The Basic New Keynesian Model

Steve Ambler
Université Mohammed VI Polytechnique
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Introduction

- Goals:
  1. Look at a couple of models with nominal price rigidity (Fischer and Taylor) which are simpler to solve than the New Keynesian model as a warmup to the New Keynesian model itself
  2. Develop the basic New Keynesian model
  3. Look at its solution
  4. Look at some of its predictions

Preliminaries

- We will be using the logarithmic version of the solution for the monopolistically-competitive firm's optimal relative price from Chapter 6 in Romer (equation (6.60) in the 2019 edition)
- We have

$$\frac{P^*}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}$$

$$\Rightarrow p_i^* - p = \ln\left(\frac{\eta}{\eta - 1}\right) + (\gamma - 1)y \equiv c + \phi y$$

- Using the simple model of aggregate demand ($Y = M/P$) gives

$$p_i^* = c + (1 - \phi)p + \phi m$$

- Simplify this to

$$p_t^* = \phi m_t + (1 - \phi)p_t$$

Fischer's model of predetermined prices

- Model of staggered price adjustment
- Firms set their price in t for $t+1$ and for $t+2$
- In any period, half of prices were set one period ago, the other half two periods ago
- All firms have the same information

$$p_t = \frac{1}{2}\left(p_t^1 + p_t^2\right) \tag{1}$$

- We make no assumptions for now about the process generating m_t

Fischer (cont.)

- We assume that p_t^1 is the expectation at $t-1$ of p_t^* and p_t^2 is the expectation of p_t^* at $t-2$

$$\begin{aligned} p_t^1 &= E_{t-1}\left[\phi m_t + (1-\phi)p_t\right] \\ &= \phi E_{t-1} m_t + (1-\phi)\frac{1}{2}\left(p_t^1 + p_t^2\right) \end{aligned} \tag{2}$$

- and

$$\begin{aligned} p_t^2 &= E_{t-2}\left[\phi m_t + (1-\phi)p_t\right] \\ &= \phi E_{t-2} m_t + (1-\phi)\frac{1}{2}\left(E_{t-2} p_t^1 + p_t^2\right) \end{aligned} \tag{3}$$

- We want to solve for output and prices

Fischer (cont.)

- Solve (2) for p_t^1

$$p_t^1 + \frac{1}{2}(\phi - 1)p_t^1 = \frac{1}{2}(1 + \phi)p_t^1 = \phi E_{t-1} m_t + \frac{1}{2}(1 - \phi)p_t^2$$

$$\Rightarrow p_t^1 = \frac{2\phi}{1+\phi} E_{t-1} m_t + \frac{1-\phi}{1+\phi} p_t^2 \tag{4}$$

- Take expectations at $t-2$ of both sides

$$E_{t-2} p_t^1 = \frac{2\phi}{1+\phi} E_{t-2} m_t + \frac{1-\phi}{1+\phi} p_t^2$$

Fischer (cont.)

- Substitute this into (3) and simplify

$$p_t^2 = \phi E_{t-2} m_t + (1-\phi)\frac{1}{2}\left(\frac{2\phi}{1+\phi} E_{t-2} m_t + \frac{1-\phi}{1+\phi} p_t^2 + p_t^2\right)$$

$$\Rightarrow p_t^2 = E_{t-2} m_t \tag{5}$$

Fischer (cont.)

- Substitute this into (4) and rearrange:

$$p_t^1 = \frac{2\phi}{1+\phi} E_{t-1} m_t + \frac{1-\phi}{1+\phi} E_{t-2} m_t$$

$$\Rightarrow p_t^1 = E_{t-2} m_t - \frac{1+\phi}{1+\phi} E_{t-2} m_t + \frac{2\phi}{1+\phi} E_{t-1} m_t + \frac{1-\phi}{1+\phi} E_{t-2} m_t$$

$$\Rightarrow p_t^1 = E_{t-2} m_t + \frac{2\phi}{1+\phi} \left(E_{t-1} m_t - E_{t-2} m_t \right) \qquad (6)$$

Fischer (cont.)

- Finally, substitute (5) and (6) into $p_t = \left(p_t^1 + p_t^2\right)/2$ and $y_t = m_t - p_t$:

$$p_t = \frac{1}{2}\left(E_{t-2}m_t + \frac{2\phi}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right) + E_{t-2}m_t\right)$$

$$\Rightarrow p_t = E_{t-2}m_t + \frac{\phi}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right) \qquad (7)$$

$$y_t = m_t - E_{t-2}m_t - \frac{\phi}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right)$$

$$= m_t - E_{t-1}m_t + \frac{1+\phi}{1+\phi}E_{t-1}m_t - E_{t-2}m_t - \frac{\phi}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right)$$

$$\Rightarrow y_t = \left(m_t - E_{t-1}m_t\right) + \frac{1}{1+\phi}\left(E_{t-1}m_t - E_{t-2}m_t\right) \qquad (8)$$

Consequences

- Equation (8) is the main result. Unanticipated changes in m_t are passed through one-for-one to output
- Changes in m_t that become anticipated between $t-2$ and $t-1$ also affect output because not all prices (set for time t) are flexible at time $t-1$
- ϕ is the sensitivity of price setters' relative prices to aggregate output. Smaller ϕ means greater **real rigidity**. Price setters do not allow their prices to differ much from their competitors', and the real effects of monetary shocks are large
- Output does **not** depend directly on $E_{t-2}m_t$, just indirectly on $(E_{t-1}m_t - E_{t-2}m_t)$ and $(m_t - E_{t-1}m_t)$. This means that the effects of monetary shocks are **not very persistent**

Taylor's model of price rigidity

- What happens if price-setters must set the **same** price for both periods?
- The algebra here is a bit tricky. We will be able to reduce the system to a dynamical system we can solve with phase diagrams
- The book (Chapter 7) uses an undetermined coefficients approach and lag operators to solve the model. This may be of interest to those of you going on to do more work in macroeconomics or macro-econometrics

Taylor (cont.)

- First, we simplify the money supply process to be a random walk:

$$m_t = m_{t-1} + u_t, \tag{9}$$

with u_t white noise

- Now let χ_t be the price chosen by firms able to reset their price at t (the same for two periods). We have

$$\chi_t = \frac{1}{2}\left(p^*_{it} + E_t p^*_{it+1}\right)$$

$$= \frac{1}{2}\left(\left[\phi m_t + (1-\phi)p_t\right] + \left[\phi E_t m_{t+1} + (1-\phi)E_t p_{t+1}\right]\right), \tag{10}$$

using the same optimal pricing equation as before

Taylor (cont.)

