ECO511 Eric Sims’ RBC Model

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1 Introduction

I want to go through the Sims model for a few reasons. [1]

1. We can solve analytically for the steady state.
2. We can look at how to use properties of the steady state to calibrate the model or examine how changes in the parameter values affect the steady state.
3. We can calculate a linearized version by hand and do a qualitative analysis using phase diagrams. The linearized dynamic conditions will tell us the slopes of “isoclines” (curves along which either consumption is stationary or the capital stock is stationary), and what is the direction of change of these variables on either side of these isoclines.

The model is stationary to begin with (no trend in technological progress and no population growth), so he is following the first of the two broad strategies I described in cycleres.pdf. “Build a model of the cyclical components of macro time series. Generate predictions, most likely by numerical simulation. The series will be stationary by construction. Use a standard method to remove trends from the data series. Many methods will work … Compare the predictions with the equivalent statistics in the data.”

1. I’m writing this in the form of notes rather than slides since it’s more for reading than for in-class presentation.

2 Model Specification

Sims runs through a specification where firms own the capita stock and one where the representative agent owns capital and rents it to firms. He then shows that the two are equivalent. Showing this is useful, but I will look only at the second specification.

There is no population growth and technology is stationary, so there is no growth. The aggregate production function is given by

$$Y_t = A_t F\left(K_t, N_t\right) \tag{1}$$

with technology A_t following and AR(1) process in logs :

$$\ln\left(A_t\right) = \rho \ln\left(A_{t+1}\right) + \varepsilon_t \tag{2}$$

The representative household has the following utility function :

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t\right) + \nu\left(1 - N_t\right)\right). \tag{3}$$

The household's budget constraint is given by

$$C_t + K_{t+1} - (1-\delta)K_t + B_{t+1} - B_t = w_t N_t + R_t K_t + r_t B_t + \Pi_t. \tag{4}$$

The household's income is in the form of wages $w_t N_t$, income from renting capital to firms $R_t K_t$, interest payments on bond holdings $r_t B_t$ and dividend payments from firms' profits Π_t. The household consumes, invests, and can change its holdings of bonds (from its point of view).

Since there is a representative agent and there is no government (and we assume that firms do not borrow by issuing debt), B_t must actually be zero in equilibrium. Everyone in the economy will want either to borrow or lend, and there will be nobody on the other side of the market. From the point of view of the individual he/she can lend or borrow, and leaving this as part of the individual's problem will allow us to evaluate the riskless real interest rate r_t.

Similarly, we will be assuming a constant returns to scale (CRS) production function and perfect competition, so Euler's theorem implies that payments to factors of production (labour and capital) will be exactly equal to output, so profits will be zero in every period. I will henceforth assume $\Pi_t = 0$.

2.1 Household

The household's problem can be written as

$$max_{\lambda_t, C_t, N_t, K_{t+1}, B_{t+1}} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t\right) + \varphi\left(1 - N_t\right) \right)$$

$$+\lambda_t \left[w_t N_t + R_t K_t + r_t B_t - C_t - K_{t+1} + (1-\delta) K_t - B_{t+1} + B_t \right] \quad (5)$$

The FOCs give

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow u'\left(C_t\right) = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow \nu'\left(1 - N_t\right) = \lambda_t w_t \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Rightarrow \lambda_t = \beta E_0 \left(\lambda_{t+1} \left(R_{t+1} + (1-\delta) \right) \right) \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \Rightarrow \lambda_t = \beta E_0 \left(\lambda_{t+1} \left(1 + r_{t+1} \right) \right) \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 \Rightarrow \text{Budget constraint holds} \quad (10)$$

2.2 Firm

Sims has a long discussion of the appropriate **discount rate** for the firm. If the household is the owner of the firm, it will want to weight future profits by marginal utility. So he introduces a so-called "stochastic discount rate." However, if the firm just rents capital and hires workers each period and has no profits to retain or invest, then its problem is static. In any case the stochastic discount rate will collapse to one. We can just write down the FOCs for the firm's optimal choice of labour and capital :

$$w_t = A_t F_n \left(K_t, N_t \right) \quad (11)$$

$$R_t = A_t F_k \left(K_t, N_t \right) \quad (12)$$

In the usual way, the real wage equals the marginal product of labour, and the rental rate of capital equals the marginal product of capital.

