Inflation Targeting, Price-Level Targeting and the Zero Lower Bound*

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Abstract

We evaluate inflation targeting and price-level targeting in a simulation model that explicitly takes into account the central bank's zero lower bound constraint on its policy rate. We find that the economy is much less likely to hit the zero bound under price-level targeting for a given rate of inflation. Trend inflation is optimally positive under both regimes because of the zero lower bound, but is lower under price-level targeting, which delivers an enhanced level of economic welfare. Monetary policy is modeled using simple rules. We model the lower bound constraint with a smooth function that can approximate a kink.

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1 Introduction

Previous studies have shown that price-level targeting (henceforth PT) can lead to improved macroeconomic stabilization and economic welfare compared to inflation targeting (IT).[1] PT may be particularly beneficial when short-term nominal interest rates are stuck at or near their lower bound of zero. Under IT, if the interest rate is expected to remain at the lower bound for some time, and if the economy is experiencing deflation, the ante real interest rate may be high and aggregate demand may remain depressed. Under PT, if the commitment to restore the price level to its target path is credible, expected inflation over the medium term must equal the target rate of inflation so that the ex ante real interest rate is negative, potentially giving a boost to aggregate demand and output.

We study the impact of IT versus PT in a simulation model that takes into account the zero lower bound constraint on the central bank's policy rate. We show that the economy is much less likely to hit the zero bound under PT for a given trend inflation rate. Trend inflation is optimally positive because of the zero lower bound under both IT and PT, but lower under PT. Economic welfare is significantly enhanced in a PT regime.

We solve the model using higher-order perturbation techniques[2] that preserve its essential nonlinearities in order to make valid welfare comparisons.[3] We model

[1] See Ambler (2009) for a survey.

[2] We use *Dynare* (Adjemian et al., 2014) and *Dynare*++ (Kamenik, 2011).

[3] For a detailed discussion of why linearized models can give misleading welfare comparisons, see Kim and Kim (2003). Linearized models ignore the possibility that changes in policy or in policy rules can affect the stochastic means of variables due to convexities or concavities in their response to shocks.

monetary policy using simple rules. Under IT, the central bank's desired interest rate follows a Taylor rule. Under PT, its desired interest rate follows a modified Taylor rule that depends on deviations of the price level from its target path. The actual (net) short term interest rate is the maximum of the desired rate and zero. We use a smooth function that approximates the kink in the interest rate reaction function at zero arbitrarily well.[4]

Previous studies have analyzed the impact of the zero lower bound on monetary policy in models that are linear except for the zero bound constraint itself. Examples include Adam and Billi (2006, 2007), Aksoy et al. (2006), Billi (2011), Coenen, Orphanides and Wieland (2004), Coibion, Gorodnichenko and Wieland (2012), Jung, Teranishi and Watanabe (2005), Levin, López-Salido, Nelson and Yun (2009), Kato and Nishiyama (2005), Nakov (2006) and Reifschneider and Williams (2000).[5] Exceptions to this rule are the papers of Eggertsson and Woodford (2003), Fernandez-Villaverde et al. (2012), and Wolman (2005). Eggertsson and Woodford's model is simple enough to solve analytically. Wolman's model is the closest in spirit to our own. He solves a simple New Keynesian model with two-period nominal price rigidity using projection methods. Fernandez-Villaverde et al. also use projection methods with sparse grids.[6]

[4]Kim, Kim and Kollmann (2011) use a barrier-function approach in a related context in order to approximate a decision rule with a kink.

[5]A small group of papers considers the use of the exchange rate as a monetary policy instrument when the interest rate is stuck at or near the zero bound. These models are also linear. See Coenen and Wieland (2004), McCallum (2006), and Svensson (2001). Jeanne and Svensson (2007) study the role of quantitative easing and the central bank's balance sheet at or near the lower bound. Buiter and Panigirtzoglu (2002) consider the payment of a negative rate of interest on currency.

[6]The sparse grid approach is a tractable way of avoiding the curse of dimensionality associated with projection methods, and allows for the tractable solution of models with many state variables.

The paper is organized as follows. The next section develops the model, including the monetary policy rules under IT and PT. The third section describes the calibration of the model's structural parameters. The fourth section discusses the results. Conclusions are in the fourth section.

2 Model Description

The economy consists of a representative household with an infinite planning horizon, a representative final good firm, a collection of monopolistically competitive firms that produce differentiated intermediate goods, and a monetary authority that sets the short-term nominal interest rate following a Taylor rule under IT and following a modified Taylor rule under PT. Following Adam and Billi (2006, 2007), we abstract from the demand for money, which implies that the optimal rate of trend inflation would be close to zero without taking the zero lower bound into account.[7] Amano and Shukayev (2012) study a model with real balances in the utility function. In their paper, the implied functional form of the money demand function gives an infinite demand for real balances at a zero nominal interest rate so that the nominal interest rate can approach but never reach a zero value.