- Half of firms set prices this period, half set their price for this period last period, so

$$p_t = \frac{1}{2}\left(\chi_t + \chi_{t-1}\right)$$

- Substituting in (10) and using the assumption of a random walk in the money supply, we get

$$\chi_t = \frac{1}{2}\left(\left[\phi m_t + (1-\phi)\frac{1}{2}\left(\chi_t + \chi_{t-1}\right)\right] \right.$$

$$\left. + \left[\phi m_t + (1-\phi)\frac{1}{2}\left(E_t\chi_{t+1} + \chi_t\right)\right] \right)$$

$$\Rightarrow \chi_t = \phi m_t + \frac{1}{4}(1-\phi)\left[\chi_{t-1} + 2\chi_t + E_t\chi_{t+1}\right] \qquad (11)$$

Taylor (cont.)

- Equation (11) is a second-order expectational difference equation. We can write it in matrix form as

$$\begin{bmatrix} \chi_t \\ E_t \chi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2\frac{1+\phi}{1-\phi} \end{bmatrix} \begin{bmatrix} \chi_{t-1} \\ \chi_t \end{bmatrix} + \begin{bmatrix} 0 \\ -4\frac{\phi}{1-\phi} \end{bmatrix} m_t. \qquad (12)$$

- The first equation is just an identity
- Here, looking at the vector on the right hand side, χ_{t-1} is a state variable (it is **predetermined**) and χ_t is a non-predetermined variable which depends, among other things, on the conditional expectation (at time t) of p^*_{it+1} and can react to new information

Taylor (cont.)

- Subtract the vector

$$\begin{bmatrix} \chi_{t-1} \\ \chi_t \end{bmatrix}$$

from both sides to get

$$\begin{bmatrix} \chi_t - \chi_{t-1} \\ E_t \chi_{t+1} - \chi_t \end{bmatrix} =$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 2\frac{1+\phi}{1-\phi} - 1 \end{bmatrix} \begin{bmatrix} \chi_{t-1} \\ \chi_t \end{bmatrix} + \begin{bmatrix} 0 \\ -4\frac{\phi}{1-\phi} \end{bmatrix} m_t. \qquad (13)$$

Taylor (cont.)

- In the long run, with $\chi_{t-1} = \chi_t = \chi_{t+1}$ equation (11) gives

$$\chi = \phi m + \frac{1}{4}(1 - \phi)\left[\chi + 2\chi + \chi\right]$$

$$\Rightarrow \chi = m$$

- This is not surprising. Money is neutral in the long run

Taylor (cont.)

- We need saddlepoint stability
- The determinant of the 2×2 matrix in equation (13) is equal to

$$1 - 2\frac{1+\phi}{1-\phi} + 1 = -4\frac{\phi}{1-\phi} < 0$$

- The determinant is the **product** of the characteristic roots of the system, so it has one positive and one negative root
- The phase diagram of the system is given in Figure 1 below
- The long-run equilibrium of the system must lie on the 45-degree line, with a constant price level χ

Taylor (cont.)

- Equation (13) shows that when $\chi_t = \chi_{t-1}$, then

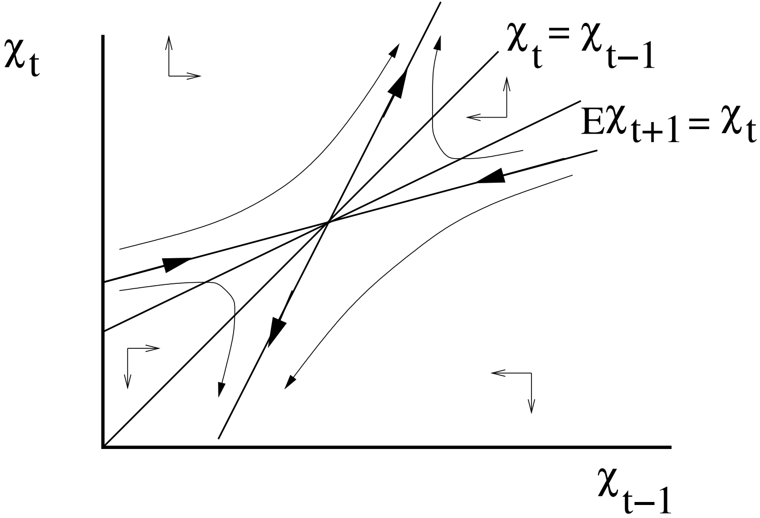
$$\frac{\partial \chi_t}{\partial \chi_{t-1}} = 1$$

and when $E_t \chi_{t+1} = \chi_t$, then

$$\frac{\partial \chi_t}{\partial \chi_{t-1}} = \frac{1}{2\frac{1+\phi}{1-\phi} - 1} < 1$$

- These two loci are illustrated on Figure 1
- Equation (13) shows that the rate of change of χ_{t-1} depends negatively on its level. This gives the horizontal arrows on Figure 1
- Equation (13) also shows that the rate of change of χ_t depends positively on its level. This gives the vertical arrows on Figure 1

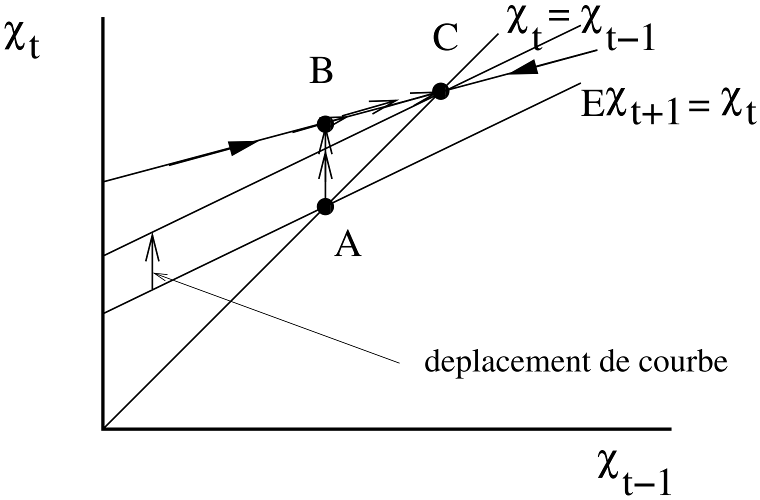
Figure 1: Gradual price adjustment



Taylor (cont.)