2.3 Equation system

We want to write down a complete system of equations which must hold in G.E. (general equilibrium).

First, substitute out λ_t using the FOC w.r.t. C_t :

$$\nu'(1-N_t) = u'(C_t)\, w_t$$

Now substitute out λ_t from the FOCs w.r.t. K_{t+1} and B_{t+1} :

$$u'(C_t) = \beta E_0 \left(u'(C_{t+1})\left(R_{t+1} + (1-\delta)\right)\right)$$

$$u'(C_t) = \beta E_0 \left(u'(C_{t+1})\left(1 + r_{t+1}\right)\right)$$

Essentially, we are trying to eliminate some of the variables of the model by subsitution.

We can then write down the following equation system.

$$u'(C_t) = \beta E_0 \left(u'(C_{t+1})\left(A_t F_k(K_t, N_t) + (1-\delta)\right)\right) \tag{13}$$

$$\nu'(1-N_t) = u'(C_t)\, A_t F_N(K_t, N_t) \tag{14}$$

$$K_{t+1} = A_t F(K_t, N_t) - C_t + (1-\delta)K_t \tag{15}$$

$$\ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t \tag{16}$$

$$Y_t = C_t + I_t \Rightarrow I_t = Y_t - C_t \tag{17}$$

$$u'(C_t) = \beta E_t \left(u'(C_{t+1})\left(1 + r_{t+1}\right)\right) \tag{18}$$

$$w_t = A_t F_N(K_t, N_t) \tag{19}$$

$$R_b = A_t F_K(K_t, N_t) \tag{20}$$

This gives nine equations for the following nine unknowns :

1. the predetermined state variables K_t and A_t;
2. the non-predetermined or forward-looking dynamic variable C_t;
3. N_t , r_t, Y_t, I_t, w_t, R_t.

We know which variables are **state** and **dynamic** variables because they appear in the equation system with different dates : t and $(t+1)$. This means they define an explicit or implicit difference equation. There are six static variables which only appear with one date. Of the six, we can see from equations (16), (19), and

(20) that Y_t, w_t and R_t are determined **recursively**. They only appear on the left-hand side of those three equations. If we have solved for the other variables of the system, we can then compute their values. Also, equation (17) shows that I_t is semi-recursive. If we solve for the other variables of the system we can compute Y_t and then we can compute I_t.

Finally, equation (18) allows us to determine the real interest rate in the economy. From equations (13) and (18) we see that the expected rates of return on investment and on increasing bond holdings have to be equal. But we know from our reasoning that B_t does not appear in the equation system because it must be equal to zero in equilibrium.

If we get rid of the variables which are determined recursively and of r_t, we are left with a four-equation system to determine the following unkowns : the dynamic variables K_t, C_t, A_t; and N_t, which is determined by the static FOC given by equation (14).

When we get to looking at analyzing the dynamics using a phase diagram, we will consider A_t to be exogenous. Otherwise we would need a three-dimensional phase diagram (beyond my abilities to trace freehand or to program).

2.4 Functional forms

Sims then imposes functional forms on the utility function and the production function :

$$u\left(C_t\right) = \ln\left(C_t\right)$$

$$\nu\left(1-N_t\right) = \theta \ln\left(1-N_t\right)$$

$$Y_t = A_t {K_t}^{\alpha} {N_t}^{(1-\alpha)}$$

We are now ready to start talking about the steady state.

3 Steady State

We impose constant values of the variables in the model. We use asterisks to denote these values. In particular, $A_t = A_{t-1} = A^*$. From equation (16) it's clear that the long-run value of the log of A must be zero, so we have $A^* = 1$. We also impose $K_{t+1} = K_t = K^*$ and $C_{t+1} = C_t = C^*$

We can now find the long-run value of capital to employment ratio, which we will then use to solve for a bunch of other stuff. From equation (13) we get

$$1 = \beta\left(\alpha\left(\frac{K^*}{N^*}\right)^{(\alpha-1)} + (1-\delta)\right)$$

$$\Rightarrow \frac{1}{\beta} - (1-\delta) = \alpha\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}$$

$$\Rightarrow \left(\frac{K^*}{N^*}\right) = \left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta)}\right)$$

The strategy will be to try to simplify as many of the other conditions as possible by expressing them in terms of the capital to employment ratio.

