Welfare Benefits of Stabilization Policy

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Introduction

- We will look at the following topics:
  1. the objectives of stabilization;
  2. the welfare costs of cyclical fluctuations;
  3. the costs of unanticipated versus anticipated inflation; and
  4. the costs of fluctuations with and without distortions.

Welfare loss function

- We will work with the loss function given by

$$SL = \frac{a_y}{2}(y - \bar{y})^2 + \frac{a_\pi}{2}(\pi - \pi^*)^2., \tag{1}$$

where a_y snd a_π give the relative costs of output v inflation fluctuations.

- The expected loss is given by

$$\mathrm{E}(SL) = \frac{a_y}{2}\sigma_y^2 + \frac{a_\pi}{2}\sigma_\pi^2, \tag{2}$$

with $\sigma_y^2 \equiv \mathrm{E}\left[(y_t - \bar{y})^2\right]$, $\sigma_\pi^2 \equiv \mathrm{E}\left[(\pi_t - \pi^*)^2\right]$.

- It is possible to derive this from the utility function of a representative agent. See Galí (2008) or Woodford (2003).
- We neglect the costs of changing distributions of income/wealth.

Welfare loss function (cont.)

1. What are reasonablle values of a_y et a_π?
2. Should we stabilize y independent of the type of shock?
3. What is the appropriate inflation target?
4. Is there always a trade-off between stabilizing output and stabilizing inflaiton? Are there policies which reduce the fluctuations of both at the same time?
5. What are the properties of optimal stabilization policies?

Welfare costs of fluctuation in C and L

- Risk aversion $\Rightarrow$ fluctuations of C are costly.
- An **individual** can smooth idsoyncratic fluctuations of C through **insurance** (as long as moral hazard is not a problem).
- If aggregate C fluctuates these fluctuations are **non-diversifiable**.
- Small open economies have fluctuations in aggregate C which can be diversifiable in the context of the world economy.
- But there are potential problems.
  1. Moral hazard: if income is insured and individual may not have a strong incentive not to lose his/her job.
  2. Adverse section: it's mostly individuals who have higher risks of losing their jobs who have an incentive to insure their income.

Welfare costs of fluctuations in C et L (cont.)

- Lucas (1987) argued that the welfare costs of fluctuations were trivially small.
- He develops an analysis with a representative agent, perfect competition, and perfect competition.
- See *Wikipedia*, "Welfare Costs of Business Cycles" for more details.
- We will look in detail at the welfare costs of fluctuations with distortions.

Algebraic model

- The representative agent has the utility function given by

$$U(C, L) = \frac{C^{(1-\theta)}}{1-\theta} - \frac{L^{(1+\mu)}}{1+\mu}.$$

- θ: relative risk aversion. μ: measures how quickly the marginal disutility of work increases.
- Budget constraint:

$$C = \frac{W}{P}(1-\tau)L + I.$$

- There is no saving. I gives income other than labour income.

Algebraic model (cont.)

- Substitute $\frac{W}{P}(1-\tau)L + I$ for C.
- The household's problem is

$$\max_{L}\left\{\frac{\left(\frac{W}{P}(1-\tau)L+I\right)^{(1-\theta)}}{1-\theta}-\frac{L^{(1+\mu)}}{1+\mu}\right\}.$$

- The first order condition (W/P is exogeneous) is

$$\frac{\partial U}{\partial C}\frac{\partial C}{\partial L}+\frac{\partial U}{\partial L}=0$$

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$$\Rightarrow C^{-\theta}\frac{W}{P}(1-\tau)-L^{\mu}=0.$$

Algebraic model (cont.)

- $\frac{W}{P}(1-\tau) = \frac{L^{\mu}}{C^{-\theta}} \equiv MRS(C:L)$.
- $MRS(C:L)$ is the marginal rate of substitution between consumption and labour supply.
- $MRS(C:L) = -\frac{\partial U}{\partial L} / \frac{\partial U}{\partial C} = C^{\theta} L^{\mu}$.
- The total differential of the utility fonction d'utilité is

$$dU = 0 = \frac{\partial U}{\partial C} dC + \frac{\partial U}{\partial L} dL = C^{-\theta} dC - L^{\mu} dL$$

- $$\Rightarrow MRS(C:L) = \left(\frac{\partial C}{\partial L} | (dU = 0) \right) = -\frac{\partial U}{\partial L} / \frac{\partial U}{\partial C} = C^{\theta} L^{\mu}.$$

Algebraic model (cont.)