[7] Price dispersion is eliminated in the steady state when trend inflation is zero. The average markup is minimized when trend inflation is slightly positive, but the quantitative effects of price dispersion on welfare are greater than variations in the average markup, so that the optimal trend inflation rate is positive but very close to zero. See Wolman (2001) for a detailed explanation.

2.1 Households

The representative household maximizes expected utility given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, H_t\right), \tag{1}$$

where C_t is consumption, H_t denotes hours worked, and $\beta \in (0,1)$ is a subjective discount factor. The functional form of period utility is given by

$$U\left(C_t, H_t\right) = \log\left(C_t\right) - \frac{\gamma}{(1+\chi)}\left(H_t\right)^{(1+\chi)}. \tag{2}$$

The representative household's budget constraint in period t in nominal terms is

$$P_t C_t + P_t I_t + P_t F_t + \frac{B_t}{R_t} \leq$$

$$P_t w_t H_t + P_t\left(q_t - \tau_{t-1}\right) K_t - T_t + D_t + B_{t-1}, \tag{3}$$

where P_t is the price level, w_t is the real wage, q_t is the real rental rate of capital, τ_{t-1} is an exogenous risk premium shock, T_t is a lump-sum tax, D_t denotes nominal dividend payments received from monopolistically competitive firms, I_t is real investment, K_t is the stock of capital, F_t is a capital adjustment cost (defined below), and R_t is the gross nominal interest rate on debt between t and $t+1$. It is assumed that R_t is part of the household's information set at time t, so that the nominal rate of return on bonds between t and $t+1$ is certain.

Capital is included in the model in order to give households a nontrivial asset allocation decision, and to allow for a risk premium that drives a wedge between the riskless return on bonds and the risky return to capital. Shocks to this risk premium are the main factor responsible for driving the risk-free nominal interest rate to the lower bound. For plausible parameterizations of their variance, the other shocks in the model (a monetary policy shock and an aggregate productivity shock) would almost never drive the short-term nominal interest rate to its zero lower bound. The risk premium shock follows the stochastic process given by

$$\tau_t = (1 - \rho_\tau)\,\tau + \rho_\tau \tau_{t-1} + \varepsilon_t^\tau, \tag{4}$$

where $\rho_\tau \in (0, 1)$ and $\varepsilon_t^\tau \sim N\left(0, \sigma_{\varepsilon^\tau}^2\right)$.

Investment increases the household's stock of capital according to

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{5}$$

where $\delta \in (0, 1)$ is the depreciation rate of capital. Investment is subject to convex adjustment costs of the following form:

$$F_t = \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{6}$$

where φ is a positive parameter. This equation states that adjustment costs depend on the divergence of gross investment from the level needed to replace depreciating capital. This implies that in the steady state, with a constant capital stock, aggregate adjustment costs are zero.

The first-order conditions associated with the optimal choice of C_t, B_t, H_t, and K_{t+1} are given by:

$$\frac{1}{C_t} = \lambda_t; \tag{7}$$

$$\lambda_t = \beta E_t \left(\lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right), \tag{8}$$

$$\gamma {H_t}^{\chi} = \lambda_t w_t, \tag{9}$$

$$\lambda_t \left[1 + \varphi \left(\frac{I_t}{K_t} - \delta \right) \right] = \beta \mathrm{E}_t \left\{ \lambda_{t+1} \left[1 + (q_{t+1} - \tau_t) - \delta \right. \right.$$

$$\left. \left. + \varphi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) + \frac{\varphi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\}, \tag{10}$$

where λ_t is the Lagrange multiplier associated with the period-t budget constraint expressed in real terms and π_{t+1} is the gross inflation rate between t and $t+1$, that is to say $\pi_{t+1} \equiv P_{t+1}/P_t$.

2.2 Firms

There is collection of monopolistically competitive intermediate good producers that sell their output to a competitive sector that produces final output, which is used for both consumption and investment purposes.

2.2.1 Representative final good firm

The representative competitive final good firm, uses $Y_t(l)$ units of each type of intermediate good to produce Y_t units of the final good using a constant returns to scale production function given by:

$$Y_t = \left[\int_0^1 Y_t(l)^{\frac{\theta-1}{\theta}} dl \right]^{\frac{\theta}{\theta-1}}, \tag{11}$$

where $\theta > 1$ is a parameter denoting the elasticity of substitution between types of differentiated intermediate goods. The final good firm sells its output at a nominal price P_t and chooses Y_t and $Y_t(l)$ for all $l \in [0, 1]$ to maximize its profits, given by:

$$P_t Y_t - \int_0^1 P_t(l) Y_t(l) dl, \tag{12}$$

subject to (11) in each period. The first-order conditions for this problem are the constraint and:

$$Y_t(l) = \left[\frac{P_t(l)}{P_t} \right]^{-\theta} Y_t. \tag{13}$$

Equation (13) expresses the conditional demand for intermediate good l as a decreasing function of its relative price and an increasing function of total output. The exact price index for final output is given by:

$$P_t = \left[\int_0^1 P_t(l)^{1-\theta} dl \right]^{\frac{1}{1-\theta}} \tag{14}$$

2.2.2 Intermediate goods firms

Each intermediate good firm, indexed by l, uses $K_t(l)$ units of capital, $H_t(l)$ units of labor, and aggregate technology A_t to produce $Y_t(l)$ units of the intermediate good l. Its production function is:

$$Y_t(l) = A_t K_t(l)^{1-\alpha} H_t(l)^{\alpha}. \tag{15}$$

The level of technology A_t follows a stationary AR(1) process given by:

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_t^A, \tag{16}$$

where $\rho_A \in (0,1)$ and $\varepsilon_t^A \sim N(0, \sigma_{\varepsilon^A})$.