We immediately have

$$w^* = (1-\alpha)\left(\frac{K^*}{N^*}\right)^{\alpha}$$

$$R^* = \alpha\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}$$

We also have from the FOCs for B_{t+1} and K_{t+1} that

$$1 = \beta\left(R^* + (1-\delta)\right) = \beta(1+r^*) \Rightarrow R* = r* + \delta$$

Equations (15) and (17) together imply

$$I^* = \delta K^*$$

From the production function and the accounting identity (17) we have

$$K^{*\alpha} N^{*(1-\alpha)} = C^* + I^*$$

$$\Rightarrow \left(\frac{K^*}{N^*}\right)^{\alpha} N^* = C^* + \delta K^*$$

$$\Rightarrow \frac{C^*}{N^*} = \left(\frac{K^*}{N^*}\right)^{\alpha} - \delta\left(\frac{K^*}{N^*}\right)$$

We can get a second expression for C^*/N^* using equation (14) (the static condition for labour supply) and then equate them. We have

$$\frac{\theta}{1-N^*} = \frac{1}{C^*}(1-\alpha)\left(\frac{K^*}{N^*}\right)^{\alpha}$$

$$\Rightarrow \frac{C^*}{N^*} = \frac{1-N^*}{N^*}\frac{1-\alpha}{\theta}\left(\frac{K^*}{N^*}\right)^{\alpha}$$

Equating the two expressions for C^*/N^* gives

$$\frac{1-N^*}{N^*}\frac{1-\alpha}{\alpha}\left(\frac{K^*}{N^*}\right)^{\alpha} = \left(\frac{K^*}{N^*}\right)^{\alpha} - \delta\left(\frac{K^*}{N^*}\right)$$

Multiplying through by N^* gives

$$\Rightarrow \frac{1-\alpha}{\alpha}\left(\frac{K^*}{N^*}\right)^{\alpha} - N^*\frac{1-\alpha}{\alpha}\left(\frac{K^*}{N^*}\right)^{\alpha} = N^*\left(\frac{K^*}{N^*}\right)^{\alpha} - N^*\delta\left(\frac{K^*}{N^*}\right)$$

Solving for N^* gives

$$N^* = \frac{\frac{1-\alpha}{\theta}\left(\frac{K^*}{N^*}\right)^{\alpha}}{\frac{\theta+1-\alpha}{\theta} - \delta\left(\frac{K^*}{N^*}\right)} \tag{21}$$

This is pretty messy, but we've managed to find employment as a function of the capital to employment ratio and parameters. The rest of the steady state solutions are now pretty easy. For I^* we have

$$I^* = \delta K^* = \delta\left(\frac{K^*}{N^*}\right)N^* \tag{22}$$

For Y^* we have

$$Y^* = K^{*\alpha}N^{(1-\alpha)} = \left(\frac{K^*}{N^*}\right)^{\alpha}N^* \tag{23}$$

For C^* we have

$$C^* = Y^* - I^* = \left(\frac{K^*}{N^*}\right)^{\alpha}N^* - \delta\left(\frac{K^*}{N^*}\right)N^*$$

We are ready to talk about calibration.

4 Calibration

The underlying parameters of the model are : β, α, θ, δ. This is not a lot. It's a pretty simple model.

4.1 Calibrate β

From the long-run level of the real interest rate (see why we needed it ?), we have

$$1 = \beta (1 + r^*) \Rightarrow r^* = \frac{1}{\beta} - 1$$

With any $\beta < 1$, we have a positive real (riskless) interest rate. If we think the average (annualized) value of the real interest rate is about two percent on average, or about 0.005 for quarterly data, this implies a value of β of about 0.995

4.2 Calibrate α

As usual, we note that the share of profits (or revenue to capital) in the model is α. This is around 1/3 in the national accounts data for many countries. Enough said.