- Assume a small increase in the net real wage ($(W/P)(1-\tau)$) with an adjustment of I to keep C constant. From the FOC we have

$$d\left(\frac{W}{P}(1-\tau)\right)C^{-\theta} - \theta\frac{W}{P}(1-\tau)C^{-(\theta+1)}dC = \mu L^{(\mu-1)}dL.$$

- Suppose $dC = 0$. We get

$$d\left(\frac{W}{P}(1-\tau)\right)C^{-\theta} = \mu L^{(\mu-1)}dL.$$

$$\Rightarrow \frac{dL}{d\left(\frac{W}{P}(1-\tau)\right)} = \frac{C^{-\theta}}{\mu L^{(\mu-1)}} = \frac{L^{\mu}/(w(1-\tau))}{\mu L^{(\mu-1)}} = \frac{L}{w(1-\tau)\mu}$$

because $C^{-\theta} = \frac{L^{\mu}}{w(1-\tau)}$ (FOC).

Algebraic model (cont.)

- We get

$$\frac{dL}{d\left(\frac{W}{P}(1-\tau)\right)} = \frac{L}{w(1-\tau)\mu}$$

$$\Rightarrow \frac{dL/L}{d\left(w(1-\tau)\right)/\left(w(1-\tau)\right)} = \frac{1}{\mu}.$$

- $1/\mu$ is labour supply elasticity **net of the income effect.**
- See "Frisch Elasticity of Labor Supply" in Wikipedia.
- It measures the substitution effect of a change in the real wage, keeping marginal utility.
- Caclulating $MRS(C, L)$ and the elasticity of labour supply are a bit peripheral to the calculation of the welfare loss of fluctuations. Let's return to that.

Algebraic model (cont.)

- Now introduce a friction: the real wage is greater than $MRS(C : L)$:

$$\frac{W}{P} = m^w \frac{MRS}{(1-\tau)} = m^w \frac{C^\theta L^\mu}{(1-\tau)}.$$

- This is like the condition in Chapter 17. It's a markup over the opportunity cost of leisure and not b.
- We have

$$m^w = \frac{(W/P)(1-\tau)}{C^\theta L^\mu} > 1.$$

Algebraic model (cont.)

- The firm's problem:

$$\max_{L} \left\{ BL^{(1-\alpha)} - \left(\frac{W}{P}\right) L \right\}.$$

- FOC (perfect competition):

$$(1-\alpha)BL^{-\alpha} - \left(\frac{W}{P}\right) = 0 \Rightarrow (1-\alpha)BL^{-\alpha} = \left(\frac{W}{P}\right) \equiv MPL.$$

- With imperfect competition:

$$P = m^p \frac{W}{(1-\alpha)BL^{-\alpha}} \Rightarrow \frac{W}{P} = \frac{1}{m^p}(1-\alpha)BL^{-\alpha} \equiv \frac{MPL}{m^p}.$$

Algebraic model (cont.)

- Equilibrium unemployment is sub-optimal. In fact we have

$$\frac{MPL}{MRS} = \frac{m^p m^w}{(1-\tau)} \equiv \bar{\delta}.$$

- $\bar{\delta}$ measures by how much the economy diverges from the competitive equilibrium.
- Let's transform the equations to log form.

$$w - p = mpl - u^p, \quad u^p \equiv \ln\left(m^p\right)$$

$$w - p = u^w + mrs - \ln(1-\tau), \quad u^w \equiv \ln\left(m^w\right)$$

$$\Rightarrow mpl - u^p = u^w + mrs - \ln(1-\tau)$$

$$\Rightarrow mpl - mrs = u^w + u^p - \ln(1-\tau).$$

Algebraic model (cont.)

- Consider symmetrical fluctuations around this equilibrium.
- We proceed as follows:
  1. Calculate a 2nd order approximation of the utility function.
  2. Use the production function (Y as a function of L) and the assumption of zero savings ($C = Y$) to express utility loss as aa function of Y.
  3. Calcuate the expected loss.
  4. The 2nd order approximation allows us to decompose the loss into a first-order and a second-order effect.

Algebraic model (cont.)

- Here's the 2nd order expansion of an arbitrary function:

$$f(v,z) \approx f(\bar{v},\bar{z}) + \frac{\partial f(\bar{v},\bar{z})}{\partial v}(v-\bar{v}) + \frac{\partial f(\bar{v},\bar{z})}{\partial z}(z-\bar{z})$$

$$+\frac{1}{2}\frac{\partial^2 f(\bar{v},\bar{z})}{\partial v \partial v}(v-\bar{v})^2 + \frac{1}{2}\frac{\partial^2 f(\bar{v},\bar{z})}{\partial z \partial z}(z-\bar{z})^2$$

$$+\frac{\partial^2 f(\bar{v},\bar{z})}{\partial v \partial z}(v-\bar{v})(z-\bar{z}).$$

- Applied to the utility function:

$$U(C,L) \approx U(\bar{C},\bar{L}) + \bar{C}^{-\theta}\left(C-\bar{C}\right) - \bar{L}^{\mu}\left(L-\bar{L}\right)$$

$$-\frac{\theta \bar{C}^{-(\theta+1)}}{2}\left(C-\bar{C}\right)^2 - \frac{\mu \bar{L}^{(\mu-1)}}{2}\left(L-\bar{L}\right)^2,$$

since $\frac{\partial^2 U(C,L)}{\partial C \partial L} = 0$

Algebraic model (cont.)