- Figure 2 below shows how the economy reacts to a permanent increase in m
- We know the long-run equilibrium involves a proportionate increase in χ (monetary neutrality)
- The price level immediately jumps up to the stable arm of the saddle (from A to B) as those firms which **can** adjust their prices in t do so
- Because firms don't want to set prices too far away from those of their competitors, the adjustment is partial
- Then, there is a gradual adjustment as the economy moves towards its new long-run equilibrium

Figure 2: Positive and permanent monetary shock



Consequences

- With firms setting a **constant** price for two periods, the impact of monetary shocks is much more persistent
- The effect on output is easy to derive. We have $y_t = m_t - \chi_t$, so if m_t increases by one unit, χ_t initially responds less and the difference is an increase in output
- This gradually dissipates as the price level χ_t adjusts towards its higher long-run level

The basic New Keynesian model

- Why the New Keynesian model?
  1. The model allows us to model price rigidity by changing one parameter (the probability a firm can adjust its price) instead of varying the number of equations
  2. The model is solved for the rate of inflation and the output gap, two variables which are the focus of central banks
  3. The nominal interest rate is the central bank's policy instrument, which corresponds to IT (inflation-targeting) regimes
- The development here will be a bit different from the one in Romer
- Like the model of price rigidity of Chapter 6, it will be based on monopolistically competitive firms
- There will be one or two changes in notation which I will explain as we go

- The goal is to derive the following two-equation dynamic system:

$$\pi_t = \beta E_t \pi_{t+1} + \varphi y_t, \tag{14}$$

$$y_t = E_t y_{t+1} - \frac{1}{1+r}\left(i_t - E_t \pi_{t+1}\right), \tag{15}$$

where

1. π_t is the difference between inflation and its steady-state value (typically the **inflation target**);
2. y_t is the output gap;
3. β is the household's subjective discount rate;
4. φ is a parameter which depends on structural parameters as shown below;
5. i_t is the short-run nominal interest rate (which we will later assume to be the policy instrument of the central bank)

New Keynesian (cont.)

- The model is quite simple
- Equation (14) is the **New Keynesian Phillips curve** and will be the equivalent of an aggregate supply curve. It will depend on the price-setting behaviour of firms
- Equation (15) is the **New Keynesian IS** equation. It will depend on the consumption decisions of households and is essentially a linearised Euler equation for consumption
- There is no explicit money supply. The nominal interest rate i_t is the central bank's monetary policy instrument. We could close the model with an interest rate rule such as the Taylor rule or suppose that i_t is chosen optimally to maximize a social welfare function

Intermediate goods demand

- We use exactly the same formulation as in the previous chapter on the microfoundations of nominal rigidities (with a slight change in notation to make cutting and pasting from older notes more easy)
- We have

$$c_t(z) = c_t \left(\frac{P_t(z)}{P_t} \right)^{-\mu} \tag{16}$$

- The demand for an individual good $c_t(z)$ depends on aggregate consumption c_t and on its relative price $(P_t(z)/P_t)$
  The exact price index is given by

$$P_t = \left(\int_0^1 P_t(z)^{1-\mu} dz \right)^{1/(1-\mu)} \tag{17}$$

Monopolistically competitive firm

- As before, the firm's optimal price is a markup over marginal cost:

$$\frac{P_t(z)}{P_t} = \frac{\mu}{(\mu - 1)} \psi_t(z). \tag{18}$$

- We use a more general formulation of marginal cost which allows for possibly multiple inputs. The firm's marginal cost is denoted by $\psi_t(z)$

Calvo contracts

- We use a seemingly unusual formulation of nominal rigidity first used by Calvo (1983) which says that a firm keeps its price fixed until it is allowed to do so **stochastically**. Each period it has a fixed probability of being able to reoptimize its price
- This main reason for this is that it allows (as we will see) a particularly simple way to **aggregate** across firms
- Simplifying the FOCs of the firm turns out to be somewhat long and tedious, but the reward is a very simple equation for its pricing decision

Firm's dynamic problem

- The firm z which chooses a price at time t now has an infinite horizon since there is a probability κ^i that the price it chooses will still be effective at time $(t+i)$. The firm will give the same value to its future profits as its shareholder (the representative household), which we will denote by λ_{t+i}, and using the same discount rate β^i
- $\beta^i \lambda_{t+i}$ is known as the **stochastic discount rate**, which we have (briefly) seen before
- Its maximization problem can be written

$$\max_{P_t(z)} E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \lambda_{t+i} q_{t+i}(z)$$

where $q_{t+i}(z)$ are the firm's profits in $t+i$

Firm's problem (cont.)

- Let's look at the partial derivative of each, $\frac{\partial q_{t+i}(z)}{\partial P_t(z)}$, before applying the stochastic discount factor and expectations operator We have

$$q_{t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}} c_{t+i}(z) - \phi\left(c_{t+i}(z)\right) \right]$$

$$\frac{\partial q_{t+i}(z)}{\partial P_t(z)} = \frac{1}{P_{t+i}} c_{t+i}(z) + \frac{P_t(z)}{P_{t+i}} \frac{\partial c_{t+i}(z)}{\partial P_t(z)} - \frac{\partial \phi\left(c_{t+i}(z)\right)}{\partial c_{t+i}(z)} \frac{\partial c_{t+i}(z)}{\partial P_t(z)} \tag{19}$$

Firm's problem (cont.)

- We need to simplify (19) before proceeding. Start by writing the marginal cost of the firm as

$$\frac{\partial \phi\left(c_{t+i}(z)\right)}{\partial c_{t+i}(z)} \equiv \psi_{t+i}(z)$$

- We can write

$$\frac{\partial q_{t+i}(z)}{\partial P_t(z)} = \frac{1}{P_{t+i}} c_{t+i}(z) + \frac{1}{P_{t+i}} c_{t+i}(z) \left(\frac{\partial c_{t+i}(z)}{\partial P_t(z)} \frac{P_t(z)}{c_{t+i}(z)} \right)$$

$$-\psi_{t+i}(z) \left(\frac{\partial c_{t+i}(z)}{\partial P_t(z)} \frac{P_t(z)}{c_{t+i}(z)} \right) \frac{c_{t+i}(z)}{P_t(z)}$$

$$= \frac{1}{P_{t+i}} c_{t+i}(z) \left(1 - \mu\right) + \mu \psi_{t+i}(z) \frac{c_{t+i}(z)}{P_t(z)}$$

Firm's problem (cont.)

- Given this we can write the FOC of the firm as

$$E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \lambda_{t+i} \left(\frac{1}{P_{t+i}} c_{t+i}(z) \left(1 - \mu\right) + \mu \psi_{t+i}(z) \frac{c_{t+i}(z)}{P_t(z)} \right)$$

$$= 0 \tag{20}$$

Firm's problem (cont.)

- First substitute out $c_{t+i}(z)$ using the conditional demand equations:

$$E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \lambda_{t+i} \left((1-\mu) \frac{c_{t+i}}{P_{t+i}} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\mu} + \mu \psi_{t+i} \frac{1}{P_t^*} c_{t+i} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\mu} \right)$$
$$= 0$$

where P_t^* the optimal choice of P_t by the firm

- Now assume separable log utility so $\lambda_{t+i} = \frac{1}{c_{t+i}}$:

$$E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \left(\frac{1}{P_{t+i}} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\mu} + \frac{\mu}{(1-\mu)} \psi_{t+i} \frac{1}{P_t^*} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\mu} \right) = 0$$

$$\Rightarrow E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \left(\left(\frac{P_t^*}{P_{t+i}} \right)^{1-\mu} + \frac{\mu}{(1-\mu)} \psi_{t+i} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\mu} \right) = 0 \quad (21)$$

Firm's problem (cont.)