4.3 Calibrate δ

This is an example where we could choose δ based on national accounts, which would imply a value for the share of investment in output. Or we can proceed in reverse. We use

$$\frac{I^*}{N^*} = \delta \frac{K^*}{N^*}$$

and

$$\frac{Y^*}{N^*} = \left(\frac{K^*}{N^*} \right)^{\alpha}$$

This gives

$$\frac{I^*}{Y^*} = \delta \left(\frac{K^*}{N^*} \right)^{(1-\alpha)}$$

Substituting in our solution for K^*/N^* we get

$$\frac{I^*}{Y^*} = \frac{\delta \alpha}{\frac{1}{\beta} - (1 - \delta)}$$

Now, the share of investment in aggregate demand is on average about 20 percent. Sims rounds up to 22.5 percent. Using the values of α and β which we have already calibrated, we get $\delta \approx 0.02$. Given the values for α, β and δ we get a capital/labour ratio of about 35.

4.4 Calibrate θ

This leaves θ. From the capital accumulation equation in the steady state, consumption per worker is

$$\frac{C^*}{N^*} = \left(\frac{K^*}{N^*}\right)^{\alpha} - \delta\left(\frac{K^*}{N^*}\right)$$

The FOC for labor supply gives a different expression for C^*/N^* :

$$\frac{C^*}{N^*} = \frac{1-N^*}{N^*}\frac{1-\alpha}{\theta}\left(\frac{K^*}{N^*}\right)^{\alpha}$$

Equating the two gives

$$\frac{1}{\theta}\frac{1-N^*}{N^*}(1-\alpha)\left(\frac{K^*}{N^*}\right)^{\alpha} = \left(\frac{K^*}{N^*}\right)^{\alpha} - \delta\left(\frac{K^*}{N^*}\right)$$

which gives a solution for θ :

$$\theta = \frac{\frac{1-N^*}{N^*}(1-\alpha)\left(\frac{K^*}{N^*}\right)^{\alpha}}{\left(\frac{K^*}{N^*}\right)^{\alpha} - \delta\left(\frac{K^*}{N^*}\right)}$$

We have solved for K^*/N^*. If we set N^* (individuals spend about a third of their time endowment working) we get $\theta \approx 1.75$ as a solution.

4.5 Stochastic process for A_t

Sims gets a measure of total factor productivity from the data. The TFP (A_t) in the model is stationary, so Sims detrends the TFP in the data by estimating a linear time trend :

$$\ln\left(\hat{A}_t\right) = \phi_0 + \phi_1 t + u_t$$

He gets estimates of $\hat{\phi}_0 = 0.145$ and $\hat{\phi}_1 = 0.003$. This gives quarterly growth 0.3 percent, or an annualized growth rate of 1.2 percent. He then takes the residual of the equation and estimates the following regression :

$$\hat{u}_t = \rho\hat{u}_{t-1} + e_t$$

He gets a $\hat{\rho}$ of 0.974 and an estimated standard error of 0.009.

5 Phase diagram

Sims solves algebraically for the "isoclines" of the model, curves (straight lines after we linearize the model) along which either C_t is not changing of K_t is not changing.

Note that for these purposes, we take A_t to be fixed or exogenous. Otherwise we would have a three-dimensional system and would need a three-dimensional phase diagram to analyzed the dynamics.

The procedure is as follows :

1. linearize the model ;
2. calculate the isoclines ; and
3. find the directions of motion of C and K on either side of the isoclines

Sims goes through the algebra of linearization in great detail. I will summarize very briefly here.

The two main dynamic equations of the model come from :

1. the law of motion for the capital stock ; and
2. the consumption Euler equation.

To construct a phase diagram it is necessary to eliminate all endogenous variables other than C_t and K_t from the equations by substitution. This is essentially what Sims demonstrates in detail. The main variable to substitute out is N_t.