If allowed to reoptimize its price in period t, the firm maximizes the discounted sum of expected future profits:

$$\max \mathbf{E}_t \sum_{i=0}^{\infty} d_{t+i} \left(\beta^i \frac{\lambda_{t+i}}{\lambda_t}\right)\left(\frac{D_{t+i}(l)}{P_{t+i}}\right), \tag{17}$$

where D_{t+i} represents nominal dividends in period $t+i$, $\left(\beta^i \frac{\lambda_{t+i}}{\lambda_t}\right)$ is the stochastic discount factor used by shareholders to value profits at date $t+i$, and d_{t+i} is the probability that the price set in time t will still be in force at time $t+i$. Under Calvo pricing, firms have a constant probability $(1-d) \in (0,1)$ of being able to reset their price optimally in any given period, so that

$$d_{t+i} = d^i, \quad 0 \le i \le \infty.$$

Nominal dividends $D_{t+i}(l)$, are given by:

$$D_{t+i}(l) = P_t^*(l)Y_{t+i}(l) - w_{t+i}P_{t+i}H_{t+i}(l) - q_{t+i}P_{t+i}K_{t+i}(l), \tag{18}$$

where $P_t^*(l)$ is the price set by the firm in period t, w_t is the real wage rate, and q_t is the real rental rate of capital. The first-order conditions of the firm's problem with respect to $K_t(l)$, $H_t(l)$ and $P_t^*(l)$ are given by:

$$q_t = (1-\alpha)\psi_t(l)\frac{Y_t(l)}{K_t(l)}, \tag{19}$$

$$w_t = \alpha\psi_t(l)\frac{Y_t(l)}{H_t(l)}, \tag{20}$$

$$P_t^*(l) = \left(\frac{\theta}{\theta-1}\right)\frac{\mathrm{E}_t\sum_{i=0}^{\infty}(d\beta)^i\frac{\lambda_{t+i}}{\lambda_t}\psi_{t+i}(l)Y_{t+i}(P_{t+i})^{\theta}}{\mathrm{E}_t\sum_{i=0}^{\infty}(d\beta)^i\frac{\lambda_{t+i}}{\lambda_t}Y_{t+i}(P_{t+i})^{\theta-1}}, \tag{21}$$

where $\psi_t(l)$ denotes the real marginal cost at date t associated with firm l's maximization problem. According to equations (19) and (20), the marginal products of labor and capital both exceed their respective marginal costs. Equation (21) is the firm's optimal price equation, derived from the equalization of marginal cost with marginal revenue in a dynamic context.

As is well known, this equation can be linearized under the assumption of zero steady-state inflation to obtain the standard New Keynesian Phillips curve, or under non-zero steady-state inflation to obtain an extended version of the New Keynesian Phillips curve.[8] However, linearizing the model can give misleading

[8]See Galí (2008) for a detailed derivation for the basic New Keynesian model and Bakhshi et al. (2007) for the case of positive trend inflation.

comparisons of economic welfare across monetary policy regimes, in addition to ignoring the fundamental nonlinearity of the zero lower bound constraint itself. We show (see Appendix B) that the representative firm's optimal pricing equation can be written in the form of two first-order forward-looking nonlinear difference equations, with a static relationship linking the two. Higher-order perturbation techniques can be used the model that includes these nonlinear equations.

2.3 Aggregation

Capital is perfectly mobile across firms. Therefore, all firms share the same capital to labor ratio and have identical real marginal costs. All firms that optimize their price in a given period will choose the same price, so we can also drop the (l) argument after P_t^*. Firms setting prices at different dates will in general have different relative prices.

Each intermediate firm that sets its relative price accepts to supply demand at that price. Integrating over the conditional demand functions for firms' output given in (13) gives the following aggregate resource constraint:

$$Y_t^s = \left(C_t + K_{t+1} + (1-\delta)K_t + \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \right) \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di. \quad (22)$$

We define

$$S_t \equiv \int_0^1 \left(\frac{P_t(l)}{P_t}\right)^{-\theta} dl, \quad (23)$$

and we have:

$$Y_t^s = A_t {K_t}^{(1-\alpha)} {H_t}^{\alpha}, \tag{24}$$

with Y_t^s defined as aggregate supply and where K_t and H_t are, respectively, the aggregate capital stock and aggregate hours worked.