- We can rewrite as

$$U(C,L) - U(\bar{C},\bar{L}) \approx \bar{C}^{(1-\theta)}\left(\frac{C-\bar{C}}{\bar{C}}\right) - \bar{L}^{(1+\mu)}\left(\frac{L-\bar{L}}{\bar{L}}\right)$$

$$-\frac{\theta\bar{C}^{(1-\theta)}}{2}\left(\frac{C-\bar{C}}{\bar{C}}\right)^2 - \frac{\mu\bar{L}^{(1+\mu)}}{2}\left(\frac{L-\bar{L}}{\bar{L}}\right)^2.$$

- Dividing both sides by $\bar{C}^{(1-\theta)}$:

$$\frac{U(C,L) - U(\bar{C},\bar{L})}{\bar{C}^{(1-\theta)}} \approx$$

$$\left(\frac{C-\bar{C}}{\bar{C}}\right) - \frac{\theta}{2}\left(\frac{C-\bar{C}}{\bar{C}}\right)^2$$

$$-\frac{\bar{L}^{(1+\mu)}}{C^{(1-\theta)}}\left(\frac{L-\bar{L}}{\bar{L}}\right) - \frac{\mu}{2}\frac{\bar{L}^{(1+\mu)}}{C^{(1-\theta)}}\left(\frac{L-\bar{L}}{\bar{L}}\right)^2.$$

Algebraic model (cont.)

- Interpretation: we have written the welfare loss as

$$\frac{U(C,L) - U(\bar{C},\bar{L})}{\bar{C}^{(1-\theta)}}$$

$$= \frac{\left(U(C,L) - U\left(\bar{C},\bar{L}\right)\right)/\bar{C}^{-\theta}}{\bar{C}}.$$

- The numerator gives the loss in consumption units.
- Why? $\left(U(C,L) - U\left(\bar{C},\bar{L}\right)\right)$ is in units of utility. $\bar{C}^{-\theta}$ gives units of utility per unit of consumptin. The ration is therefore units of consumptiion.
- Dividing through by $\bar{C}$, we can write the loss in units of consumption **relative** to the long-term level of consumption.

Algebraic model (cont.)

- We have

$$\hat{c} \equiv \ln(C) - \ln\left(\bar{C}\right) \approx \frac{C - \bar{C}}{\bar{C}}$$

and

$$\hat{l} \equiv \ln(L) - \ln\left(\bar{L}\right) \approx \frac{L - \bar{L}}{\bar{L}},$$

- Using this, we get:

$$\frac{\Delta/\bar{U}_C}{\bar{C}} \approx \hat{c} - \frac{\theta \hat{c}^2}{2} - \left(\frac{\bar{C}^\theta \bar{L}^{1+\mu}}{\bar{C}}\right)\left(\hat{l} + \frac{\mu \hat{l}^2}{2}\right),$$

where

$$\Delta \equiv U(C, L) - U(\bar{C}, \bar{L}).$$

Algebraic model algébrique (suite)

- All of output is consumed, so $C = Y = BL^{1-\alpha}$, et

$$\hat{c} = \hat{y} = (1-\alpha)\hat{l}.$$

- Using $\bar{C} = \bar{Y}$, $\overline{MRS} = \bar{C}^{\theta}\bar{L}^{\mu}$, and $\overline{MPL} = (1-\alpha)\bar{Y}/\bar{L}$, we have

$$\frac{\bar{C}^{\theta}\bar{L}^{1+\mu}}{\bar{C}} = \frac{\overline{MRS}\,\bar{L}}{\bar{C}} = \frac{\overline{MRS}(1-\alpha)\bar{Y}}{\overline{MPL}\,\bar{C}} = \frac{\overline{MRS}(1-\alpha)}{\overline{MPL}} = \frac{(1-\alpha)}{\bar{\delta}}.$$

- We can rewrite $\Delta\bar{U}_C/\bar{C}$ as

$$\frac{\Delta/\bar{U}_C}{\bar{C}} \approx \hat{c} - \frac{\theta\hat{c}^2}{2} - \left(\frac{1-\alpha}{\bar{\delta}}\right)\left(\hat{l} + \frac{\mu\hat{l}^2}{2}\right).$$

Algebraic model (cont.)