5.1 Linearized equation for N_t

He first writes the static condition for labour supply in log form. This will allow him to substitute out employment from the two main dynamic equations. We have

$$\ln(\theta) - \ln(1 - N_t) = -\ln(C_t) + \ln(1-\alpha) + \ln(A_t) + \alpha\ln(K_t) - \ln(N_t)$$

Rewriting this in terms of proportional deviations from the long run gives

$$\frac{N_t - N^*}{1 - N^*} = \frac{C_t - C^*}{C^*} + \frac{A_t - A^*}{A^*} + \alpha\frac{K_t - K^*}{K^*} - \alpha\frac{N_t - N^*}{N^*}$$

Use a more compact notation, defining for any variable X_t, $\tilde{X}_t \equiv \frac{X_t - X^*}{X^*}$:

$$\frac{N^*}{1 - N^*}\frac{N_t - N^*}{1 - N^*} \equiv \frac{N^*}{1 - N^*}\tilde{N}_t = -\tilde{C}_t + \tilde{A}_t + \alpha\tilde{K}_t - \alpha\tilde{N}_t$$

Define $N^*/\left(1-N^*\right) \equiv \gamma$ to make the notation even more compact. We have (solving for $\tilde{N}_t$)

$$\tilde{N}_t = -\frac{1}{\gamma+\alpha}\tilde{C}_t + \frac{1}{\gamma+\alpha}\tilde{A}_t + \frac{\alpha}{\gamma+\alpha}\tilde{K'}_t, \tag{24}$$

allowing us to substitute out $\tilde{N}_t$ in the Euler equation and the law of motion for capital.

5.2 Euler equation

Writing the Euler equation in logs gives

$$-\ln\left(C_t\right) = \ln(\beta) - \ln\left(C_{t+1}\right) + \ln\left(\alpha A_{t+1} {K_{t+1}}^{(\alpha-1)} {N_{t+1}}^{(1-\alpha)} + (1-\delta)\right)$$

Linearize, ignoring zero-order terms (they cancel out anyway) and making use of $\alpha K^{*(\alpha-1)} N^{*(1-\alpha)} + (1-\delta) = 1/\beta$. I leave out a couple of steps here :

$$-\tilde{C}_t = -\tilde{C}_{t+1} + \beta\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\left(\tilde{A}_{t+1} + (\alpha-1)\tilde{K}_{t+1} + (1-\alpha)\tilde{N}_{t+1}\right)$$

Then we use equation (24), forwarded by one period, to eliminate $\tilde{N}_{t+1}$. The algebra is tedious, and it leads to

$$\begin{aligned}
-\tilde{C}_t = -&\left(1+\beta(1-\alpha)\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)\tilde{C}_{t+1} \\
&+\left(\beta\alpha\frac{K^*}{N^*}\right)^{(\alpha-1)}\left(\frac{1+\gamma}{\gamma\alpha}\right)\tilde{A}_{t+1} \\
&-\left(\beta\alpha(1-\alpha)\frac{K^*}{N^*}\right)^{(\alpha-1)}\left(\frac{\gamma}{\gamma\alpha}\right)\tilde{K}_{t+1}
\end{aligned} \tag{25}$$

This is quite messy. We can set $\tilde{C}_t = \tilde{C}_{t+1}$ to calculate the "isocline," the curve (straight line for a linearized system) along which $\tilde{C}$ doesn't change. Once again, the algebra is messy, but can be simplified to yield

$$\tilde{C}_{t+1} = \left(\frac{1+\gamma}{1-\alpha}\right)\tilde{A}_{t+1} - \gamma\tilde{K}_{t+1}$$

We are doing everything in discrete time, but for very small time intervals we can replace this with an equation with variables dated t :

$$\tilde{C}_t = \left(\frac{1+\gamma}{1-\alpha}\right)\tilde{A}_t - \gamma\tilde{K}_t \tag{26}$$

This gives a negatively-sloped relationship between $\tilde{C}$ and $\tilde{K}$ along which $\tilde{C}$ is not changing. Also, if there is an exogenous increase in $\tilde{A}$, the relationship shifts up. This gives the downward-sloping curve on page 14 of Sims' paper.

We can also stare at equation (25) to figure out what happens on either side of the isocline. I'll just quote Sims here.

> "Quite intuitively, if $\tilde{K}_t$ is "too big" relative to what it would be when $\tilde{C}_{t+1} - \tilde{C}_t = 0$ — i.e. we are to the right of the $\tilde{C}_{t+1} - \tilde{C}_t = 0$ isocline — then consumption will be expected to decline over time."