The aggregate resource constraint takes into account the inefficiency in resource allocation induced by price dispersion across firms. Because individual intermediate goods enter symmetrically and with equal weight in the production function for the final good given by equation (11), efficient resource allocation would dictate producing the same amount of each intermediate good. Price dispersion causes the macroeconomic equilibrium to deviate from this optimum. It can be shown that S_t is bounded from below by one.[9] Under Calvo pricing, equation (23) simplifies to the following law of motion for S_t:

$$S_t = (1-d)\left(\frac{P_t^*}{P_t}\right)^{-\theta} + d\left(\frac{P_t}{P_{t-1}}\right)^{\theta} S_{t-1}. \tag{25}$$

The price level also follows a nonlinear first-order difference equation:

$$ {P_t}^{(1-\theta)} = (1-d){P_t^*}^{(1-\theta)} + d{P_{t-1}}^{(1-\theta)} \tag{26}$$

2.4 Monetary Policy

We model monetary policy using simple rules. The monetary authority sets the short-term nominal interest rate in accordance with a Taylor-type rule. Under IT,

[9] See Schmitt-Grohé and Uribe (2005).

the rule is

$$R_t^* = (1-\rho_R)R^* + \rho_R R_{t-1}^* + \rho_\pi \log\left(\frac{\pi_t}{\widetilde{\pi}}\right)$$
$$+ \rho_y \log\left(\frac{Y_t}{Y_t^*}\right) + \varepsilon_t^R, \tag{27}$$

where $0 \leq \rho_R \leq 1$, $0 < \rho_\pi$, and $0 < \rho_y$. Variables without time subscripts denote deterministic steady-state values. Y_t^* is the natural level of output (the level of output that would prevail in the absence of nominal price rigidities given the current aggregate capital stock),[10] and ε_t^R is a monetary policy shock with $\varepsilon_t^R \sim N\left(0, \sigma_{\varepsilon^R}^2\right)$. R^* is the steady-state desired nominal interest rate and is given by the Fisher equation

$$\widetilde{R} = r\widetilde{\pi} - wedge,$$

where r is the gross steady-state natural riskless real interest rate and $wedge$ is the (small) wedge between the desired interest rate and the actual interest rate that comes from our approximation of the kink in the central bank's reaction function (see (32) below). One long-run solution to this equation immediately implies that in the deterministic steady state the rate of inflation will be equal to $\widetilde{\pi}$.[11] Therefore, it is natural to interpret $\widetilde{\pi}$ as the target level of inflation as well as its deterministic steady state level.

[10]The central bank takes as given the loss in output due to the monopoly power of firms. Justiniano and Primiceri (2008) among others distinguish between the natural level of output and potential output in which the monopoly distortion is eliminated.

[11]See the subsection on the model's steady state for one other possible solution.

Under PT, the modified Taylor rule is given by:

$$R_t^* = (1-\rho_R^p)R^* + \rho_R^p R_{t-1}^* + \rho_p \log\left(\frac{P_t}{\widetilde{P}_t}\right)$$

$$+ \rho_y^p \log\left(\frac{Y_t}{Y_t^*}\right) + \varepsilon_t^p,, \tag{28}$$

with $0 \leq \rho_R^p \leq 1$, $0 < \rho_p$, and $0 < \rho_y^p$, and where $\varepsilon_t^p \sim N(0, \sigma_{\varepsilon^p}^2)$. We do not necessarily impose $\rho_R = \rho_R^p$, $\rho_\pi = \rho_p$, $\rho_y = \rho_y^p$ or $\sigma_{\varepsilon^R}^2 = \sigma_{\varepsilon^p}^2$. Here, $\widetilde{P}_t$ has the interpretation of a deterministic price-level path. It evolves according to

$$\widetilde{P}_t = P_0(\widetilde{\pi})^t, \tag{29}$$

so that once again $\widetilde{\pi}$ is the deterministic steady-state rate of inflation. Here, P_0 is an arbitrary initial value for the price level.

Under IT the price level is difference-stationary, while under PT the price level is stationary around a deterministic trend. For the purposes of numerical simulation, the deviation of the price level from the target path follows the simple law of motion given by:

$$\left(\frac{P_t}{\widetilde{P}_t}\right) = \frac{\pi_t}{\widetilde{\pi}}\left(\frac{P_{t-1}}{\widetilde{P}_{t-1}}\right), \tag{30}$$

so that the proportional deviation of the price level from its target path is an additional predetermined state variable in the model.

Given the central bank's desired nominal interest rate, the actual short-term nominal interest rate is given by

$$R_t = 1 + \max\left(0\ ,\ \left(R_t^* - 1\right)\right). \tag{31}$$

This is a kinked function. In order to use perturbation methods to solve the model, we approximate it by the following function:[12]

$$R_t = 1 + \left(\sqrt{\left(R_t^* - 1\right)^2 + \epsilon^2} + \left(R_t^* - 1\right)\right)/2, \tag{32}$$

where ϵ is a small positive constant. The maximum error of this function occurs at $(R_t^* - 1) = 0$ and is given by $\epsilon/2$, so that it can be made arbitrarily small. The function is continuously differentiable over the real line. Using perturbation methods introduces a further approximation error, which can be reduced by using higher and higher order approximations.