- Now we need a first-order Taylor expansion. As usual, drop the zero-order terms which cancel out anyway
- Also, as usual, linearise in levels and then divide by steady-state levels to express everything in terms of proportional deviations from steady state

Firm's problem (cont.)

- Let's consider a single term in the summation dated $t+i$

$$\left(\frac{P_t^*}{P_{t+i}}\right)^{1-\mu} + \frac{\mu}{(1-\mu)}\psi_{t+i}\left(\frac{P_t^*}{P_{t+i}}\right)^{-\mu}$$

$$\approx (1-\mu)\left(\frac{P^*}{P}\right)^{-\mu}\left(\frac{1}{P}\left(P_t^* - P^*\right) - \frac{P^*}{P^2}\left(P_{t+i} - P\right)\right)$$

$$-\frac{\mu}{1-\mu}\psi\mu\left(\frac{P^*}{P}\right)^{-(1+\mu)}\left(\frac{1}{P}\left(P_t^* - P^*\right) - \frac{P^*}{P^2}\left(P_{t+i} - P\right)\right)$$

$$+\frac{\mu}{1-\mu}\left(\frac{P^*}{P}\right)^{-\mu}\left(\psi_{t+i} - \psi\right)$$

Firm's problem (cont.)

- We can simplify this taking into account the following restrictions
  1. If inflation is zero in the long run, $P^* = P$
  2. Without shocks, in the long run, we have $1 = \frac{\mu}{\mu - 1}\psi$
  3. Continuing with the algebra ...

Firm's problem (cont.)

$$
\begin{aligned}
&= (1-\mu)\left(\frac{(P_t^* - P)}{P} - \frac{(P_{t+i} - P)}{P}\right) \\
&+\mu\left(\frac{(P_t^* - P)}{P} - \frac{(P_{t+i} - P)}{P}\right) + \frac{\mu}{\mu - 1}\left(\psi_{t+i} - \psi\right) \\
&= \left(\frac{(P_t^* - P)}{P} - \frac{(P_{t+i} - P)}{P}\right) - \frac{(\psi_{t+i} - \psi)}{\psi} \\
&= \left(\frac{(P_t^* - P)}{P} - \frac{(P_t - P)}{P}\right) - \left(\frac{(P_{t+i} - P)}{P} - \frac{(P_t - P)}{P}\right) \\
&\quad - \frac{(\psi_{t+i} - \psi)}{\psi} \\
&= (\hat{p}_t^* - \hat{p}_t) - (\hat{p}_{t+i} - \hat{p}_t) - \hat{\psi}_{t+i}
\end{aligned}
$$

Firm's problem (cont.)

- with hats on lower-case variables denoting proportional deviations from steady state
- Plugging back into equation (21) gives

$$E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \left((\hat{p}_t^* - \hat{p}_t) - (\hat{p}_{t+i} - \hat{p}_t) - \hat{\psi}_{t+i} \right) = 0$$

$$\Rightarrow E_t \sum_{i=0}^{\infty} (\beta\kappa)^i (\hat{p}_t^* - \hat{p}_t) = E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \left((\hat{p}_{t+i} - \hat{p}_t) + \hat{\psi}_{t+i} \right)$$

$$\Rightarrow (\hat{p}_t^* - \hat{p}_t) = (1 - \beta\kappa) E_t \sum_{i=0}^{\infty} (\beta\kappa)^i \left((\hat{p}_{t+i} - \hat{p}_t) + \hat{\psi}_{t+i} \right) \tag{22}$$

- The equation says that the optimal relative price is a weighted average of future prices (relative to the current price) plus deviations of real marginal cost

- We are almost home. We still need to eliminate $\hat{p}_t^*$ from equation (22) and express the infinite sum as a first-order difference equation
- First rewrite the exact price index (exactly the same as in the previous chapter but with the appropriate change in notation):

$$P_t = \left(\int_0^1 P_t(z)^{1-\mu} dz \right)^{1/(1-\mu)}$$

$$\Rightarrow {P_t}^{(1-\mu)} = (1-\kappa){P_t^*}^{(1-\mu)} + \kappa(1-\kappa){P_{t-1}^*}^{(1-\mu)} + \kappa^2(1-\kappa){P_{t-2}^*}^{(1-\mu)} + \ldots$$

- Take the first lag of this and multiply by κ to get

$$\kappa {P_{t-1}}^{(1-\mu)} = \kappa(1-\kappa){P_{t-1}^*}^{(1-\mu)} + \kappa^2(1-\kappa){P_{t-2}^*}^{(1-\mu)} + \ldots .$$

- Subtract from the previous equation to get

$$ {P_t}^{(1-\mu)} - \kappa {P_{t-1}}^{(1-\mu)} = (1-\kappa){P_t^*}^{(1-\mu)}$$

Firm's problem (cont.)

- This is a nonlinear first-order difference equation. We could just plug it into *DYNARE* and let the program deal with it. Alternatively, we can linearise:

$$
\begin{aligned}
(1-\mu)P^{-\mu}(P_t - P) = \\
(1-\kappa)(1-\mu)P^{-\mu}(P_t^* - P) + \kappa(1-\mu)P^{-\mu}(P_{t-1} - P) \\
\Rightarrow \frac{(P_t - P)}{P} = (1-\kappa)\frac{(P_t^* - P)}{P} + \kappa\frac{(P_{t-1} - P)}{P} \\
\Rightarrow \hat{p}_t = (1-\kappa)\hat{p}_t^* + \kappa\hat{p}_{t-1},
\end{aligned}
\tag{23}
$$

- Use

$$
\hat{p}_t^* = \frac{1}{(1-\kappa)}\hat{p}_t - \frac{\kappa}{(1-\kappa)}\hat{p}_{t-1}
$$

- Substituting,

$$\frac{1}{(1-\kappa)}\hat{p}_t - \frac{\kappa}{(1-\kappa)}\hat{p}_{t-1} = (1-\beta\kappa)E_t\sum_{i=0}^{\infty}(\beta\kappa)^i\left(\hat{p}_{t+i}+\hat{\psi}_{t+i}\right)$$

$$= (1-\beta\kappa)E_t\left(\hat{p}_t+\hat{\psi}_t+(\beta\kappa)\left(\hat{p}_{t+1}+\hat{\psi}_{t+1}\right)+\dots\right).$$

- Lead this by one period, multiply by $(\beta\kappa)$ and apply the conditional expectations operator E_t to get

$$\frac{\beta\kappa}{(1-\kappa)}E_t\hat{p}_{t+1} - \frac{\beta\kappa^2}{(1-\kappa)}\hat{p}_t$$

$$= (1-\beta\kappa)E_t\left((\beta\kappa)\left(\hat{p}_{t+1}+\hat{\psi}_{t+1}\right)+(\beta\kappa)^2\left(\hat{p}_{t+2}+\hat{\psi}_{t+2}\right)+\dots\right)$$

Firm's problem (cont.)