This gives the downward-pointing arrows to the right of the isocline, and the upward-pointing arrows to the left.

Sims compares this isocline to what we get in the Ramsey model we looked at in the previous section of the course. This hinges on the value of γ. Recall that we defined this as $\gamma \equiv N^* / (1 - N^*)$. We can rearrange equation (26) to give

$$\tilde{K}_t = \frac{1+\gamma}{\gamma}\frac{1}{1-\alpha}\tilde{A}_t - \frac{1}{\gamma}\tilde{C}_t$$

As $\tilde{N}_t$ approaches one, γ approaches ∞. In this case, $\tilde{C}_t$ drops out of the equation and we get a value of $\tilde{K}_t$ which depends on the value of $\tilde{A}_t$. This is a vertical line, as it was in the Ramsey model.

5.3 Law of motion for capital

We now have to go through the same process for the capital accumulation equation. I will go through this even more quickly. First, take logs :

$$\ln(K_{t+1}) = \ln(A_t F(K_t, N_t) - C_t + (1-\delta)K_t)$$

Now, linearize using a Taylor expansion (and ignore the zeroth order terms because they will cancel out). Then use the compact notation and substitute out $\tilde{N}_t$ as before to get

$$\tilde{K}_{t+1} = \left(\frac{1}{\beta} + \frac{1-\gamma}{\gamma+\alpha}\alpha\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)\tilde{K}_t + \left(\frac{1+\gamma}{\gamma+\alpha}\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)\tilde{A}_t$$

$$-\left(\frac{C^*}{K^*}\frac{1-\alpha}{\gamma+\alpha}\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)\tilde{C}_t \tag{27}$$

When $\tilde{K}_{t+1} = \tilde{K}_t$ this gives a relationship (curve) between $\tilde{K}$ and $\tilde{C}$ along which the capital stock is constant. We get

$$\tilde{C}_t = \left(\frac{C^*}{K^*}\frac{1-\alpha}{\gamma+\alpha}\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)^{-1}\left[\left(\frac{1}{\beta}-1+\frac{1-\gamma}{\gamma+\alpha}\alpha\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)\tilde{K}_t\right.$$

$$\left.+\left(\frac{1+\gamma}{\gamma+\alpha}\left(\frac{K^*}{N^*}\right)^{(\alpha-1)}\right)\tilde{A}_t\right] \tag{28}$$

This is positively-sloped. It is possible to show that the slope is less than one. The argument is somewhat convoluted : see Sims' paper starting on page 13.

So we have the upward-sloping isocline on page 14. Above the isocline consumption is increasing, so savings and investment must be less and the capital stock must be decreasing. Below the isocline the capital stock must be increasing. This gives the arrows pointing to the left and right on the phase diagram.

To prove saddlepoint stability we would have to write this in matrix form with $\tilde{C}_{t+1} - \tilde{C}_t$ and $\tilde{K}_{t+1} - \tilde{K}_t$ as a 2×1 vector on the left hand side. We would show that the determinant of the matrix on the right hand side is negative, implying one negative root or eigenvector (the stable root) and one positive root or eigenvector (the unstable one). However, by staring at the arrows we can deduce the nature of the convergent arm of the saddle, which is the dotted line on the phase diagram.

5.4 Shocks

Starting on page 18, Sims looks at the effects of a permanent increase in A_t. The phase diagram is on page 19. The long-run equilibrium level of the capital stock and consumption increase, and both of the isoclines shift up and to the right. The convergent arm of the saddle shifts up. Consumption jumps up immediately to place the economy on the convergent arm, and then consumption and capital slowly increase along the path towards the new long-run equilibrium.

The time paths of consumption and the capital stock are illustrated on page 19. Sims goes on to look at what this implies for the dynamics of the real wage and employment.

6 Conclusion

We have analyzed Sims' version of the baseline RBC model.

We review how to solve the model analytically for the steady state and discuss the relationship between the steady state solution and the parameter values.

We briefly discuss how to linearize the model and analyze its dynamics using phase diagrams.

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