2.5 Equilibrium

Equilibrium in the model is characterized in the usual way. The first order conditions of households and firms are satisfied, the law of motion for aggregate capital is satisfied, and the goods market is in equilibrium, with aggregate output equal to consumption plus investment (inclusive of adjustment costs) and with the wedge between aggregate demand and the aggregate production function given in (24)

[12] See General Algebraic Modeling System (2007) for further details.

given by:

$$Y_t = Y_t^s / S_t.$$

Firms that are not allowed to reoptimize their price supply the quantity of their intermediate good that is demanded.

2.6 Steady State

Because firms do not index their prices to past inflation or trend inflation, the steady state of the model depends on trend inflation. A higher level of trend inflation means more price dispersion across firms in the steady state, which leads to a larger wedge S between aggregate supply and aggregate demand.

Benhabib, Schmitt-Grohé and Uribe (2001) show that the IT model must have two deterministic steady states. In one steady state, inflation is equal to its target rate. There is also a second steady state with the nominal interest rate equal to zero and inflation equal to the negative of the steady-state real rate of interest (so that the Fisher equation is satisfied).

Under PT, there is only one possible steady state. If inflation is different from the slope of the target price path, the price level diverges from the target path and the interest rate eventually responds by more than the divergence of inflation, even for small values of ρ_{pt}. The Taylor principle is automatically satisfied as long as ρ_{pt} is positive. The price level must converge in the long run to its target path.[13]

[13] See Ambler and Lam (2013) for further details.

3 Solution and Calibration

The model can be reduced to the set of nonlinear expectational difference equations summarized in Appendix B.

Parameter values used for the numerical simulations are given in Table 1.

- The paramaterization of the risk premium process follows Amano and Shukayev (2012).
- The weight on leisure in the utility function (γ) is a normalization chosen so that households spend one third of their time endowment working in the steady state.
- The rest of the parameter values are standard.

4 Results

We present preliminary results from simulations using Dynare (Adjemian et al. 2014). Dynare approximates around the steady state equilibrium with inflation equal to $\widetilde{\pi}$. Benhabib, Schmitt-Grohe and Uribe (2001) show that, because of the zero lower bound, all models with Taylor rules have a second steady state equilibrium in which the nominal interest rate is at its lower bound of zero and the inflation rate is equal to minus the steady state real interest rate. Monetary policy in the steady state with negative inflation is "passive" in that it does not satisfy the Taylor principle, according to which the nominal interest rate must respond more

Table 1: Model Calibration (Base Case)

Parameter	Meaning	Value
β	discount rate	0.995
α	labor share	0.667
d	probability of no price adjustment	0.750
χ	marginal disutility of labor	1.000
δ	capital depreciation	0.025
θ	elasticity of substitution	6.000
H	steady-state hours	0.333
ρ_A	technology shock persistence	0.950
σ_{ε^A}	technology shock variance	0.010
A	steady-state technology level	1.000
ρ_τ	risk premium shock persistence	0.840
$\sigma_{\varepsilon^\tau}$	risk premium shock variance	0.01^2
τ	steady-state risk premium	0.016
ρ_R	interest rate persistence	0.900
ρ_π	Taylor rule	0.200
ρ_y	Taylor rule	0.100
σ_{ε^R}	monetary policy shock variance	0.01^2
ρ_{Rp}	interest rate persistence	0.050
ρ_p	modified Taylor rule	0.050
ρ_{yp}	modified Taylor rule	0.100
σ_{ε^p}	monetary policy shock variance	0.01^2
ϵ	smoothing parameter	0.010

than one-to-one to changes in inflation. Dynare automatically checks that saddle-point stability is satisfied in the neighborhood of the high-inflation steady state. Dynare is of course unable to solve the model globally. We check our results to make sure that inflation does not stray too close to the negative-inflation steady state.

Tables 2 and 3 show results in which the lower bound constraint is not imposed, i.e. in which $\rho_0 = 0$. This of course means that the zero lower bound is violated with positive probability in each period.

In Table 2 we check, for different rates of trend inflation, the relative frequency of hitting the zero lower bound under IT versus PT, the average number of periods that the economy remains at the zero bound under the two regimes, and the maximum duration that the economy remains stuck at the lower bound. The rules of the policy functions are held constant across policy regimes, with the values given in Table 1. We consider rates of trend inflation equal to 2%, 1% and 0.5%. In each case, we generate one replication with a sample size of 1,000 observations.

Table 2: Results: Frequency at Lower Bound

Trend Inflation	2.0%	1.0%	0.5%
Inflation Targeting:			
Frequency	0.110	0.183	0.309
Maximum Duration	15	22	45
Price-Level Targeting:			
Frequency	0.000	0.000	0.027
Maximum Duration	0	0	3

Table 3 reports moment statistics for different variables. We measure unconditional welfare as the the average of period utility $\mathrm{E}(U)$. By construction, the deterministic steady states of the model are identical under IT and PT: this is to facilitate comparison of the model's properties across monetary policy regimes. We find the following results.