- Subtract this equation from the preceding one to get

$$\frac{1}{(1-\kappa)}\hat{p}_t - \frac{\kappa}{(1-\kappa)}\hat{p}_{t-1} - \frac{\beta\kappa}{(1-\kappa)}E_t\hat{p}_{t+1} + \frac{\beta\kappa^2}{(1-\kappa)}\hat{p}_t$$

$$= (1-\beta\kappa)\left(\hat{p}_t + \hat{\psi}_t\right)$$

$$\Rightarrow \hat{p}_t - \kappa\hat{p}_{t-1} - \beta\kappa E_t\hat{p}_{t+1} + \beta\kappa^2\hat{p}_t = (1-\kappa)(1-\beta\kappa)\left(\hat{p}_t + \hat{\psi}_t\right)$$

$$\Rightarrow \kappa\left(\hat{p}_t - \hat{p}_{t-1}\right) = \beta\kappa E_t\left(\hat{p}_{t+1} - \hat{p}_t\right) + (1-\kappa)(1-\beta\kappa)\hat{\psi}_t$$

$$+ \left(1 - \beta\kappa - \kappa + \beta\kappa^2 - \beta\kappa^2 - 1 + \kappa\right)\hat{p}_t$$

$$\Rightarrow \left(\hat{p}_t - \hat{p}_{t-1}\right) = \beta E_t\left(\hat{p}_{t+1} - \hat{p}_t\right) + \frac{(1-\kappa)(1-\beta\kappa)}{\kappa}\hat{\psi}_t$$

or

$$\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\kappa)(1-\beta\kappa)}{\kappa}\hat{\psi}_t$$

- This is **almost** of the same form as equation (14) except that we have real marginal cost on the right-hand side instead of the output gap
- Let ρ be the elasticity of marginal cost with respect to output, so we have

$$\hat{\psi}_t = \rho y_t \tag{24}$$

Marginal cost and output gap

- We need to make some assumptions about the production and utility functions to analyze this link.
- As a byproduct, we will discover the important role of **price dispersion**
- For production,

$$Y_t(z) = A_t N_t(z)^{\alpha},$$

- We want to find the **aggregate** production function so we can calculate the **average marginal cost** in the economy. We have

$$N_t = \int_0^1 N_t(z)dz = \int_0^1 \left(\frac{Y_t(z)}{A_t}\right)^{1/\alpha} dz$$

$$= \left(\frac{Y_t}{A_t}\right)^{1/\alpha} \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\mu/\alpha} dz$$

$$\Rightarrow \log(N_t) = \frac{1}{\alpha}\log(Y_t) - \frac{1}{\alpha}\log(A_t) + \log\left(\int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\mu/\alpha} dz\right)$$

Marginal cost and output gap (cont.)

- which leads to

$$\Rightarrow \log(N_t) = \frac{1}{\alpha}\log(Y_t) - \frac{1}{\alpha}\log(A_t) + \log(D_t) \qquad (25)$$

where

$$D_t \equiv \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\mu/\alpha} dz.$$

- D_t measures **dispersion** across different firms' prices. We can show:
  1. $D_t \geq 1$. See Galí (2008, Annexe 3.3)
  2. In symmetriic equilibirum w/o price rigidity, $D_t = 1$. This is immediate because $P_t(z) = P_t \quad \forall z$. With price rigidity and outside the steady state, $D_t > 1$
  3. Up to a first-order approximation, D_t is constant

- Transforming back to levels gives

$$Y_t = A_t {N_t}^{\alpha} / {D_t}^{\alpha}$$

- This is to be compared with firms' individual production functions. D_t is a "wedge" which reduces output (and therefore consumption) for a given level of inputs
- D_t plays an important role in New Keynesian macro. It is an additional distortion arising from nominal rigidities (in addition to the distortion from monopoly power)
- It is the principal cost of inflation in New Keynesian models (Ambler 2008)

Marginal cost and output gap (cont.)

- Now let's derive an expression for aggregate marginal cost. Think of minimizing the cost of producing Y_t as if one firm produced aggregate output. The minimization problem is

$$\min_{N_t} \left(w_t N_t + MC_t \left(Y_t - A_t {N_t}^{\alpha} / {D_t}^{\alpha} \right) \right),$$

where w_t is the real wage. Taking D_t as given, the CPO is

$$w_t = MC_t \alpha \frac{Y_t}{N_t}.$$

- Take logs of both sides and simplify to get

$$\ln(MC_t) = \ln(w_t) - (\ln(Y_t) - \ln(N_t)) - \log(\alpha). \tag{26}$$

Marginal cost and output (cont.)

- Now use the labour supply curve to express the real wage as a function of employment (and therefore indirectly as a function of output). Suppose utility is

$$U = E_t \sum_{i=0}^{\infty} \beta^i \left(\log(C_{t+i} - \frac{\gamma}{(1+\psi)} N_{t+i}{}^{(1+\psi)} \right)$$

- Without an explicit budget constraint, we know the FOC for the choice of hours (employment) has the form

$$\lambda_t w_t = \gamma N_t{}^{\psi}$$

- Knowing that in equilibrium $c_t = Y_t = 1/\lambda_t$ we have in logs

$$\ln(w_t) = \ln(\gamma) + \psi \ln(N_t) + \ln(Y_t) \tag{27}$$

Marginal cost and output (cont.)

- Now substitute this expression for the real wage in equation (27) in equation (26):

$$\ln(MC_t) = (\ln(\gamma) + \psi \ln(N_t) + \ln(Y_t)) - \ln(Y_t) + \ln(N_t) - \ln(\alpha)$$

$$\Rightarrow \ln(MC_t) = (\psi + 1)\left(\frac{1}{\alpha}\ln(Y_t) - \frac{1}{\alpha}\ln(A_t) + \ln(D_t)\right) + \ln(\gamma) - \ln(\alpha)$$

$$\Rightarrow \ln(MC_t) = \frac{\psi + 1}{\alpha}\ln(Y_t) - \frac{\psi + 1}{\alpha}\ln(A_t) + \ln(\gamma) + (\psi + 1)\ln(D_t) - \ln(\alpha) \tag{28}$$

Marginal cost and output (cont.)

