

- Substituting out $\left(l-\bar{l}\right)$ and simplifying gives

$$\left(y-\bar{y}\right)=\frac{(1-\alpha)}{\alpha}\left(p-p_{-1}\right)-\frac{(1-\alpha)}{\alpha}\left(p^{e}-p_{-1}\right)$$

$$-\frac{(1-\alpha)}{\alpha}\ln\left(m^{w}/\bar{m}^{w}\right)-\frac{(1-\alpha)}{\alpha}\ln\left(m^{p}/\bar{m}^{p}\right)+\frac{1}{\alpha}\ln\left(B/\bar{B}\right)$$

where we have also subtracted and added the (log of) the lagged price level

AS (cont.)

- Using $(p - p_{-1}) \equiv \hat{\pi}$ and $(p^e - p_{-1}) \equiv \hat{\pi}^e$ we get

$$\hat{\pi} = \hat{\pi}^e + \frac{\alpha}{(1-\alpha)}\hat{y} + s \tag{26}$$

where

$$s \equiv \hat{m}^w + \hat{m}^p - \frac{1}{(1-\alpha)}\hat{B}$$

- The aggregate supply shock s (notice that a positive shock according to this definition decreases $(y - \bar{y})$ relative to π) is a function of wage markup shocks, price markup shocks, and technology shocks
- This looks qualitatively like the NKPC except that inflation expectations are not fully forward-looking

Appendix C: AD

- We use a relatively ad hoc AD specification,

$$Y = D\left(Y\ ,\ T\ ,\ r\ ,\ \varepsilon\right) + G$$

- With a balanced government budget restriction $(G = T)$ we have

$$Y = D\left(Y\ ,\ G\ ,\ r\ ,\ \varepsilon\right) + G$$

- Taking a linear approximation as follows:

$$Y \approx \bar{Y} + D_Y\left(Y - \bar{Y}\right) - C_Y\left(G - \bar{G}\right) + D_r\left(r - \bar{r}\right) + D_\varepsilon\left(\varepsilon - \bar{\varepsilon}\right) + \left(G - \bar{G}\right)$$

$$\Rightarrow \left(1 - D_Y\right)\left(Y - \bar{Y}\right) = \left(1 - C_Y\right)\left(G - \bar{G}\right) + D_r\left(r - \bar{r}\right) + D_\varepsilon\left(\varepsilon - \bar{\varepsilon}\right)$$

AD (cont.)

- Here, D_Y is the marginal propensity to spend (consumption plus investment) C_Y is the marginal propensity to consume, D_r is the sensitivity of spending (consumption plus investment) and D_ϵ is the sensitivity of spending to the aggregate demand shock ϵ
- Defining $\bar{m} \equiv \frac{1}{1-D_Y}$ we get

$$\frac{\left(Y-\bar{Y}\right)}{\bar{Y}} = \bar{m}\frac{\bar{G}}{\bar{Y}}\left(1-C_Y\right)\frac{\left(G-\bar{G}\right)}{\bar{G}} + \bar{m}\frac{D_r}{\bar{Y}}\left(r-\bar{r}\right) + \bar{m}\frac{\bar{\varepsilon}D_\varepsilon}{\bar{Y}}\frac{\left(\varepsilon-\bar{\varepsilon}\right)}{\bar{\varepsilon}}$$

- With the appropriate definitions of coefficients this gives (in logs)

$$\hat{y} = \alpha_1\hat{g} - \alpha_2\hat{r} + v \tag{27}$$

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