4.1 Discussion

- Reduced inflation leads to benefits even in the absence of stochastic shocks. Because of staggered pricing by firms, there is price dispersion across firms in the steady state, and this is an increasing function of trend inflation (see Ascari, 2004 and Amano, Ambler and Rebei 2007). Less price dispersion means brings the marginal products of intermediate goods in final production closer together, which is an increase in production efficiency.

- For all trend inflation rates, unconditional welfare (measured by the average value of period utility) is higher under PT than under IT.

- Under PT, unconditional welfare is slightly higher than the steady-state level of period utility. This can be attributed to the fact that the stochastic mean of consumption is higher under PT. The stochastic mean of hours worked is almost the same for different levels of trend inflation, and there is little or no change in the standard deviations of consumption and hours. The effect can therefore be referred to as a "level effect" (change in the stochastic mean) as opposed to a "stabilization effect" (coming from a change in the second moments of consumption and/or hours).

- The change in average hours is small across regimes. Under both IT and PT, hours are insignificantly different from their value in the deterministic steady state. Most of the effects on welfare come from the differing effects of IT and PT on the stochastic mean of consumption.

- Under IT, the economy is at or below the lower bound 11% of the time when trend inflation is equal to 2%. This increases to 18% of the time when trend inflation drops to 1%, and 31% of the time when trend inflation is equal to 0.5%.

- At low rates of trend inflation (both 1% and 0.5% under IT, it is common for the economy to spend extended periods of time at the lower bound. The maximum length of time for which the economy remains at the lower bound is 45 periods.[14]

- The results understate the average length of time an economy would spend at the lower bound. Because the reported results are based on perturbation methods, and because we set $\rho_0 = 0$, the algorithm allows the central bank's policy rate to become negative at the lower bound, which is a policy stance that is more expansionary than could be achieved if the constraint were explicitly taken into account.

- For this reason, the results also understate the advantages of PT in terms of its effects on economic welfare.

- The short-term nominal interest rate is less variable under PT than under IT, even though the coefficients in the modified Taylor rule are identical across regimes. The variability of output across the two regimes is very similar. The lower variability of the interest rate is explained by the lower

[14]Note that this is based on one draw of 1,000 observations.

variability of the deviation of the price level from its target path under PT compared to the variability of inflation.

4.2 To Do

- Measure unconditional welfare in terms of compensating variations.
- Compare results under second-order approximations (Dynare) and greater-than-second-order approximations (Dynare++).
- Optimize over the ρ coefficients to find optimal rules under IT and PT.
- Compare welfare under IT and PT with optimized coefficients in the two regimes.
- Optimize over $\widetilde{\pi}$ to find optimal trend inflation under IT and PT. item Solve for the optimal (Ramsey) interest rate rules under IT and PT. This would be another paper. We could go back to using barrier methods, which make more sense in an optimizing context.

5 Conclusion

With rational expectations, PT helps keep the short-term nominal interest rate above its zero lower bound. When inflation deviates below its target path, individuals rationally expect the central bank to move inflation above its target path to restore the price **level** to its target path. Therefore, any given level of the short-term nominal interest rate implies a lower real interest rate under PT than IT and is more expansionary. The reduction in the frequency with which the economy is at the zero lower bound directly affects the stochastic mean of output, consumption, and hours worked. The impact of policy on the stochastic means of these variables is crucial when evaluating the impact of the monetary policy regime on economic welfare.

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Appendices

A Stationary Variables (Not for Publication)

Nominal variables are difference stationary under IT in the model and trend stationary under PT. The model can be rewritten in terms of stationary variables via suitable normalizations. For example, the exact price level given in (14) can be transformed by noting that it is just a weighted average of the price set by firms that can reoptimize their prices in the current period and the price level of last period. We have:

$$P_t = \left[\int_0^1 P_t(l)^{1-\theta} dl \right]^{\frac{1}{1-\theta}}$$

$$\Rightarrow {P_t}^{(1-\theta)} = d{P_t^*}^{(1-\theta)} + (1-d){P_{t-1}}^{(1-\theta)}$$

$$\Rightarrow 1 = d{p_t^*}^{(1-\theta)} + (1-d){\pi_t}^{(\theta-1)}$$

where we define $p_t^* \equiv \frac{P_t^*}{P_t}$.