- The last three terms are constant ($\ln(D_t)$ is constant up to a first-order approximation). This equation holds independently of nominal rigidities. We can use this equation to find the "natural" level of output, since without rigidities we have

$$MC_t = \frac{\mu - 1}{\mu}$$

- Note that the value of MC_t w/o price rigidities is its value in the steady state. Using Y_t^n for this natural level,

$$\ln\left(\frac{\mu - 1}{\mu}\right) =$$

$$\frac{\psi + 1}{\alpha}\log(Y_t^n) - \frac{\psi + 1}{\alpha}\log(A_t) + \log(\gamma) + (\psi + 1)\log(D_t) - \log(\alpha)$$

Marginal cost and output (cont.)

- Subtracting this from equation (28) gives

$$\log(MC_t) - \log\left(\frac{\mu - 1}{\mu}\right) = \frac{\psi + 1}{\alpha}\left(\log(Y_t) - \log(Y_t^n)\right)$$

which we write as

$$\hat{\psi}_t = \frac{\psi + 1}{\alpha} y_t.$$

- So, the ρ of equation (24) is equal to $\frac{\psi+1}{\alpha}$. Finally, we get the following Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(\psi + 1)}{\alpha} \frac{(1 - \kappa)(1 - \beta\kappa)}{\kappa} y_t$$

so

$$\varphi = \frac{(\psi + 1)}{\alpha} \frac{(1 - \kappa)(1 - \beta\kappa)}{\kappa}.$$

Special cases

1. When $\kappa \to 0$, all firms adjust their prices each period and we have a special case of the model without nominal rigidity. Inflation adjusts immediately to eliminate the output gap
2. When $\psi \to \infty$, inflation is very sensitive to the output gap. Labour supply elasticity is very small. Like Section 6.5 in Romer. With a negative demand shock, employment must fall, the real wage must decrease a lot, marginal costs decrease a lot, and firms which can lower prices a lot. The dynamics are similar to the case where $\kappa \to 0$
3. When $\kappa \to 1$, firms adjust their prices rarely. Inflation is insensitive to the output gap. In the limit

$$\pi_t = \beta E_t \pi_{t+1}$$

   Because $\beta \approx 1$, inflation behaves almost like a random walk and is very persistent
4. When $\psi = 0$ we have the indivisible labour model of Hansen (1985) which we saw briefly in the RBC section. The elasticity of marginal cost to the output gap is $1/\alpha$

Discussion

- Even without extreme cases, for plausible values of κ and ψ inflation is not very persistent. To explain persistence, researchers have explored several different specifications
  1. Introduce persistence direcly by supposing firms adjust inflation partly based on lagged inflation. Let g be the fraction of such firms. The Phillips curve becomes

     $$\pi_t = g\pi_{t-1} + (1-g)\left(\beta E_t \pi_{t+1} + \varphi y_t\right)$$

     This has had some empirical success, but is somewhat ad hoc
  2. We can get almost the same thing by supposing that firms partially index their prices to inflation even when they don't reoptimize. See Christiano, Eichenbaum and Evans (2007)

Discussion (cont.)

- One can break the link between labour supply and output
  1. Introduce wage rigidity, with households who sell specialized labour to firms and set wages in advance. See Huang et Liu (2002)
  2. Different assumptions about the working of the labour market such as efficiency wages.
  3. Another possibility is search and matching models, where workers and potential employers have to find each other and set wages through wage bargaining. The volatility of real wages in such models is very sensitive to assumptions: See Blanchard et Galí (2007) for a discussion
- Question the very idea of inflation persistence. Benati (2008) shows that since 1991 (the beginning of the inflation-targeting era), there is very little inflation persistence

Romer's derivation

- Romer's derivation of the NKPC is actually quite succinct, if we accept several shortcuts. Here we reproduce the derivation in Section 7.4 (fifth edition)
- Start off with the following definition of the price level:

$$p_t = \alpha x_t + (1 - \alpha) p_{t-1} \tag{29}$$

This is the equivalent of equation (23) with $\alpha \equiv (1 - \kappa)$ and $x_t = \hat{p}_t^*$. This cuts through the complicated linearization we went through to get to (23)

- Subtract p_{t-1} from both sides to get

$$\pi_t = \alpha \left(x_t - p_{t-1} \right) \tag{30}$$

Romer's derivation (cont.)

- We have XX

New Keynesian IS curve

- The IS curve is given by equation (15), which follows directly from the Euler equation for a household which hold a one-period bond with a certain rate of return. The Euler equation is

$$\lambda_t = \beta E_t \left(\lambda_{t+1}(1 + r_{t+1}) \right),$$

where λ_t is the marginal utility of consumption. With log utility and a closed economy, with no investment of government, we have $c_t = Y_t = 1/\lambda_t$, so

$$\frac{1}{Y_t} = \beta E_t \left(\frac{1}{Y_{t+1}}(1 + r_{t+1}) \right).$$

New Keynesian IS curve (cont.)

- Calculating a first-order approximation gives

$$-\frac{1}{Y^2}\left(Y_t - Y\right) = -\beta(1+r)\frac{1}{Y^2}E_t\left(Y_{t+1} - Y\right) + \beta\frac{1}{Y}E_t\left(r_{t+1} - r\right)$$

$$\Rightarrow \hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{(1+r)}E_t\left(r_{t+1} - r\right)$$

where $\hat{y}_t$ is the gap between output and its steady-state value. This holds for equilibrium output with price rigidities and also for natural output

New Keynesian IS curve (cont.)

- In the latter case we have

$$\Rightarrow \hat{y}_t^n = E_t \hat{y}_{t+1}^n - \frac{1}{(1+r)} E_t \left(r_{t+1}^n - r \right)$$

  where r_t^n is the natural interest rate, compatible with equilibrium without nominal rigidities.
- Subtracting this from the previous equation,

$$y_t = E_t y_{t+1} - \frac{1}{(1+r)} E_t \left(r_{t+1} - r_{t+1}^n \right)$$

$$\Rightarrow y_t = E_t y_{t+1} - \frac{1}{(1+r)} \left(i_t - E_t \pi_{t+1} - E_t r_{t+1}^n \right) \qquad (31)$$

  where y_t is the output gap $(\hat{y}_t - \hat{y}_t^n)$
- This equation is slightly different from equation (15) because of the presence of the natural interest rate.
- The last term is relevant if we want to analyze the impact of technology shocks. Otherwise we can drop it

Closing the model: monetary policy

- We have two equations, (14) and either (15) or (31). There are three unknowns: y_t, π_t et i_t. If we use the IS curve with r_t^n, we also have to model the flexible-price equilibrium.
- We need another equation to complete the model, one for the interest rate
- There are several possibilities

Monetary policy (cont.)