- Using $\hat{c} = \hat{y} = (1-\alpha)\hat{l}$, rewrite in terms of the output gap:

$$\frac{\Delta/\bar{U}_C}{\bar{C}} \approx \hat{y} - \frac{(1-\alpha)}{\bar{\delta}\,(1-\alpha)}\hat{y} - \frac{\theta}{2}\hat{y}^2 - \frac{1-\alpha}{\bar{\delta}}\frac{\mu}{2}\frac{\hat{y}^2}{(1-\alpha)^2}$$

$$= \left(\frac{\bar{\delta}-1}{\bar{\delta}}\right)\hat{y} - \left(\theta + \frac{\mu}{\bar{\delta}(1-\alpha)}\right)\frac{\hat{y}^2}{2}.$$

- We can divide the welfare loss into:
  1. a first-order effect which depends on $\hat{y}$; and
  2. a second-order effect which depends on $\hat{y}^2$.
- The first-order effect is symmetrical around potential output. We have

$$\mathrm{E}\left[\frac{\Delta/\bar{U}_C}{\bar{C}}\right] \approx -\left(\theta + \frac{\mu}{\bar{\delta}(1-\alpha)}\right)\frac{\sigma_y^2}{2}.$$

Algebraic model (cont.)

- The expected value of the first-order effect is zero.
- The consumer is better off if Y and L are stabilized at the long-term values.
- $\frac{\theta \sigma_y^2}{2}$ gives the welfare loss from fluctuations in consumption.
- $\left(\frac{\mu}{\bar{\delta}(1-\alpha)}\right)\frac{\sigma_y^2}{2}$ give the welfare loss from fluctuations in employment.

Algebraic model (cont.)

- Galí, Gertler and López-Salido (2007) calculate the gains during expansions and the losses during recessions for the US economy.
- Their utility function is slightly more general.

Algebraic model (cont.)

Table 1: Benefits of expansions and costs of recessions

Benefits and costs in % of annual consumption					
	Period		Expansion	Recession	Net
Start	Peak	End			
68: 2	70: 2	72: 3	+6.50	-9.40	-2.90
72: 4	74: 4	77: 3	+7.30	-22.20	-14.90
77: 4	80: 2	83: 4	+8.70	-16.80	-8.10
87: 4	90: 4	94: 1	+9.30	-14.80	-5.50

The appropriate output target

- We have seen (graphically and algebraically) that on average output is sub-optimal.
- Why not try to raise output systematically?
- The OA curve with static expecctations can be written as

$$\pi_t = \pi_{t-1} + \gamma\left(y_t - \bar{y}\right) + s_t$$

- Any effort to do this comes at the cost of an **accelerating** inflation rate.

Impact of demand shocks

- Our analysis has been based on markup fluctuations.
- With demand shocks, it is the case that $\pi \neq \pi^e$.
- Therefore the *ex post* markup is not equal to the *ex ante* markup.

The advantages of stable inflation

- What are the consequences of unanticipated changes in π?
  1. Unanticipated changes in the real rate of return on capital and/or the real wage rate.
  2. Unanticipated redistribution of wealth between debtors and creditors.
  3. A larger dispersion of relative prices between firms who set their price for several periods, but not at the same time. This is not part of our model. (See Ambler 2008.)
- The indexation of all contracts could prevent these effects.
- However: we observe very little indexation of contracts in the real world.

The appropriate inflation target

- Costs of inflation even when it is perfectly anticipated.
  1. *Shoe leather costs*. More frequent trips to the bank to economize on holdings of cash.
  2. *Menu costs*. Firms must adjust their prices more frequently.
  3. *Relative price distortions*. Firms don't adjust their prices at the same time. Firms which haven't adjusted their price for several periods will have lower *relative* prices. (See Ambler 2008.)
  4. *Fiscal impact*. There will be a wedge between real rates of return and nominal rates of return. To the extent that tax rates apply to nominal rates of return, we could see higher effective tax rates simply because of inflation.

The appropriate inflation target (cont.)

- This would suggest a low (or zreo) inflation target.
- However, with a low or zero target the central bank could be faced more frequently with a policy interest rate at its **zero lower bound**.
- This could hamper its ability to react to negative shocks.
- For example, this was the case in Japan during the 1990s and much of the 2000s.
- This was the case of several countries between the 2008 financial crisis and the pandemic.
- Nominal wages may be downwardly sticky. (Workers do not easily accept wage cuts.) This idea is controversial.

Important concepts

- In general, central banks try to minimize loss functions which depend on fluctuations in inflation and the output gap.
- With perfect competition fluctuations are not very costly.
- With imperfect competition, the welfare costs of fluctuations are greater (cf. Galí, Gertler et López-Salido).
- With imperfect competition, the natural level of output (and its long-run level) are **too low**.
- This gives the central bank an incentive to try to raise the average level of output by accepting a higher inflation rate. This is not possible in the long run.
- Inflation fluctuations (both expected and unexpected) are costly.
- It can be optimal to target a positive rate of inflation.

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