B Pricing Decision of Firms (Not for Publication)

The optimal pricing equation given by (21) involves two infinite sums, so it is not amenable to numerical simulation. It is well known that a linear approximation to this equation leads to the New Keynesian Phillips curve, but we need to simulate our model using higher-order approximations in order to make accurate welfare comparisons that take into account the effect of shocks on the stochastic means of endogenous variables. It is possible to express each of the two infinite sums in equation (21) recursively, in terms of two artificial variables. We drop the (l) argument and rewrite equation (21) as

$$\frac{P_t^*}{P_t} \equiv p_t^* = \left(\frac{\theta}{\theta-1}\right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \psi_{t+i} Y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} Y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}.$$

We now work with the numerator of this transformed equation. Define

$$x_t \equiv \mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \psi_{t+i} Y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}$$

$$\Rightarrow \lambda_t {P_t}^{\theta} x_t = \mathrm{E}_t \sum_{i=0}^{\infty} (d\beta)^i \lambda_{t+i} \psi_{t+i} Y_{t+i} {P_{t+i}}^{\theta}.$$

Leading this equation by one period, multiplying both sides by $d\beta$, and taking expectations conditional on information available at time t gives

$$d\beta \mathrm{E}_t \left(\lambda_{t+1} {P_{t+1}}^{\theta} x_{t+1}\right) = \mathrm{E}_t \sum_{i=1}^{\infty} (d\beta)^i \lambda_{t+i} \psi_{t+i} Y_{t+i} {P_{t+i}}^{\theta}.$$

Subtracting this equation from the preceding one and simplifying gives

$$x_t = Y_t \psi_t + d\beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} {\pi_{t+1}}^{\theta} x_{t+1} \right\}.$$

Manipulating the denominator in similar fashion gives

$$z_t = Y_t + d\beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} {\pi_{t+1}}^{(\theta-1)} z_{t+1} \right\},$$

with

$$\frac{\theta}{\theta-1}x_t = p_t^* z_t.$$

C Equation System (Not for Publication)

This appendix gives the complete equation systems used to simulate the model under inflation targeting and price level targeting.

Equations common to the two versions of the model are:

$$\lambda_t = \frac{1}{C_t}; \tag{A-1}$$

$$\lambda_t = \beta \mathrm{E}_t \left(\lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right); \tag{A-2}$$

$$\lambda_t w_t = \gamma {H_t}^{\chi}; \tag{A-3}$$

$$\begin{aligned} \lambda_t \left[1 + \varphi \left(\frac{I_t}{K_t} - \delta \right) \right] = \beta \mathrm{E}_t \Bigg\{ \lambda_{t+1} \Bigg[& 1 + (q_{t+1} - \tau_t) - \delta \\ & + \varphi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) + \frac{\varphi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \Bigg] \Bigg\}; \end{aligned} \tag{A-4}$$

$$q_t = \psi_t (1-\alpha) \frac{Y_t^s}{K_t}; \tag{A-5}$$

$$w_t = \psi_t \alpha \frac{Y_t^s}{H_t}; \tag{A-6}$$

$$x_t = \psi_t Y_t + d\beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} {\pi_{t+1}}^{\theta} x_{t+1} \right); \tag{A-7}$$

$$z_t = Y_t + d\beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} {\pi_{t+1}}^{(\theta-1)} z_{t+1} \right); \tag{A-8}$$

$$p_t^* z_t = \frac{\theta}{\theta - 1} x_t; \tag{A-9}$$

$$Y_t = C_t + I_t + \frac{\varphi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t; \tag{A-10}$$

$$K_{t+1} = (1-\delta) K_t + I_t; \tag{A-11}$$

$$Y_t^s = A_t {K_t}^{(1-\alpha)} {H_t}^{\alpha}; \tag{A-12}$$

$$Y_t^s = Y_t S_t; \tag{A-13}$$

$$1 = (1-d) p_t^{*(1-\theta)} + d{\pi_t}^{(\theta-1)}; \tag{A-14}$$

$$S_t = d{\pi_t}^{\theta} S_{t-1} + (1-d) p_t^{*-\theta}; \tag{A-15}$$

$$\log(A_t) = (1-\rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{A,t}; \tag{A-16}$$

$$\tau_t = (1-\rho_\tau)\, \tau + \rho_\tau \tau_{t-1} + \varepsilon_t^{\tau}. \tag{A-17}$$

The endogenous variables are λ_t, C_t, R_t, π_t, w_t, H_t, I_t, K_t, Y_t, Y_t^s, S_t, q_t, τ_t, ψ_t, p_t^*, A_t, x_t, and z_t. To close the system, we need one more equation. Under IT, we add the Taylor rule equation (27), which introduces the extra endogenous variable R_t^*, so we add equation (32) to close the system. Under PT, we add the modified Taylor rule (28). This introduces an extra stationary variable which measures the deviation of the price level from the target path, given in equation (30). We have:

$$\widetilde{p}_t = \frac{\pi_t}{\widetilde{\pi}} \widetilde{p}_{t-1}, \tag{A-18}$$

where $\widetilde{p}_t \equiv P_t/\widetilde{P}_{t-1}$. This equation viewed in isolation appears to convey a unit root to $\widetilde{p}_t$. The equation can be rewritten as:

$$\log\left(\widetilde{p}_t\right) = -\log\left(\widetilde{\pi}\right) + \log\left(\widetilde{p}_{t-1}\right) + \log\left(\pi_t\right),$$

which looks very much like a random walk with drift. A shock to inflation appears to have a permanent impact on the price level, but the modified Taylor rule ensures that the central bank drives down inflation in response to a positive divergence between the price level and its target path. This endogenous reaction of π_t to movements in $\widetilde{p}_t$ ensures that the latter is stationary.