1. Could we treat i_t as exogenous? No, because of the "Taylor principle." If the interest rate does not react to inflation, there can be a problem of dynamic stability. **Any** price level can be compatible with equilibrium in this case
2. We could suppose that i_t is chosen optimally by the central bank to maximize welfare. We will look at this in the chapter on monetary policy
3. We could add a reaction function for i_t which fits the data. In fact we often use a version of the "Taylor rule" which takes the form

$$i_t = i + \rho_\pi \pi_t + \rho_y y_t$$

with $\rho_\pi > 0$ and $\rho_y > 0$. In fact for stability we need $\rho_\pi > 1$ to satisfy the "Taylor principle"

Monetary policy (cont.)

- The intuition for this restriction is that, in order to counter an increase in inflation due to a positive demand shock (the argument for a negative shock is symmetric), the central bank must increase the **real** interest rate in order to reduce aggregate demand, increase the output gap, and reduce pressure on inflation
- For this to happen, the **nominal** interest rate must increase by more than inflation
- Technically, we need this to get the right number of stable and unstable roots to the two-equation dynamic system.
- For the exact conditions, see Galí (2008)
- Without this, we can get **sunspot equilibria**

Monetary policy (cont.)

- If we treat i_t as exogenous, we can write the system as

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{1+r} & 1 \end{bmatrix} \begin{bmatrix} E_t\pi_{t+1} - \pi_t \\ E_t y_{t+1} - y_t \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1-\beta}{\beta} & -\frac{\varphi}{\beta} \\ -\frac{1}{1+r} & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{1+r} \end{bmatrix} i_t \tag{32}$$

- The determinant of the matrix on the left hand side is positive, and the determinant of the matrix on the right had side is negative, so the determinant of the dynamic system is negative, with one positive and one negative root
- We need two **positive** roots for saddlepoint stability with two non-predetermined variables
- Se we indeed have a potential problem of multiple and/or sunspot equilibria

Monetary policy (cont.)

- If we replace i_t with the Taylor rule and ignore i (we are only interested in stability) we get

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{1+r} & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} - \pi_t \\ E_t y_{t+1} - y_t \end{bmatrix} = \begin{bmatrix} \frac{1-\beta}{\beta} & -\frac{\varphi}{\beta} \\ \frac{\rho_\pi - 1}{1+r} & \frac{\rho_y}{1+r} \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} \tag{33}$$

- The determinant of the matrix on the right hand side is now

$$\left(\frac{1-\beta}{\beta}\right)\left(\frac{\rho_y}{1+r}\right) + \left(\frac{\rho_\pi - 1}{1+r}\right)\left(\frac{\rho_y}{1+r}\right)$$

- $\rho_\pi > 1$ is a **sufficient condition** for this to be positive, compatible with two positive (unstable) roots

Monetary policy (cont.)

- The Taylor rule has several advantages from a modelling standpoint
  1. It is compatible with inflation-targeting (IT) regimes with a target value for inflation and where the central bank's policy instrumetn is the short-term nominal interest rate
  2. It gives a reasonable empirical fit for several industrialized countries (good statistical fit, good out-of-sample predictive power)
  3. In more elaborate New Keynesian models and with an appropriate choice of ρ_π and ρ_y, it yields welfare levels comparable to optimal monetary policies

Shocks

- The model contains no shocks except possibly a technology shock which would affect the natural interest rate. How could we add shocks?
  1. Add an error term to the Phillips curve. This can be justified if the elasticity of substitution μ is time varying. See Steinsson (2003). This would be a "cost push" shock or "supply shock". Such shocks can be important for analyzing optimal monetary policy. For a detailed discussion of "cost push" shocks, see Clarida, Galí and Gertler (1999)
  2. Add an error term to the Taylor rule. This can be justified as a "non-systematic" component, perhaps exogenous shocks affecting the financial/banking sector
  3. Add and error term to the IS curve. This could represent governments spending shocks or shocks to the marginal utility of consumption

Persistence

- To summarize, we have equations (14) and either (15) or (31) plus something like a Taylor rule. Romer (5th edition) adds a shock u_t^{IS} to the NKIS equation, a shock u_t^{π} to the NKPC and a shock u_t^{MP} to the Taylor rule
- Romer then specifies AR(1) processes for the three shocks
- Note that, once we subsitute out i_t using the Taylor rule, both y_t and π_t are forward-looking (non-predetermined) variables
- If we rule out sunspot equilibria ($\rho_\pi > 1$ in the Taylor rule)
- This means that the only source of persistence in the model is the shocks themselves

Persistence (cont.)

- Several modifications of the basic framework have attempted to rectify this
- As noted above, Christiano, Eichenbaum and Evans (2005, reference in Romer) introduce **indexation**, whereby even firms which do not re-optimize change prices based on **past** inflation
- Mankiw and Reis (2002, reference in Romer) go back to Fischer by assuming that when firms optimize they set a (non-constant) path for prices, so past beliefs about what inflation would be affect price changes

Persistence (cont.)

- An alternative way of introducing persistence would be to assume that monetary policy reacts with a lag to inflation and the output gap, say

$$i_t = i + \rho_\pi \pi_{t-1} + \rho_y y_{t-1}$$

- Combine this with equations (14) and (15) to get a two-equation system

$$\pi_t = \beta E_t \pi_{t+1} + \varphi y_t \tag{34}$$

$$y_t = E_t y_{t+1} - \frac{1}{1+r}\left(i + \rho_\pi \pi_{t-1} + \rho_y y_{t-1}\right) + \frac{1}{1+r} E_t \pi_{t+1} \tag{35}$$

- Since we are only interested in stability we can drop i from the system

Persistence (cont.)

- Rewrite to get

$$E_t \pi_{t+1} = \frac{1}{\beta} \pi_t - \frac{\varphi}{\beta} y_t \tag{36}$$

$$E_t y_{t+1} + \frac{1}{1+r} E_t \pi_{t+1} = y_t + \frac{\rho_\pi}{1+r} \pi_{t-1} + \frac{\rho_y}{1+r} y_{t-1} \tag{37}$$

- By introducing lagged output and lagged inflation we have created **second order** difference equations
- However we can write the system down as a system of four **first order** difference equations using the appropriate identities

Persistence (cont.)

- We have

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{1+r} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ e_t y_{t+1} \\ \pi_t \\ y_t \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\beta} & -\frac{\varphi}{\beta} & 0 & 0 \\ 0 & 1 & \frac{\rho_\pi}{1+r} & \frac{\rho_y}{1+r} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \pi_{t-1} \\ y_{t-1} \end{bmatrix} \tag{38}$$

$$\Rightarrow \begin{bmatrix} E_t \pi_{t+1} \\ e_t y_{t+1} \\ \pi_t \\ y_t \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{1+r} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\beta} & -\frac{\varphi}{\beta} & 0 & 0 \\ 0 & 1 & \frac{\rho_\pi}{1+r} & \frac{\rho_y}{1+r} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \pi_{t-1} \\ y_{t-1} \end{bmatrix} \quad (39)$$

- The last two equations are the identities
- We now have two predetermined variables on the right hand side and two non-predetermined variables

Persistence (cont.)

- For saddlepoint stability we require two roots less than one in absolute value and two greater than one in absolute value
- The following ".m" file (MATLAB or Octave) calculates the eigenvalues and their absolute values. Parameters values are from Calvert Jump and Levine (2019)

```
beta = 0.99; rreal = 0.02; rhopi = 1.5; rhoy = 0.125;
varphi = 0.1;
aa = [ 1 0 0 0; 1/(1+rreal) 1 0 0; 0 0 1 0; 0 0 0 1]
bb = [1/beta -varphi/beta 0 0; 0 1 rhopi/(1+rreal)
rhoy/(1+rreal); 1 0 0 0; 0 1 0 0]
cc = inv(aa)*bb;
dd = eig(cc); disp(dd);
ee = abs(dd); disp(ee);
```

Persistence (cont.)

- The following are the eigenvalues and their absolute values

eigenvalue	abs(eig)
-0.2013	0.2013
0.0000	0.0000
1.1552 + 0.1347i	1.1631
1.1552 - 0.1347i	1.1631

- We have saddlepoint stability
- The **largest** stable root gives an idea of the rate of return towards long run equilibrium. Its value, 0.2013, is not that large, an indication that there is not much persistence

Taylor principle not satisfied

- Let's rerun the same script but with $\rho_\pi = 0.5$, so the Taylor principle is not satisfied
- The following are the eigenvalues and their absolute values

eigenvalue	abs(eig)
1.3673	1.3673
0.8850	0.8850
-0.1432	0.1432
0.0000	0.0000

- As we would predict, saddlepoint stability is not satisfied
- There are one too many stable roots, opening up the possibility of multiple dynamic equilibria and sunspot equilibria

Basic Model with Shock Persistence

- Romer (Section 7.8) looks at the basic New Keynesian model and adds shocks with persistence
- The equation system can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \varphi y_t + u_t^{\pi}$$

$$y_t = E_t y_{t+1} - \frac{1}{1+r}\left(i_t - E_t \pi_{t+1}\right) + u_t^{US}$$

$$i_t = i + \rho_{\pi} \pi_t + \rho_y y_t + u_t^{MP}$$

$$u_t^{\pi} = \alpha_{\pi} u_{t-1}^{\pi} + e_t^{\pi}$$

$$u_t^{IS} = \alpha_{IS} u_{t-1}^{IS} + e_t^{IS}$$

$$u_t^{MP} = \alpha_{MP} u_{t-1}^{MP} + e_t^{MP}$$

Model with Shock Persistence (cont.)

- Once we substitute out i_t using the Taylor rule, we have a system of five first-order difference equations

$$\pi_t = \beta E_t \pi_{t+1} + \varphi y_t + u_t^{\pi}$$

$$y_t = E_t y_{t+1} - \frac{1}{1+r}\left(i + \rho_\pi \pi_t + \rho_y y_t + u_t^{MP} - E_t \pi_{t+1}\right) + u_t^{US}$$

$$u_t^{\pi} = \alpha_\pi u_{t-1}^{\pi} + e_t^{\pi}$$

$$u_t^{IS} = \alpha_{IS} u_{t-1}^{IS} + e_t^{IS}$$

$$u_t^{MP} = \alpha_{MP} u_{t-1}^{MP} + e_t^{MP}$$

- We rewrite this in matrix form on the following slide

Model with Shock Persistence (cont.)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{1+r} & 1 & 0 & 0 & -\frac{1}{1+r} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ e_t y_{t+1} \\ u_t^{\pi} \\ u_t^{IS} \\ u_t^{MP} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{\beta} & -\frac{\varphi}{\beta} & -\frac{1}{\beta} & 0 & 0 \\ \frac{\rho_\pi}{1+r} & \frac{1+r+\rho_y}{1+r} & 0 & -1 & 0 \\ 0 & 0 & \alpha_\pi & 0 & 0 \\ 0 & 0 & 0 & \alpha_{IS} & 0 \\ 0 & 0 & 0 & 0 & \alpha_{MP} \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ u_{t-1}^{\pi} \\ u_{t-1}^{IS} \\ u_{t-1}^{MP} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ e_t^{\pi} \\ e_t^{IS} \\ e_t^{MP} \end{bmatrix}$$

Model with Shock Persistence (cont.)

- Consider the same parameter values we used above to look at persistence in the model with lagged inflation and the lagged output gap in the Taylor rule
- The table below gives the absolute values of the eigenvalues as the persistence parameters are increased from 0.3 to 0.6 to 0.9

Persistence	0.3	0.6	0.9
abs(eig)	1.1324	1.1324	1.1324
	1.1324	1.1324	1.1324
	0.3000	0.6000	0.9000
	0.3000	0.6000	0.9000
	0.3000	0.6000	0.9000

- We have saddlepoint stability. The eigenvalues of the unstable roots are not **at all** affected by changes in the shock persistence parameters
- **All** of the persistence in the model is coming from the persistence of the shocks

The evolution of macro models

1. Traditional Keynesian (IS-LM) model
   - Ad hoc IS curve, LM curve which depends on equilibrium in the money market, the monetary base is the central bank's policy instrument, prices and the price level are **fixed**, graphical analysis in i/Y space
2. The traditional AD/AS framework
   - Ad hoc IS curve, graphical analysis in P/Y space, downward-sloping AD curve which most often depends on a **real-balance** or **wealth** effect, money stock is still the central bank's policy instrument, monetary policy is integrated into AD, upward-sloping AS curve where an increase in P means that some firms benefit from a higher relative price or a lower real wage (if there is nominal price rigidity)

Evolution of macro models (cont.)

3. The Romer/Taylor model
    - The interest rate is the instrument of the central bank, which changes the real interest rate in response to changes in inflation, $\Uparrow \pi \Rightarrow \Uparrow r \Rightarrow \Downarrow \mathrm{AD}$, AS is derived from price-setting firms and depends on inflation (similar to the New Keynesian model)
4. The New Keynesian model
    - We have analyzed its components in detail. It gives dynamic relationships between inflation, expected inflation, the output gap, and the expected output gap. A graphical analysis is not easy (or possible) under rational expectations because it reduces to a system of difference equations.
      The best detailed presentations of the New Keynesian model are Woodford (2003) and Galí (2015)

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