D Steady State (Not for Publication)

The following procedure can be used to find the economy's steady state. If we impose $H = 1/3$ in steady state equilibrium, the system can be solved recursively, and equation (A-3) can be used to back out the value of γ compatible with this level of hours in the long run. The Taylor rule and the modified Taylor rule imply that, in the long run

$$R = \widetilde{R} = r\widetilde{\pi}.$$

The Euler equation for bonds then implies that

$$\frac{1}{\beta} = \frac{R}{\widetilde{\pi}}$$

so that $1/\beta = r$ in the steady state. Equation (A-4) then gives

$$\frac{1}{\beta} = 1 + q - \tau - \delta$$

so that $q = (r-1) + \tau + \delta$. In the long run, the real rental rate equals the net risk-free real interest rate plus a compensation for depreciation and for the long-term value of the risk premium.

Equations (A-17) and (A-16) immediately give the long run levels of technology and the risk premium.

In the steady state, the law of motion for the price level (A-14) gives

$$p^{*(1-\theta)} = \frac{1 - d\widetilde{\pi}^{(\theta-1)}}{(1-d)}$$

$$\Rightarrow p^{*} = \left(\frac{(1-d)}{1 - d\widetilde{\pi}^{(\theta-1)}}\right)^{1/(\theta-1)}.$$

When steady-state inflation is equal to zero ($\widetilde{\pi} = 1$), this gives $p^{*} = 1$ so firms that can reoptimize their price set it equal to the price level. As steady-state inflation increases, firms that can reoptimize set a price higher than the price level since they know that their relative price will be eroded over time.

Given this solution for p^{*}, equation (A-15) gives a solution for the long-run level of the wedge between production and aggregate demand:

$$S = \frac{(1-d)p^{*-\theta}}{1 - d\widetilde{\pi}^{\theta}}.$$

Once again, when steady-state inflation is zero, firms' prices are all equal in the steady state so the wedge is equal to one.

Equation (A-9) can be solved for x:

$$x = \frac{\theta - 1}{\theta} p^* z$$

Then, substituting SY for Y^s, equations (A-5), (A-7), (A-8) and (A-12) can be reduced to the following four equations in transformed variables:

$$q = (1 - \alpha) S (\psi Y/K);$$

$$\frac{\theta - 1}{\theta} p^* z = (\psi Y/K)(K/H) H + d\beta \frac{\theta - 1}{\theta} \widetilde{\pi}^{\theta} p^* z;$$

$$z = (\psi Y/K)(K/H) H/\psi + d\beta \widetilde{\pi}^{(\theta - 1)} z;$$

$$(\psi Y/K)(K/H) = \psi A (K/H)^{-\alpha}.$$

The first equation can be used to solve for $(\psi Y/K)$. The last equation can then be solved for ψ and used to substitute out ψ from the third equaton. This equation can be solved for z and substituted into the second equation, which is nonlinear in (K/H) but which has a unique analytical solution. K/H gives the steady-state capital stock for a given level of H. It is then possible to solve for the steady-state level of aggregate demand Y. Equation (A-13) then gives Y_t^s, and C follows from the aggregate budget constraint (A-10). Given the level of consumption, equation (A-1) gives the steady-state level of λ. Finally, equation (A-3) gives the steady-state value of γ compatible with $H = 1/3$.

Table 3: Results: Steady States and Stochastic Means

Trend Inflation	2.0%	1.0%	0.5%
Steady State:			
U	-0.6122	-0.6110	-0.6107
C	0.7783	0.7791	0.7793
H	0.3333	0.3333	0.3333
R	1.0100	1.0075	1.0063
Inflation Targeting:			
E(U)	-0.6264	-0.6163	-0.6101
(σ_U)	(0.0356)	(0.0381)	(0.0341)
E(C)	0.7670	0.7755	0.7802
(σ_C)	(0.0244)	(0.0260)	(0.0232)
E(H)	0.3328	0.3333	0.3333
(σ_H)	(0.0091)	(0.0083)	(0.0084)
E(R)	1.0119	1.0077	1.0054
(σ_R)	(0.0088)	(0.0087)	(0.0086)
Price-Level Targeting:			
E(U)	-0.6082	-0.6074	-0.6009
(σ_U)	(0.0322)	(0.0348)	(0.0246)
E(C)	0.7821	0.7823	0.7880
(σ_C)	(0.0224)	(0.0242)	(0.0185)
E(H)	0.3334	0.3332	0.3336
(σ_H)	(0.0089)	(0.0088)	(0.0085)
E(R)	1.0100	1.0077	1.0061
(σ_R)	(0.0028)	(0.0030)	(0.0030)